EFFICIENT TRAINING FOR FINGERPRINT BASED POSITIONING USING MATRIX COMPLETION

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ABSTRACT

Fingerprint localization methods are extensively used in many location-aware applications in pervasive computing. In this paper, we propose a new framework in order to reduce the exhaustive calibration procedure during the training phase in fingerprint-based systems. In particular, we minimize the number of Received Signal Strength (RSS) fingerprints by sensing a subset of the available channels in a WLAN. We exploit the spatial correlation structure of the RSS fingerprints to reconstruct the signature map. The problem is formulated according to the recently introduced Matrix Completion (MC) framework, which provides a new paradigm for reconstructing low rank data matrices from a small number of randomly sampled entries. Analytical studies and simulations are provided to show the performance of the proposed technique in terms of reconstruction and location error.

Index Terms— Fingerprint-based positioning, received signal strength, matrix completion

1. INTRODUCTION

Wireless communication systems have entered the realms of consumer applications to provide various types of services. The new paradigm in mobile services lies in providing a location-based user experience. Health care monitoring, personal tracking and guiding, context dependent information services, and self organising sensor networks are some of the numerous possible application areas.

Localization or *location-sensing* refers to determining the physical position of a mobile user and can be achieved through the efficient gathering of network data [1]. A typical localization scenario consists of a set of access points (APs) placed at known positions and a mobile station (MS) carried by a person that needs to be located.

IEEE 802.11 is currently the dominant local wireless networking standard as APs are installed in a large number of buildings. This extensive deployment makes 802.11 appealing to be used for localization as well since the infrastructure is already available, although currently unused for positioning purposes. Received Signal Strength (RSS)-based methods, as opposed to traditional time of arrival (ToA) or angle of arrival (AoA) positioning techniques, exploit the existing wireless infrastructure and avoid the additional cost of deploying localization-specific hardware [2].

The majority of signal strength-based systems can be classified into two categories, namely the *map*- or *fingerprint*-based and the *distance prediction*-based. Distance prediction-based systems estimate the position of the MS by computing its distances from at least three reference points (*e.g.*, APs, anchor nodes) by applying a known RF propagation model [3]. The main challenge arising in these systems is the difficulty to formulate a reliable radio propagation model due to multipath fading, shadowing, the low probability of line-of-sight (LOS) path and specific parameters such as floor layout and moving objects.

On the other hand, fingerprint based systems create signature maps to represent the physical space by capturing the variations of the dynamic nature of indoor radio propagation [1], [4]. RSS-based fingerprinting methods share a common characteristic which is their implementation in two distinct phases, the *offline* or *training* phase and the *online* or *runtime* phase. During the training phase, location fingerprints are collected at different places in the position estimation area to create the so-called *signature* or *radio* map. In the runtime phase, the observed runtime signal strength measurements are compared with the training fingerprints to perform localization. The estimation methods used for predicting locations from the radio map can be classified into three categories, the deterministic [5], the probabilistic [6] and the recently introduced approaches based on spatial sparsity [7], [8]. Recently, we proposed a compressed sensing (CS) technique that outperforms traditional localization techniques in terms of location accuracy [9]. Although, fingerprint-based systems achieve the best performance, the time required for the calibration phase remains their major disadvantage [10].

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Thus, in this paper, we propose a novel training scheme based on the recently developed framework of Matrix Completion (MC) [11]. Particularly, we exploit the inherent correlation structure of the signal strength fingerprints to reduce the training phase by reconstructing the signature map from fewer measurements.

The organization of the paper is as follows: Section 2 presents a short overview of the related work. Section 3 discusses the motivation of the proposed framework. In Section 4, we present the MC background, while in Section 5 we develop the proposed framework. Experimental results are provided in Section 6, while we conclude in Section 7.

2. RELATED WORK

Consider a typical WLAN positioning scenario where a set of APs are connected and a user carries a mobile device equipped with a wireless network interface card (NIC). The wireless device that listens to a channel receives the beacons sent by APs (at that channel) periodically and records their Received Signal Strength (RSS) values. The radio frequency channels of IEEE 802.11b/g are in the 2.4 GHz which is divided into 13 channels spaced 5MHz apart. The number of non-overlapping channels for 802.11b/g is four [12]. Wireless devices that run fingerprint-based positioning systems, are required to scan all the available channels to generate statistical fingerprints for a position using these measurements. The result of this phase is the *signature map*.

The major problem of fingerprint-based systems is the exhaustive survey to create the signature map, a task that requires substantial cost and labour. It is obvious that representative signatures of the physical space result in higher accuracy during the runtime phase. Thus, a recalibration is mandatory every time that environmental changes, which affect the power or the number of APs, occur to the position estimation area. Current literature shows a growing interest for training techniques that attempt to shorten the calibration phase of fingerprint-based systems.

Authors in [13] propose a training procedure where the whole area is divided into rooms, thereby limiting the location estimates to room-level granularity. This approach leads to a reduction in training time at the expense of less precise location estimations. Some localization protocols adopt traditional interpolation methods to complete the training map using fingerprints taken at a small number of training points [5]. The interpolation is based either on intuitive guidelines or on linear regression techniques.

Recently, a compressed sensing-based training technique was proposed in [8]. Considering that the RSS readings per AP have a sparse nature in the frequency domain, the radio map is reconstructed based on a small number of measurements. The reconstruction procedure is performed independently for each AP and consequently the correlation structure among various APs is not taken into consideration.

3. MOTIVATION

To address the above issues, we propose to perform random sampling of the environment, where the mobile device collects RSS measurements from a randomly selected channel at each position of the physical space. Obviously, sub-sampling reduces training time and consequently energy consumption at the MS. While random sensing has numerous benefits, recovering such data is only possible if there exists a correlation between the measurements.

In an indoor environment, signal strength measurements depend on the properties of the APs. They are also affected by path loss and shadowing effects which represent the signal degradation due to the distance travelled and the obstacles, respectively. Thus, the training RSS measurements are spatially correlated in the sense that training locations in proximity should have similar feature vectors.

In this work, we exploit the spatial correlations of RSS measurements collected from various APs when we move from one position to another. Formally, we consider a matrix, the signature map, where each element corresponds to the average RSS measurements from AP i at location j. Based on the assumption that measurements are correlated, the degrees of freedom of this matrix are much lower than its dimensions. The limited number of degrees of freedom results in a matrix that exhibits a low rank property. To demonstrate the low rank nature of the signature map, Fig. 1 illustrates an example of simulated RSS fingerprints from 20 APs at 64 different positions. In this example, we adopt the channel model for signal propagation presented in [14].

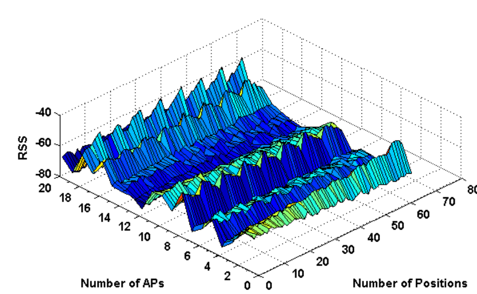

Fig. 1. Recorded RSS samples from 20 APs at 64 different positions.

Although the underlying trend in the measurements is evident, indoor radio propagation is affected by dense multipath environment and propagation effects, *i.e.*, reflection, diffraction, and scattering. This result can be seen in Fig. 2 where the corresponding normalized singular values of the signature map are shown. The gathered training map is characterized as low rank, while the transient phenomena result in additional singular values that exhibit much lower energy.

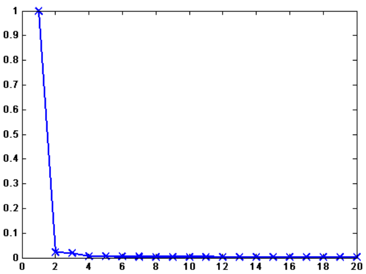

Fig. 2. Singular values of the signature map.

4. LOW RANK MATRIX COMPLETION

Matrix Completion (MC) builds on the observation that a matrix which is low rank or approximately low rank can be recovered using just a subset of randomly observed data [11].

Denote the matrix $\boldsymbol{X} \in \mathbb{R}^{J \times D}$ that we would like to recover as precisely as possible. Clearly, the recovery of the $J \times D$ entries of the matrix $\boldsymbol{X}$ is impossible from a number of measurements m, where $m \ll J \times D$. The MC framework shows that such a recovery is possible, in the case where the *rank* of matrix $\boldsymbol{X}$ is small enough compared to its dimensions. Specifically, the recovery of the unknown matrix is feasible from $m \geq cD^{6/5} r \log(D)$ random measurements, where $D > J$ and $rank(\boldsymbol{X}) = r$.

When the above conditions hold, the original matrix $\boldsymbol{X}$ can be recovered as the solution of following optimization problem:

$$\min\{\|\boldsymbol{X}\|_\star \quad : \quad \mathcal{A}(\boldsymbol{X}) = \mathcal{A}(\boldsymbol{M})\}, \qquad (1)$$

where the nuclear norm is defined as the sum of the singular values of $\boldsymbol{X}$, *i.e.*$\|\boldsymbol{X}\|_\star = \sum_{k=1}^{\min\{J,D\}} \sigma_k(\boldsymbol{X})$ and $\sigma_k(\boldsymbol{X})$ is the kth largest singular value. $\mathcal{A}$ is a linear map from $\mathbb{R}^{J \times D} \to \mathbb{R}^m$ and $\boldsymbol{M}$ defines the sub-sampled matrix of measurements $X_{i,j}$. Particularly, the linear map $\mathcal{A}$ must satisfy a modified Restricted Isometry Property (RIP), *i.e.*, uniform random sampling in both rows and columns.

The problem in (4) can be treated as a general convex optimization problem and solved by any off-the-shelf interior point solver (*e.g.*, CVX [15]), after being reformulated as a semidefinite program. Recently, efficient algorithms have been proposed for solving the low rank matrix completion problem for massive data sets. Singular Value Thresholding (SVT), introduced in [16], was one of the first MC algorithms developed. It is an iterative algorithm where in each step, first a singular value decomposition algorithm is applied and followed by a projection onto the known elements.

5. PROPOSED FRAMEWORK

In this work, we propose a new calibration procedure that can be adopted during the training phase of the fingerprint-based systems. Unlike other calibration techniques, this one minimizes the number of collected data by exploiting the spatial correlations of the RSS fingerprints among various APs.

To perform localization, a grid of cells with fixed structure is used to represent the physical space, where each cell of the grid corresponds to a physical position of the spatial space. During the training phase a mobile user moves to various sample grids, the so-called Reference Points (RPs), to collect RSS signatures from J APs to built up the *signature* map. Specifically, the signature map $\boldsymbol{X}$ is represented as:

$$\mathbf{X} = \begin{pmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,D} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,D} \\ \vdots & \vdots & \ddots & \vdots \\ P_{J,1} & P_{J,2} & \cdots & P_{J,D} \end{pmatrix}_{J \times D}, \qquad (2)$$

where $P_{j,i}$ corresponds to the mean value of RSS measurements received from the jth AP at location i and D is the number of RPs.

The MC formulation is able to recover a matrix by making no assumption about the process that generates the matrix, except that it is low rank. In the proposed method the mobile device, instead of sensing all P available channels, randomly selects k channels with equal probability. Once the selection is performed, the beacons transmitted from J' APs ($J' < J$) that broadcast at these channel are recorded. It is obvious that sub-sampling over channels at each cell of the grid will result in the overall reduction of calibration time.

Sampling $P_{j,i}$ RSS measurements result in an incomplete signature map. Particularly, the MS receives a subset $\Omega \subseteq [J] \times [D]$ of $\boldsymbol{X}$'s entries, where $|\Omega| = \frac{k}{P}(D \times J)$. We define the sampling map $\mathcal{A}$ by

$$[\mathcal{A}_\Omega(\boldsymbol{M})]_{j,i} = \begin{cases} P_{j,i}, & (j,i) \in \Omega \\ 0, & \text{otherwise.} \end{cases} \qquad (3)$$

Effective location sensing requires the recovery of the signature map (2) that will be used during the runtime phase. Thus, observing that the signature map satisfies the property required by the MC theory, we recover the unobserved data of $\boldsymbol{X}$ by solving the following minimization problem,

$$\min\{\|\hat{\boldsymbol{X}}\|_\star \quad : \quad \|\mathcal{A}_\Omega(\hat{\boldsymbol{X}}) - \mathcal{A}_\Omega(\boldsymbol{M})\|_F^2 < \epsilon\}, \qquad (4)$$

where $\hat{\boldsymbol{X}}$ is the recovered signature map and ϵ is the noise level and $\|\cdot\|_F$ denotes the *Frobenius* norm.

6. EXPERIMENTAL RESULTS

The effectiveness and properties of the proposed training technique based on MC formulation are studied and analysed through simulations. We consider two aspects of the problem, reconstruction quality of the recovered matrix with respect to the original one and final localization error. To explore the robustness of the proposed scheme, we investigated with both measurement noise and channel noise. The effectiveness of the proposed MC training technique was compared to the traditional interpolation technique [5] that served as a baseline. Regarding the reconstruction quality, the error was measured, with respect to the number of the sensed channels (13 for IEEE 802.11 b/g), as $\|\hat{\boldsymbol{X}} - \boldsymbol{X}\|_F / \|\boldsymbol{X}\|_F$. To investigate the application of the proposed training technique to location estimation, the K-nearest Neighbour localization (KNN, K = 3) technique [5] was employed. Location error refers to the euclidean distance between the real position of the user and the estimated one. To generate the simulation data, we adopt the following channel model for signal propagation [14]. The received power at the ith position from the jth AP is given by

$$P_r = P_t - \overline{PL}(d_0) - 10n \log \frac{d_{ij}}{d_0} - s_i, \qquad (5)$$

where $\overline{PL}(d_0)$ is the path loss at the reference distance $d_0 = 1m$, n denotes the path loss exponent, d_{ij} is the euclidean distance between the AP j and the ith RP and s_i describes the shadowing effect. The $\overline{PL}(d_0) = 37.5dBm$ and $n = 3$ for non-line-of sight propagation (NLOS) [4]. The shadowing variable is a zero-mean Gaussian distributed random variable (in dB) with standard deviation $\sigma(dB)$. The P_t is the transmit power of APs which is fix at 15dBm for IEEE 802.11b based WLANs. We consider an $144m^2$ area of $1m \times 1m$ grid-spacing. Each data point in the results is averaged over 20 independent trials.

We first introduce and discuss the scenario where no shadowing effect is present. Fig.3-4 show the recovery of the signature map based on (4) and the corresponding location error with respect to the number of sensed channels. We observe that the performance of the proposed technique increases, both in reconstruction and localization accuracy, as the number of sensed channel increases. Furthermore, the proposed MC-based approach is able to achieve lower reconstruction and location error compared to the interpolation technique, especially for small number of sensed channels. The proposed technique results in lower reconstruction error while the localization technique masks the results in the location error. More specifically, for the KNN localization technique, the MC reconstruction technique for two channels (*i.e.* 15% of measurements) leads to improvements in terms of location error in the order of 32% (1m).

Fig. 5-6 illustrate the performance of the proposed technique for different number of APs, when the MS receives RSS measurements only from two channels. As it can be seen, the proposed MC approach presents a clear superiority against the interpolation method. We observe that as the number of APs increases, the performance of the MC technique decreases, in contrast to the interpolation. This effect occurs as the number of APs increases, more correlated RSS measurements are produced, thus the signature map presents lower rank.

Finally, Fig 7-8 demonstrate the reconstruction and the location error for different shadowing effects. In this experi-

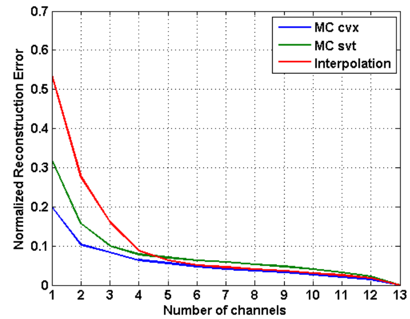

Fig. 3. Reconstruction error of signature map for 30 APs, with no shadowing, for the two different MC reconstruction algorithms vs. the interpolation method.

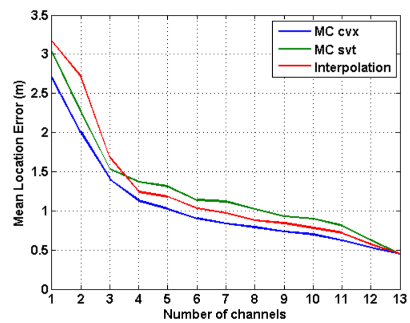

Fig. 4. Location error, during runtime phase, for 30 APs, with no shadowing, for the two different MC reconstruction algorithms vs. the interpolation method.

ment we consider 30 APs. We observe that, for complicated scenarios the proposed technique results in lower accuracy both in reconstruction and location error. Experimental results also indicate that linear interpolation technique offers less robust results, as it can be seen from the variance, compared to the proposed method.

7. CONCLUSIONS

In this paper, we present a novel training procedure for RSS fingerprint-based localization systems. The proposed technique, based on channel sub-sampling, exploits the correlation structure among the RSS fingerprints. We reduce the acquisition and communication requirements while the time required for calibration procedure is minimized. Observing that the signature map is approximately low rank, the Matrix Completion framework is used to recover the original map just from a subset of sensed measurements. The experimental results indicate that MC reconstruction method can achieve superior performance when compared to traditional interpolation technique, in terms of reconstruction error and localization accuracy. Future work will investigate the impact of the proposed technique on the localization accuracy for different localization protocols and multi-modal localization approaches.

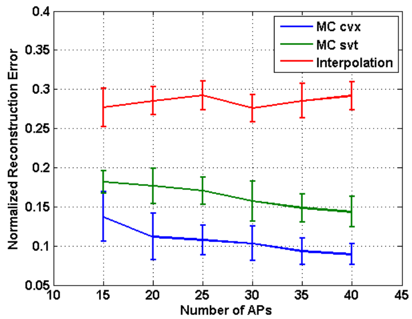

Fig. 5. Reconstruction error for the two different MC reconstruction algorithms vs. the interpolation method. No shadowing effect is considered.

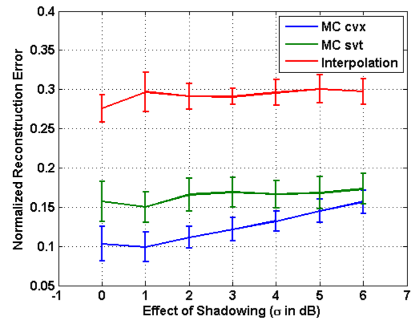

Fig. 7. Reconstruction error for the two different MC reconstruction algorithms vs. the interpolation method for different shadowing effects. We consider a set of 30 APs.

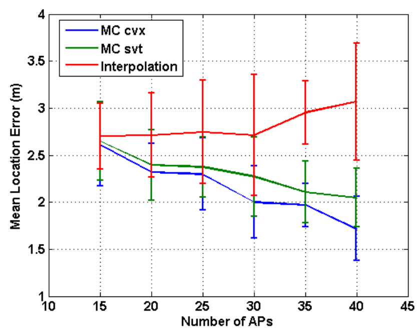

Fig. 6. Location error for the two different MC reconstruction algorithms vs. the interpolation method. No shadowing effect is considered.

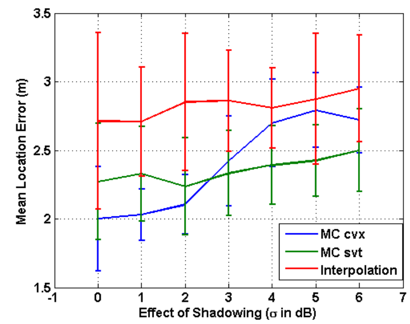

Fig. 8. Location error for the two different MC reconstruction algorithms vs. the interpolation method for different shadowing effects. We consider a set of 30 APs.

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