A New Criterion for Blind Deconvolution of Colored Input Signals

Panagiotis Tsakalides and Chrysostomos L. Nikias

Signal and Image Processing Institute
Department of Electrical Engineering
University of Southern California
Los Angeles, CA 90089

Abstract

In this paper, a new criterion with memory nonlinearity is introduced for blind deconvolution problems when the input signals are colored. The basic idea is to make use of the autocorrelation of the input sequence as the only statistical knowledge about the data. An adaptive weight algorithm is presented and tested with simulation examples of signals of known autocorrelation function. It is shown that the optimum memory size is directly related to the significant values of the autocorrelation function, and that the new algorithm converges faster than the Godard algorithm.

1 Introduction

Blind deconvolution is a signal processing technique that restores the input sequence of an unknown linear time-invariant system from its output only. When the system is non-minimum phase, blind deconvolution uses non-linear transformations or higher-order statistics of the output to extract the desired response. Traditionally the name ***blind deconvolution*** was used in the context of seismology and speech processing applications while communication engineers used the term *blind equalization*. Although blind deconvolution and blind equalization are equivalent in the sense that they both try to find the inverse of a linear, time-invariant and possibly non-minimum phase system from its output only, there are some important differences in the nature of the signals they deal with. First, in communication applications the transmitted data are discrete in time and can take a finite number of amplitude levels. On the other hand, in blind deconvolution problems the input signals are continuous waveforms. Secondly, in blind equalization the data can be assumed independent from each other and of known probability distribution. In blind deconvolution though, the underlying distribution of the data is not known and the data are statistically dependent in general, i.e., input signals are colored.

In the past, several memoryless methods ([1]-[5]), based only on statistical information about the magnitude of the input sequence, have been developed for blind equalization problems. All the above methods perform only memoryless nonlinear processing of the equalizer output in order to get the desired response. In [7], Chen and Nikias introduce a new criterion that employs the fact of statistical independence of the input data sequence. In this paper we address the blind deconvolution problem for continuous waveform signals for which we assume a priori knowledge of their autocorrelation function. We extend the CRIMNO criterion introduced in [7], by using the autocorrelation function of the transmitted data.

This paper is organized as follows: In section 2, we give the problem formulation and we introduce the criterion for blind deconvolution of colored signals. In section 3, the corresponding adaptive weight algorithm is derived. In section 4, simulation results are presented, followed by a conclusion in section 5.

2 Problem Formulation

Consider the linear, time-invariant and possibly non-minimum phase system illustrated in Fig. 1, with unknown impulse response $\{h_n\}$. The input data sequence $\{a_n\}$ is real, wide-sense stationary and has a known autocorrelation function $R(i)$. Then, from Fig. 1 we have that

$$y_n = a_n * h_n = \sum_i a_i h_{n-i} \qquad (1)$$

and

$$z_n = a_n * s_n = \sum_j a_j s_{n-j} \qquad (2)$$

where

$$s_n = h_n * c_n = \sum_k h_k c_{n-k}. \tag{3}$$

Our objective is to adjust the coefficients $\{c_n\}$ so that the output sequence $\{z_n\}$ is identical to the input sequence $\{a_n\}$ except perhaps for a constant delay, i.e., such that

$$h_n * c_n = \delta_{n-d} \tag{4}$$

where δ_n is the Kronecker delta function and d is a constant delay.

In the practical sense, we want to adjust the coefficients $\{c_n\}$ such that the output sequence $\{z_n\}$ is as close to the original data sequence as possible. Thus, if we construct a cost function $J(\mathcal{F}(\{a_n\}), \{z_n\})$, where $\mathcal{F}(\{a_n\})$ only depends on the partial knowledge we have about the original data, and it reaches a minimum if and only if the original data are restored by the inverse filter $\{c_n\}$, the problem is solved by seeking the minima of this function. In general, it is very difficult to find a convex cost function $J(\cdot)$. Known algorithms, such as the ones described in [1] and [2] make use of nonconvex functions.

2.1 Development of the New Criterion

Our criterion employs the fact that at perfect deconvolution, the statistics of the output sequence $\{z_n\}$ must be the same to the statistics of the input sequence $\{a_n\}$. In this case, the two sequences $\{a_n\}$ and $\{z_n\}$ will have the same autocorrelation function $R(i)$. Therefore,

$$E\{z_n z_{n-i}\} = E\{a_{n-d} a_{n-i-d}\} = R(i). \tag{5}$$

Using the above observation, we expand the CRIMNO cost function introduced in [7], to take the form:

$$\begin{aligned} M^{(p)} &= w_0 E(|z_n|^p - R_p)^2 + \\ &\quad w_1[E(z_n z_{n-1}) - R(1)]^2 + \cdots + \\ &\quad w_M[E(z_n z_{n-M}) - R(M)]^2 \end{aligned} \tag{6}$$

where $R_p = E(a_n)^{2p}/E(a_n)^p$, $\{w_n\}$ are the weights chosen adaptively, and M is the memory of the criterion. The parameter p is usually taken to be 2. We are going to refer to this new cost function as the *Colored* CRIMNO cost function or CCRIMNO cost function.

The CCRIMNO cost function can be seen as a generalization of the Godard cost function [1], because when $w_0 = 1$ and $w_i = 0$ for $i = 1, \ldots, M$, we obtain

$$D^p = E(|z_n|^p - R_p)^2 \tag{7}$$

From Eq. (6) we see that the CCRIMNO cost function depends on the current filter output, as well as on the M previous output samples.

3 CCRIMNO Blind Deconvolution Algorithm

Having introduced the cost function $M^{(p)}$ in Eq. (6), the problem of blind deconvolution reduces to adjusting the coefficients $\{c_n\}$ to seek for the global minimum of this function. Each iteration of the algorithm consists of the following steps:

$$\mathbf{C}_{n+1} = \mathbf{C}_n - \alpha \frac{\partial M^{(p)}}{\partial \mathbf{C}_n} \tag{8}$$

where $\mathbf{C}_n = [c_0^{(n)}, \ldots, c_{N-1}^{(n)}]^T$ is the vector of the deconvolution filter coefficients, and α is the step-size.

Using Eq. (6) and (8) and taking the derivative of each term with respect to $\mathbf{C}_n$, we obtain

$$\frac{\partial E(|z_n|^p - R_p)^2}{\partial \mathbf{C}_n} = 2pE[\mathbf{Y}_n z_n |z_n|^{p-2}(|z_n|^p - R_p)] \tag{9}$$

$$\frac{\partial [E(z_n z_{n-i} - R(i))]^2}{\partial \mathbf{C}_n} = 2E(\mathbf{Y}_n z_{n-i})E(z_n z_{n-i} - R(i)); \quad i = 1, \ldots, M \tag{10}$$

where we use the fact that $z_n = \mathbf{Y}_n^T \mathbf{C}_n = \mathbf{C}_n^T \mathbf{Y}_n$. $\mathbf{Y}_n = [y_n, \ldots, y_{n-N+1}]^T$ is the vector of the input signal. Therefore,

$$\begin{aligned} \frac{\partial M^{(p)}}{\partial \mathbf{C}_n} &= 4w_0 pE[\mathbf{Y}_n z_n z_n^{p-2}(z_n^p - R_p)] + \\ &\quad 2w_1 E(\mathbf{Y}_n z_{n-1})E(z_n z_{n-1} - R(1)) + \cdots + \\ &\quad 2w_M E(\mathbf{Y}_n z_{n-M})E(z_n z_{n-M} - R(M)) \end{aligned} \tag{11}$$

Substituting the expected values by their current unbiased estimates, and setting $p = 2$, we obtain the following coefficient adaptation formula:

$$\begin{aligned} \mathbf{C}_{n+1} &= \mathbf{C}_n - \alpha \mathbf{Y}_n \{4w_0 z_n(z_n^2 - R_2) + \\ &\quad 2w_1 z_{n-1}[z_n z_{n-1} - R(1)] + \cdots + \\ &\quad 2w_M z_{n-M}[z_n z_{n-M} - R(M)]\}. \end{aligned} \tag{12}$$

Since the CCRIMNO algorithm performs nonlinear operations on the output of the deconvolution filter, it can be placed under the general framework of the Bussgang techniques. Indeed, the output is given by:

$$z_n = \mathbf{Y}_n^T \mathbf{C}_n = a_n + e_n \tag{13}$$

where e_n is the "convolutional noise," namely, the residual ISI arising from the difference between the actual inverse filter $\{c_n\}$ and our estimation for the coefficients $\mathbf{C}_n$. Our target is to use the deconvolved sequence $\{z_n\}$ to find the best estimate of the original sequence $\{a_n\}$. If we denote this optimum estimate at the nth iteration by $g[Z_n]$, where Z_n stands for $\{z_i : 1 \leq i \leq n\}$, then at the $(n+1)$th iteration the Bussgang techniques consist of the following two equations:

$$\mathbf{C}_{n+1} = \mathbf{C}_n - \alpha \mathbf{Y}_n \breve{e} \tag{14}$$

$$\breve{e} = z_n - g[Z_n]. \tag{15}$$

The choice of $g[\cdot]$ affects the performance of the algorithm. If $g[\cdot]$ is only a function of the current output z_n of the deconvolution filter the corresponding algorithms are said to have memoryless nonlinearity. For the case of the CCRIMNO algorithm we see that the nonlinear function $g[Z_n]$ is:

$$\begin{aligned} g[Z_n] &= z_n - \{4w_0 z_n(z_n^2 - R_2) + \cdots + \\ &\quad 2w_M z_{n-M}[z_n z_{n-M} - R(M)]\}. \end{aligned} \tag{16}$$

Note that $g[\cdot]$ is a function of not only the current output z_n, but also of the previous outputs $\{z_k : n - M \leq k \leq n\}$.

3.1 Adaptive Weight Algorithm

A critical performance metric in every adaptive algorithm is its convergence speed. The convergence of the CCRIMNO algorithm depends on the update mechanism of the cost function weights $\{w_i\}$. The following strategy is used to update the weights $\{w_k;\ k = 0, \ldots, M\}$:

> In every step estimate the value of each term in the CCRIMNO cost function and set the weights proportional to the deviations of the corresponding terms from their ideal value at perfect deconvolution.

Invoking Eq. (6) once again, we can rewrite it as

$$M^{(p)} = w_0 J_0 + w_1 J_1 + \cdots + w_M J_M \tag{17}$$

where

$$J_0 = E(|z_n|^p - R_p)^2 \tag{18}$$

and

$$J_i = [E(z_n z_{n-i}) - R(i)]^2;\ \ 1 \leq i \leq M. \tag{19}$$

We then define the deviation, $D(J_i)$, of the ith term by

$$D(J_i) \stackrel{\text{def}}{=} |J_i - J_i^0| \tag{20}$$

where J_i^0 is the value of J_i at perfect deconvolution. The weights are updated by using the following expressions:

$$w_0 = \begin{cases} \gamma_0 D(J_0) & \text{if } \gamma_0 D(J_0) < \lambda \\ \lambda & \text{otherwise} \end{cases}$$

$$w_i = \begin{cases} \gamma D(J_i) & \text{if } \gamma D(J_i) < \lambda \\ \lambda & \text{otherwise} \end{cases} \tag{21}$$

where γ_0, γ are positive constants, and λ constrains the maximum value that the weights can obtain so that the algorithm remains stable.

The constants γ_0 and γ should be different because of the different nature of the first term, J_0, from the other terms, J_i, in Eq. (17). With the above adaptive scheme the weights decrease toward zero when the algorithm converges. In fact, the adaptive weight algorithm behaves as an adaptive step-size algorithm which is a highly desirable property for an adaptive filter.

4 Simulation Results

The reverberation filter we use in our simulations is shown in Fig. 2. As we can see in Fig. 3 is a nonminimum phase filter. Its frequency response is given in Fig. 4. The true inverse filter is shown in Fig. 5. The autocorrelation function of the original data has length 20. Fig. 6 and Fig. 7 show the convolution of the reverberation filter with the inverse filters as estimated by the algorithms at iterations 1 and 20,000. Ideally, at convergence, the curves should be delta functions. Fig. 8 shows the mean-square error between the actual and the estimated inverse filters. In Fig. 9-12 we plot the actual inverse filter together with the estimated by the algorithms inverse filters at iteration 20,000, both in time and frequency domains.

As we see, the new algorithm converges faster than the Godard. By increasing the memory size M we get better results. This is true for increasing M up to a maximum value which equals approximately the length of the autocorrelation function of the original data.

5 Conclusions

A new criterion has been introduced for blind deconvolution of colored input signals. The criterion makes use of a priori knowledge about the autocorrelation function of the original data and has been demonstrated to exhibit faster convergence than the Godard's algorithm. The algorithm is suitable for applications where the original data are continuous, colored waveforms as in the case of seismic or speech data.

Acknowledgments

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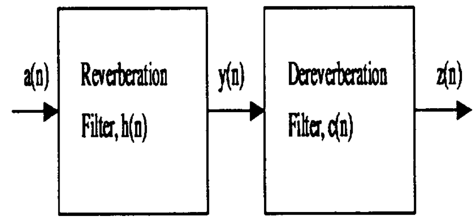

Figure 1: Block diagram of a reverberation system

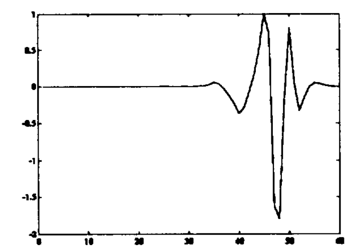

Figure 2: Reverberation filter $h(n)$

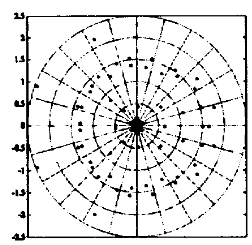

Figure 3: Roots of $h(n)$

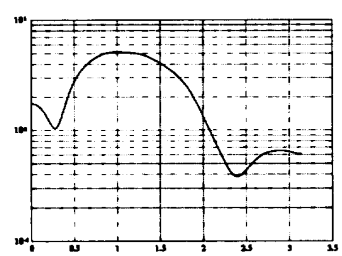

Figure 4: Frequency response $| H_1(w) |$

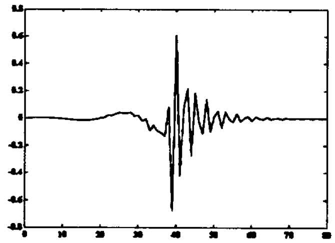

Figure 5: Inverse of $h(n)$

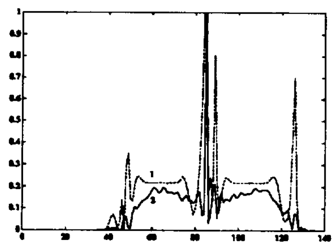

Figure 6: $h(n)$ convolved with Godard filter coefficients. (1) 1st iteration, (2) 20,000th iteration.

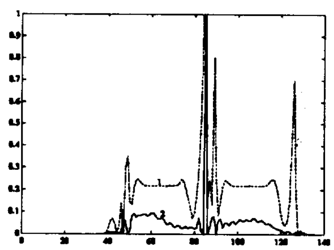

Figure 7: $h(n)$ convolved with CCRIMNO filter coefficients. Memory size $M = 20$. (1) 1st iteration, (2) 20,000th iteration.

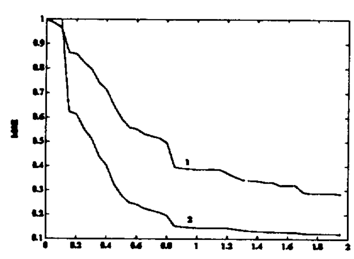

Figure 8: Convergence speed (1) Godard, (2) CCRIMNO

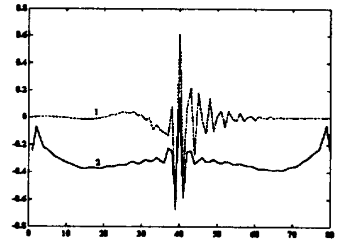

Figure 9: (1) Inverse of $h(n)$ (2) Output of Godard.

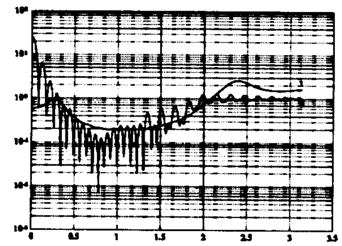

Figure 10: (1) $1/ \mid H(w) \mid$ (2) Output of Godard.

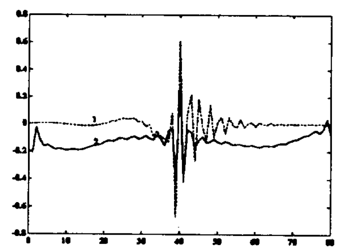

Figure 11: (1) Inverse of $h(n)$ (2) Output of CCRIMNO.

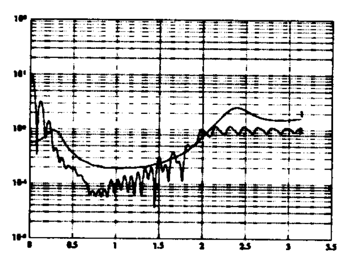

Figure 12: (1) $1/ \mid H(w) \mid$ (2) Output of CCRIMNO.