JOINT LOCATION AND DOPPLER ESTIMATION WITH FRACTIONAL LOWER-ORDER STATISTICS

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ABSTRACT

In the wireless distributed multimedia communication networks, the spatial selectivity of an array of sensors can be used in order to operate in close-frequency bands and suppress undesirable noise and interference in favor of the signal of interest. Our emphasis is in the development of methods for direction finding, null- and beam-steering, and waveform recovery in mobile networks. We develop angle and Doppler estimation techniques from measurements retrieved in the presence of impulsive noise (thermal, jamming, or clutter) modeled as a complex isotropic alpha-stable random process. The results are of great importance in the study of wireless communications and of space-time adaptive processing (STAP) applications for airborne pulse Doppler radar arrays operating in impulsive interference environments.

1. INTRODUCTION

Most of the theoretical work in detection and estimation for radar applications has focused on the case where clutter is assumed to follow the Gaussian model. The Gaussian assumption is frequently motivated by the physics of the problem and it often leads to mathematically tractable solutions. However, in many practical instances, experimental results have been reported where clutter returns are impulsive in nature and cannot be appropriately modeled by means of the Gaussian distribution [1]. A number of distributions, based on empirical as well as theoretical grounds, have been proposed for the modeling of non-Gaussian clutter and interference environments [2, 3].

Recently, a statistical model for impulsive clutter has been proposed, which is based on the theory of symmetric alpha-stable ($S\alpha S$) random processes [4]. The model is of a statistical-physical nature and has been shown to arise under very general assumptions and to describe a broad class of impulsive interference. In particular, it has been shown in [4] that the first order distribution of the amplitude of the radar return follows a SαS law, while the first-order joint distribution of the quadrature components of the envelope of the radar return follows an isotropic stable law. In addition, the theory of *multivariate sub-Gaussian* random processes provides an elegant and mathematically tractable framework for the solution of the detection and parameter estimation problems in the presence of impulsive correlated radar clutter [5].

As mentioned in [6], much of the work reported for radar systems has concentrated on target detection in Gaussian or Non-Gaussian backgrounds [5, 7, 8, 9, 10, 11, 12, 13]. In this paper, we are addressing the parameter estimation problem with a space-time adaptive processing (STAP) radar operating in impulsive clutter and interference environments. We present a new subspace-based method for joint spatial- and doppler-frequency high-resolution estimation in the presence of impulsive noise which can be modeled as a complex symmetric alpha-stable ($S\alpha S$) process. In Section 2, we present some necessary preliminaries on α-stable processes. In Section 3, we formulate the STAP problem for airborne radar. In Section 4, we define the covariation matrix of the space-time radar sensor output snapshot and we show that eigendecomposition-based methods, such as the MUSIC algorithm, can be applied to the sample covariation matrix to extract the angle/Doppler information from the measurements. Finally, in Section 5, the improved performance of the proposed source localization method in the presence of a wide range of impulsive noise environments is demonstrated via Monte Carlo experiments.

2. MATHEMATICAL PRELIMINARIES

In this section, we introduce the statistical model that will be used to describe the additive noise. The model is based on the class of *isotropic* $S\alpha S$ distributions, and is well-suited for describing impulsive noise processes [4].

Stable processes satisfy the stability property which states that linear combinations of jointly stable variables are indeed stable. They arise as limiting processes of sums of independent, identically-distributed random variables via the generalized central limit theorem. They are described by their characteristic exponent α, taking values $0 < \alpha \leq 2$. Gaussian processes are stable processes with $\alpha = 2$. Stable

The work in this paper was supported by the Office of Naval Research under Contract N00014-92-J-1034.

distributions have heavier tails than the normal distribution, possess finite pth order moments only for $p < \alpha$, and are appropriate for modeling noise with outliers.

A complex random variable (r.v.) $X = X_1 + \jmath X_2$ is isotropic $S\alpha S$ if X_1 and X_2 are jointly $S\alpha S$ and have a symmetric distribution. The characteristic function of X is given by

$$\varphi(\omega) = \mathcal{E}\{\exp(\jmath\Re[\omega X^*])\} = \exp(-\gamma|\omega|^\alpha), \qquad (1)$$

where $\omega = \omega_1 + \jmath\omega_2$. The *characteristic exponent* α is restricted to the values $0 < \alpha \leq 2$ and it determines the shape of the distribution. The smaller the characteristic exponent α, the heavier the tails of the density. The *dispersion* γ ($\gamma > 0$) plays a role analogous to the role that the variance plays for second-order processes. Namely, it determines the spread of the probability density function around the origin.

Several complex r.v.'s are jointly $S\alpha S$ if their real and imaginary parts are jointly $S\alpha S$. When X and Y are jointly $S\alpha S$ with $1 < \alpha \leq 2$, the *covariation* of X and Y is defined by

$$[X, Y]_\alpha = \frac{\mathcal{E}\{XY^{<p-1>}\}}{\mathcal{E}\{|Y|^p\}}\gamma_Y, \quad 1 \leq p < \alpha, \qquad (2)$$

where $\gamma_Y = [Y, Y]_\alpha$ is the dispersion of the r.v. Y, and we use throughout the convention $Y^{<p>} = |Y|^{p-1}Y^*$. Also, the *covariation coefficient* of X and Y is defined by

$$\lambda_{X,Y} = \frac{[X, Y]_\alpha}{[Y, Y]_\alpha}, \qquad (3)$$

and by using (2), it can be expressed as

$$\lambda_{X,Y} = \frac{E\{XY^{<p-1>}\}}{E\{|Y|^p\}}, \qquad \text{for } 1 \leq p < \alpha. \qquad (4)$$

The covariation of complex jointly $S\alpha S$ r.v.'s is not generally symmetric and has the following properties:

P1 If X_1, X_2 and Y are jointly $S\alpha S$, then for any complex constants a and b,

$$[aX_1 + bX_2, Y]_\alpha = a[X_1, Y]_\alpha + b[X_2, Y]_\alpha;$$

P2 If Y_1 and Y_2 are independent and X_1, X_2 and Y are jointly $S\alpha S$, then for any complex constants a, b and c,

$$[aX_1, bY_1 + cY_2]_\alpha = ab^{<\alpha-1>}[X_1, Y_1]_\alpha + ac^{<\alpha-1>}[X_1, Y_2]_\alpha;$$

P3 If X and Y are independent $S\alpha S$, then $[X, Y]_\alpha = 0$.

Figure 1 shows results on the modeling of the amplitude statistics of real radar clutter by means of $S\alpha S$ distributions. A comparison is made between the $S\alpha S$ amplitude probability density (APD) and the Gaussian APD on how they approximate the empirical APD corresponding to the real radar clutter time series. The estimation of the parameters of the stable distribution from the real clutter data was achieved by methods based on fractional lower-order and negative-order moments, as described in [14]. For the particular clutter series shown here, the characteristic exponent of the $S\alpha S$ distribution which best fits the data was calculated to be approximately $\alpha = 1.5$. To fit a Gaussian model, the variance was estimated by calculating the sample variance of the data. The impulsive nature of the clutter data is obvious in Figure 1 which shows that the $S\alpha S$ distribution fits the tails of the real data more accurately than the Gaussian density. Hence, it is demonstrated that the $S\alpha S$ distribution is superior to the Gaussian distribution for modeling the particular radar clutter data under study.

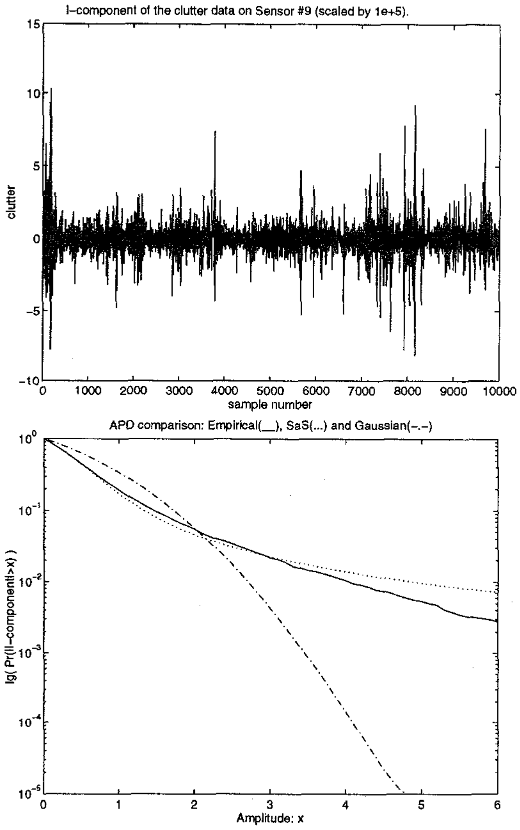

Figure 1: The in-phase component of the clutter time series (top) and the corresponding amplitude probability density curves (bottom) (Empirical: solid, $S\alpha S$: dotted, Gaussian: dash-dotted).

3. STAP PROBLEM FORMULATION

Space-time adaptive processing (STAP) refers to multidimensional adaptive algorithms that simultaneously combine the signals from the elements of an array antenna and the multiple pulses of a coherent radar waveform, to suppress interference and provide target detection [6, 15, 16].

Consider a uniformly spaced linear array radar antenna consisting of N elements, which transmits a coherent burst of M pulses at a constant pulse repetition frequency (PRF) f_r and over a certain range of directions of interest. The array receives signals generated by q narrow-band moving targets which are located at azimuth angles $\{\theta_k;\ k = 1,\ldots,q\}$ and have relative velocities with respect to the radar $\{\nu_k;\ k = 1,\ldots,q\}$ corresponding to Doppler frequencies $\{f_k;\ k = 1,\ldots,q\}$. Since the signals are narrow-band, the propagation delay across the array is much smaller than the reciprocal of the signal bandwidth, and it follows that, by using a complex envelop representation, the array output can be expressed as [15]:

$$\mathbf{x}(t) = \mathbf{V}(\Theta,\varpi)\mathbf{s}(t) + \mathbf{n}(t), \tag{5}$$

where

- $\mathbf{x}(t) = [x_1(t),\ldots,x_{MN}(t)]^T$ is the array output vector (N: number of array elements, M: number of pulses, t may refer to the number of the coherent processing intervals (CPI's) available at the receiver);
- $\mathbf{s}(t) = [s_1(t),\ldots,s_q(t)]^T$ is the signal vector emitted by the sources as received at the reference sensor 1 of the array;
- $\mathbf{V}(\Theta,\varpi) = [\mathbf{v}(\vartheta_1,\varpi_1),\ldots,\mathbf{v}(\vartheta_q,\varpi_q)]$ is the *space-time steering matrix* ($\varpi_k = \frac{f_k}{f_r}$);
- $\mathbf{v}(\vartheta_k,\varpi_k) = \mathbf{b}(\varpi_k)\otimes\mathbf{a}(\vartheta_k)$ is the space-time steering vector, where:
  - $\mathbf{a}(\vartheta_k) = [1, e^{j2\pi\vartheta_k},\ldots,e^{j(N-1)2\pi\vartheta_k}]^T$ is the spatial steering vector ($\vartheta_k = \frac{d}{\lambda_0}\cos(\theta_k)$);
  - $\mathbf{b}(\varpi_k) = [1, e^{j2\pi\varpi_k},\ldots,e^{j(M-1)2\pi\varpi_k}]^T$ is the temporal steering vector.
- $\mathbf{n}(t) = [n_1(t),\ldots,n_{MN}(t)]^T$ is the noise vector.

Assuming the availability of P coherent processing intervals (CPI's) $t_1,\ldots,t_P$, the data can be expressed as

$$\mathbf{X} = \mathbf{V}(\Theta,\varpi)\mathbf{S} + \mathbf{N}, \tag{6}$$

where $\mathbf{X}$ and $\mathbf{N}$ are the $MN\times P$ matrices

$$\mathbf{X} = [\mathbf{x}(t_1),\ldots,\mathbf{x}(t_P)], \tag{7}$$

$$\mathbf{N} = [\mathbf{n}(t_1),\ldots,\mathbf{n}(t_P)], \tag{8}$$

and $\mathbf{S}$ is the $q\times P$ matrix

$$\mathbf{S} = [\mathbf{s}(t_1),\ldots,\mathbf{s}(t_P)]. \tag{9}$$

Our objective is to jointly estimate the directions-of-arrival $\{\theta_k;\ k = 1,\ldots,q\}$ and the Doppler frequencies $\{f_k;\ k = 1,\ldots,q\}$ of the source targets.

4. THE ARRAY COVARIATION MATRIX

We will assume that the q signal waveforms are non-coherent, statistically independent, complex isotropic $S\alpha S$ ($1<\alpha\leq 2$) random processes with zero location parameter and covariation matrix $\mathbf{\Gamma}_S = \mathrm{diag}(\gamma_{s_1},\ldots,\gamma_{s_q})$. Also, the noise vector $\mathbf{n}(t)$ is a complex isotropic $S\alpha S$ random process with the same characteristic exponent α as the signals. The noise is assumed to be independent of the signals with covariation matrix $\mathbf{\Gamma}_N = \gamma_n\mathbf{I}$.

Now, we define the *covariation matrix*, $\mathbf{\Gamma}_X$, of the observation vector process $\mathbf{x}(t)$ as the matrix whose elements are the covariations $[x_i(t),x_j(t)]_\alpha$ of the components of $\mathbf{x}(t)$. By using properties P1-P3, we obtain the following expression for the covariation of the sensor measurements:

$$\begin{aligned}[x_i(t),x_j(t)]_\alpha &= \sum_{k=1}^{q} v_i(\vartheta_k,\varpi_k)v_j^{<\alpha-1>}(\vartheta_k,\varpi_k)\gamma_{s_k} + \\ &\quad \gamma_n\delta_{i,j} \quad i,j = 1,\ldots,MN.\end{aligned} \tag{10}$$

In matrix form, (10) gives the following expression for the covariation matrix of the observation vector:

$$\mathbf{\Gamma}_X \triangleq [\mathbf{x}(t),\mathbf{x}(t)]_\alpha = \mathbf{V}(\Theta,\varpi)\mathbf{\Gamma}_S\mathbf{V}^{<\alpha-1>}(\Theta,\varpi) + \gamma_n\mathbf{I}, \tag{11}$$

where the (i,j)th element of matrix $\mathbf{V}^{<\alpha-1>}(\Theta,\varpi)$ results from the (j,i)th element of $\mathbf{V}(\Theta,\varpi)$ according to the operation

$$[\mathbf{V}^{<\alpha-1>}(\Theta,\varpi)]_{i,j} = [\mathbf{V}(\Theta,\varpi)]_{j,i}^{<\alpha-1>} \tag{12}$$

Clearly, when $\alpha = 2$, i.e., for Gaussian distributed signals and noise, the expression for the covariation matrix is identical to the well-known expression for the covariance matrix:

$$\mathbf{R}_X = \mathbf{V}(\Theta,\varpi)\mathbf{\Sigma}\mathbf{V}^H(\Theta,\varpi) + \sigma^2\mathbf{I}, \tag{13}$$

where $\mathbf{\Sigma}$ is the signal covariance matrix.

When the amplitude response of the sensors equals unity, it follows that

$$[\mathbf{V}^{<\alpha-1>}(\Theta,\varpi)]_{i,j} = [\mathbf{V}(\Theta,\varpi)]_{j,i}^{*}, \tag{14}$$

and thus the covariation matrix can be written as

$$\mathbf{\Gamma}_X = \mathbf{V}(\Theta,\varpi)\mathbf{\Gamma}_S\mathbf{V}^H(\Theta,\varpi) + \gamma_n\mathbf{I}. \tag{15}$$

Observing (15), we conclude that standard subspace techniques can be applied to the covariation or the covariation coefficient matrices of the observation vector to extract the angle/Doppler information. In practice, we have to estimate the covariation matrix from a finite number of array sensor measurements. A proposed estimator for the covariation coefficient $\lambda_{x_i(t),x_j(t)}$ is called the *fractional lower order (FLOM) estimator* and is given by [17, 18]

$$\hat{\lambda}_{x_i(t),x_j(t)} = \frac{\sum_{t=1}^{n} x_i(t)x_j^{<p-1>}(t)}{\sum_{t=1}^{n}|x_j(t)|^p} \tag{16}$$

for some $0\leq p<\alpha/2$. We will refer to the new algorithm resulting from the eigendecomposition of the array covariation coefficient matrix as the **2-D Robust Covariation-Based MUSIC** or **2-D ROC-MUSIC**.

5. EXPERIMENTAL RESULTS

In this section, we show results on the resolution capability and estimation accuracy of the 2-D ROC-MUSIC and 2-D MUSIC methods. The array is linear with five sensors spaced a half-wavelength apart ($N = 5$). The number of transmitted pulses is $M = 10$. Three moving targets impinge on the array from directions $\mathbf{\Theta} = [-20^o, -40^o, 40^o]$ and they have Doppler values $\mathbf{D} = [-0.3, -0.2, 0.3]$). The number of snapshots available to the algorithms is $P =$

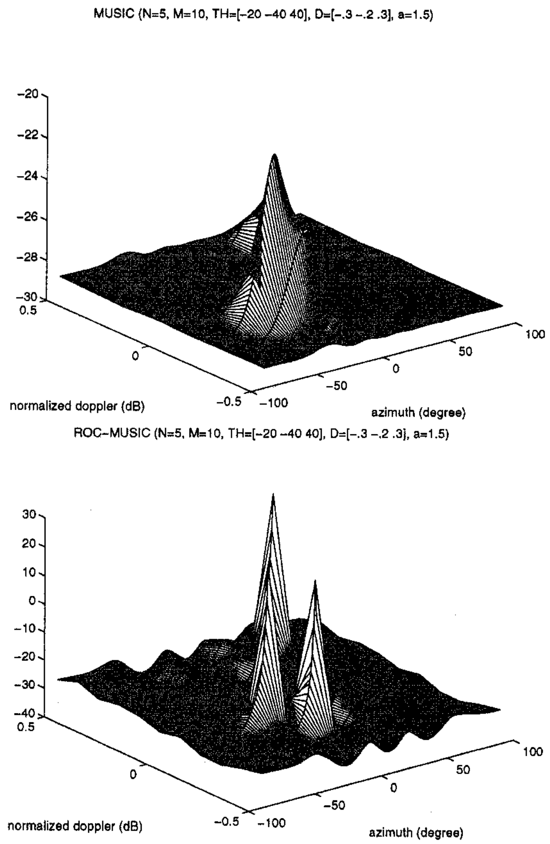

Figure 2: 2-D MUSIC (top) and 2-D ROC-MUSIC (bottom) angle-Doppler spectra ($N = 5$, $M = 10$, $\mathbf{\Theta} = [-20^\circ, -40^\circ, 40^\circ]$ $\mathbf{D} = [-0.3, -0.2, 0.3]$). Additive stable noise ($\alpha = 1.5$).

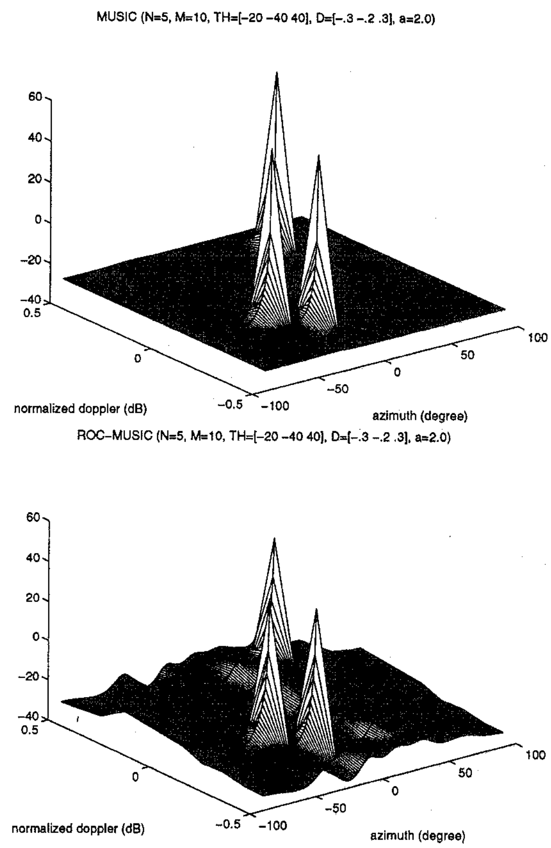

Figure 3: 2-D MUSIC (top) and 2-D ROC-MUSIC (bottom) angle-Doppler spectra ($N = 5$, $M = 10$, $\mathbf{\Theta} = [-20^\circ, -40^\circ, 40^\circ]$ $\mathbf{D} = [-0.3, -0.2, 0.3]$). Additive Gaussian noise ($\alpha = 2.0$).

1000. The noise follows the bivariate isotropic stable distribution with $\alpha = 1.5$.

Since the alpha-stable family for $\alpha < 2$ determines processes with infinite variance, we define an alternative signal-to-noise ratio. Namely, we define the *Generalized SNR (GSNR)* to be the ratio of the signal power over the noise dispersion γ_n:

$$GSNR = 10\log\left(\frac{1}{\gamma_n M}\sum_{t=1}^{M}|s(t)|^2\right). \tag{17}$$

The GSNR is 22.3 dB ($\gamma_n = 1$). The characteristic exponent α of the additive noise is unknown to the ROC-MUSIC algorithm. The parameter p in the estimation of the covariation matrix (cf. (16)): was set equal to $p = 0.8$. Clearly, MUSIC can be thought as a special case of ROC-MUSIC with $p = 2$.

In Figures 2-3, space-time spectral estimates are shown for the 2-D ROC-MUSIC and 2-D MUSIC algorithms. Two types of alpha stable noise corresponding to two values of the characteristic exponent $\alpha = 1.5$ and $\alpha = 2.0$ (Gaussian) were used. We can see that the 2-D MUSIC method exhibits high-resolution performance only for the case of additive Gaussian noise while it does not resolve the two closely moving targets when the additive noise is stable with $\alpha = 1.5$. On the other hand, the 2-D ROC-MUSIC method exhibits better resolution capabilities for non-Gaussian additive noise environments ($\alpha = 1.5$) and at the same time, performs well for Gaussian interference.

6. CONCLUSIONS

We considered the problem of target angle and Doppler estimation with an airborne radar employing space-time adaptive processing. We introduced a new joint spatial- and doppler-frequency high-resolution estimation technique based on the fractional lower-order statistics of the measurements of a radar array. We showed that the proposed 2-D ROC-MUSIC algorithm provides better angle/Doppler estimates than the 2-D MUSIC method, and it can result to improved STAP radar systems operating in impulsive interference environments.

7. ACKNOWLEDGMENTS

The authors would like to thank Dr. Jack Ma who provided the results on the modeling of real clutter data, shown in Figure 1.

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