Localization in Wireless Networks via Spatial Sparsity

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Abstract—**This paper exploits recent developments in sparse approximation and compressive sensing to efficiently perform localization in wireless networks. Particularly, we re-formulate the localization problem as a sparse approximation problem using the compressive sensing theory that provides a new paradigm for recovering a sparse signal solving an ℓ_1 minimization problem. The proposed received signal strength-based method does not require any time specific/propriatery hardware since the location estimation is performed at the Access Points (APs). The experimental results show that our proposed method, when compared with traditional localization schemes results in a better accuracy in terms of the mean localization error.**

I. Introduction

Recently, the demand for wireless communications has grown tremendously. Location and mobility management are critical issues for providing a seamless and ubiquitous computing environment for mobile users. The ability of a mobile user to determine its position through automatic means is recognized as an essential capability. This determination of the physical position is known as localization or location-sensing.

Certain localization techniques require the mobile node to compute its own position. The mobile node perceives signal strength from the APs and by applying a mathematical model determines its own location. In contrast, some systems require the object to periodically broadcast to allow the external infrastructure to locate it [1]. In cases where the localization takes place in the mobile user's device, the need of special software/driver/application in the mobile device for the data collection is essential. This leads to several problems. At first, devices that are not user serviceable, like wireless IP phones, can not be tracked unless the manufacturer has pre-installed the software and the device's processing power is adequate. Second, in an emergency situation, the user may not be able to install the proper software in order to take advantage of the tracking service.

The popularity and the low cost of IEEE 802.11 networks make them ubiquitous and therefore the majority of localization systems use signal strength measurements. Most of the signal-strength based localization systems can be classified into two categories, namely the distance prediction-based and the signature or map-based.

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Distance prediction-based systems use a prior theoretical or empirical radio propagation model to formulate the relationship between the signal strength and the position. The signal propagation between the wireless device and the AP suffers from the connectivity of line-of-sight (LOS), non line-of-sight (NLOS), and the shadow fading due to the complicated indoor environments. Therefore, using a theoretical propagation model to characterize the distance between the emitter and the receiver may result in high errors.

Map-based localization systems create a radio map that represents the physical space. They use various statistical signal strength techniques in order to find the optimal estimate of the position [2]. The K-nearest neighbors (KNN) method computes the K position estimations with the lower distances in the signal space. The estimated position of the user is the average of the coordinates of K points [3]. The Bayesian classification method is a probabilistic approach that computes the conditional distribution of a certain possible position of the mobile user given the runtime measurements. This method searches for the maximum likelihood estimator of the position [4]. Map-based systems are fairly accurate as they take advantage of the radio propagation characteristics of the physical space, but they are time consuming due to the required training phase to construct the signature map [5]–[8].

Unfortunately, most of the existing localization solutions are computationally inefficient as they require the exchange of a large number of data between the receiver and the transmitter. Moreover, as the node receives this amount of data, the administrative overhead to manage and maintain hardware and software is high. Consequently, they are frequently slow, inefficient, high-dimensional and cost-ineffective.

In order to overcome these problems, in this work we re-formulate the localization problem employing recent advances in sparse approximation and compressive sensing (CS) theory to introduce a fundamentally different approach that is efficient, linear and cost-effective. In this paper, we use the signal perceived at the APs and we propose a signal strength-based localization scheme with no pre-phases. We chose to collect measurements at the APs and to perform the localization at a central unit because in spite of improvements in energy consumption, battery capacity still grows slowly and power remains an important challenge in mobile computing.

The main idea is that under specific conditions, the localization estimates can be obtained by searching for the sparsest solution of an under-determined linear system of equations that arises in localization. Particularly, we use a grid-based representation of the physical space where each cell of the grid corresponds to a position of the physical space. The key idea is that the mobile location is sparse over the ground plane. A vector is called sparse if it contains a small number of non-zero values in a certain domain. It has been proven that minimizing the ℓ_1-norm can recover the sparse vectors [9], [10].

Recently, independently of our work, another sparse approximation approach to mobile localization has been proposed. The approach in [11] differs from our proposed method in that the localization algorithm is performed at the device of the mobile user and a two-phase signal strength CS-based algorithm is used to improve the final estimation.

The paper is organized as follows: In Section II, we present the necessary CS background, while in Section III we introduce our localization framework using the received signal strength model and CS theory. In Section IV, we compare the performance of the proposed method with other localization techniques run at the AP, while we conclude in Section V.

II. Compressive sensing background

Compressive sensing exploits sparsity to acquire high-dimensional signals using a small number of linear measurements. Denote the discrete-time signal $\boldsymbol{\zeta}$, an $N \times 1$ column vector in $\mathbf{R}^N$, and $\boldsymbol{\Psi}$ a sparsity basis matrix of $N \times 1$ vectors $\{\boldsymbol{\psi}_i\}_{i=1}^N$, such that

$$\boldsymbol{\zeta} = \boldsymbol{\Psi}\boldsymbol{b}, \tag{1}$$

where $\boldsymbol{b}$ is the $N \times 1$ column vector of weighting coefficients $b_i = \langle \boldsymbol{\zeta}, \boldsymbol{\psi}_i \rangle = \boldsymbol{\psi}_i^T \boldsymbol{\zeta}$ being the projections of $\boldsymbol{\zeta}$ to each of the basis vectors $\boldsymbol{\psi}_i$. The signal $\boldsymbol{\zeta}$ is called *K-sparse* if it is a linear combination of only K basis vectors, that is, if only K of the b_i coefficients in (1) are nonzero and $N-K$ are zero $(K << N)$.

Given that the signal is sparse, the main objective is to directly acquire a compressive signal representation without the intermediate stage of acquiring N samples. CS accomplishes a full signal acquisition by measuring a set of M linear projections of $\boldsymbol{\zeta}$ into vectors $\boldsymbol{\phi_i}, 1 \leq i \leq M$ where $M = (K \log N/K)$ [9], [10]. We can represent the measurements $\beta_i = \langle \boldsymbol{\zeta}, \boldsymbol{\phi}_j \rangle$ in a $M \times 1$ vector $\boldsymbol{\beta}$ and the measurement vectors ϕ_j^T as rows in an $M \times N$ matrix Φ. Therefore $\boldsymbol{\beta}$ can be written as:

$$\boldsymbol{\beta} = \boldsymbol{\Phi}\boldsymbol{\zeta} = \boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{b} = \boldsymbol{\Theta}\boldsymbol{b}. \tag{2}$$

When the matrix Θ obeys the so-called restricted isometry property (RIP), the original sparse vector $\boldsymbol{b}$ can be recovered exactly as the solution of the following optimization problem:

$$\widehat{\boldsymbol{b}} = \arg\min \|\boldsymbol{b}\|_1 \;\; s.t. \; \boldsymbol{\beta} = \boldsymbol{\Theta}\boldsymbol{b}. \tag{3}$$

This is a convex optimization problem that conveniently reduces to a linear problem known as Basis Pursuit.

III. Localization using CS

A. Received Signal Strength

The *received signal strength indicator* (RSSI) is the most convenient distance measurement method and has attracted a lot of attention in the recent literature. In contrast to other ranging techniques, RSSI needs no additional hardware, it is low-cost and it requires small power consumption.

In the RSSI model, each node broadcasts a signal at a maximum distance and each AP that lays in the area can estimate its distance from the node on the basis of the received signal. A commonly used model for the radio propagation path loss is

$$P_r = P_t - \overline{PL}(d_0) - 10n \log_{10}(\frac{d}{d_0}) - X_\sigma, \tag{4}$$

where P_r is the receiving power at distance d, P_t is the transmitting power and $\overline{PL}(d_0)$ is the average of the path loss value measurements at a reference distance d_0, usually set to 1 meter. The attenuation exponent, n, is a constant depending on the transmission medium (indoor, outdoors) and ranges typically from 2 to 4. It is often statistically determined to provide a best fit with measurement readings. X_σ is a zero-mean Gaussian distributed random variable (in dB) with standard deviation σ [12]. Since antilog function is used to convert RSSI values to distance values, small RSSI variation in decibel form leads to large variation in estimated distance.

B. Proposed Framework

In this paper, we propose a new localization protocol for wireless networks. Considering that the node locations form a sparse map over the ground plane, the key idea is to try to reconstruct a sparse signal from a set of appropriate RSSI measurements by applying compressive sensing. Particularly, a grid based representation of the physical space is used to create a finite set of possible positions. We formulate the localization problem as a dictionary selection problem where the dictionary entries are produced by discretizing the spatial space and then synthesizing the node's signals from each discrete cell. Sparseness, in the spatial space implies that only a few of the dictionary entries will be needed to match the measurements. We pay attention to one AP, or a central unit, that samples the reference signal. In this work we consider a system that does not require any time specific hardware and we reduce the computational overhead of the mobile device by performing the localization algorithm at the AP. This enables us to find the sparse dictionary selector vector by solving an ℓ_1 minimization problem as the one described in (3).

In the proposed system, we consider a set of APs that are wirelessly connected. The wireless card of the mobile user to be located is active and therefore the APs receive RSSI values from the beacons the mobile device transmits. The localization algorithm consists of two phases (cf. Figure 1). During the training phase, each AP acquires signal strength measurements from a node for each cell of the grid in order to construct a map of signatures of the physical space. For consistency, we receive

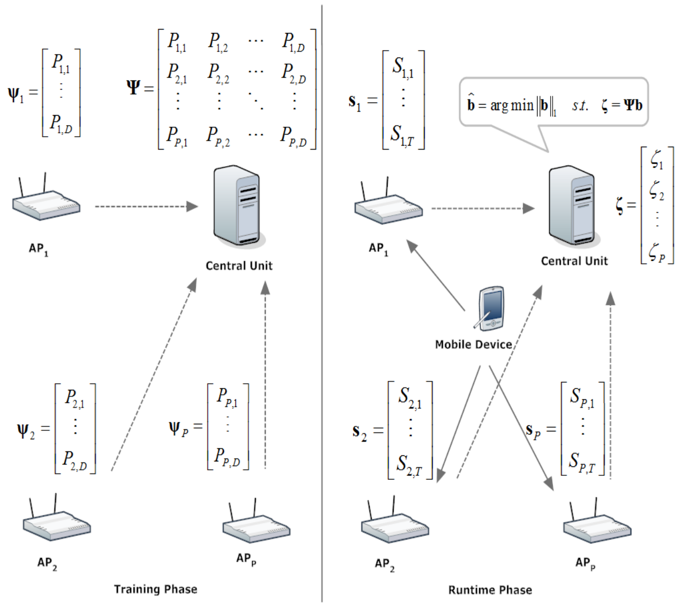

Fig. 1. Proposed localization framework.

the RSSI measurements during the training phase and runtime phase from a mobile node that has the same wireless card. In the runtime phase, each AP selects RSSI measurements in order to estimate the average signal the mobile node transmits and sends it to the central unit. The central unit collects the RSSI fingerprints from the APs and applies the CS localization protocol in order to estimate the position of the mobile device. In a general localization framework, an AP tries to locate the mobile user on its local grid by receiving signal strength measurements from the node. We consider the physical space of D dimensions and we assume that there are P APs. Our goal is to find the position $\boldsymbol{n} = [x, y]^T$ of the mobile node, using the signal strength measurements it transmits.

The physical space is discretized to form a finite set of cells $\mathcal{B} = \{p_1, p_2, \ldots, p_D\}$, where each cell corresponds to a position in the two dimensional space. The sparse vector $\boldsymbol{b} \in \mathbb{R}^D$ selects elements from $\mathcal{B}$. A non-zero component of $\boldsymbol{b}$ at the i-th position indicates the presence of a node at the cell p_i. For instance, the vector

$$\boldsymbol{b} = [0, 1, 0, \ldots, 0]^T \tag{5}$$

indicates that the node is located at the cell p_2.

During the runtime phase, each AP i creates an RSSI measurement vector $\boldsymbol{S}_i = [S_{i,1}\; S_{i,2}\; \ldots\; S_{i,T}]_{(1\times T)}$, where T is the period of RSSI collection measurements and $\boldsymbol{S}_{i,j}$ is the RSSI signal received from AP i at time j (cf. Figure 1). We can express the average value of the runtime measurements of signal ζ_i received at the i-th AP as:

$$\zeta_i = \boldsymbol{\psi}_i^T \boldsymbol{b}, \tag{6}$$

where $\boldsymbol{\psi}_i$ is the signature map of the i-th AP. Particularly, each value of $\boldsymbol{\psi}_i$ corresponds to the mean value of RSSI signals the i-th AP perceives from a node at a specific location

$$\boldsymbol{\psi}_i = [P_{i,1}\; P_{i,2}\; \ldots\; P_{i,D}]^T_{(1\times D)}, \tag{7}$$

where D is the number of the possible positions a node may occupy. We denote the average value the i-th AP receives from a node at the location j as $P_{i,j}$.

In the case where the localization process is made in a central unit, we can express the signal ensemble as a single vector $\boldsymbol{\zeta} = [\zeta_1\; \ldots\; \zeta_P]^T$ and the training matrix into a single dictionary $\boldsymbol{\Psi} = [\boldsymbol{\psi}_1 \ldots \boldsymbol{\psi}_P]^T$. As a result, the signal ensemble can be written as

$$\boldsymbol{\zeta} = \boldsymbol{\Psi}\boldsymbol{b}. \tag{8}$$

In (8), each element of vector $\boldsymbol{\zeta}$ consists of the mean values of the runtime measurements each AP perceives from the mobile node. Moreover, the matrix $\boldsymbol{\Psi}$ is the signature map that has been constructed during the training phase. Particularly, each row of $\boldsymbol{\Psi}$ represents the signature map of each AP.

Then the sparsity pattern vector $\boldsymbol{b}$ can be found from the set of samples from all the APs by solving the following ℓ_1 minimization problem:

$$\widehat{\boldsymbol{b}} = \arg\min \|\boldsymbol{b}\|_1 \;\; s.t.\; \boldsymbol{\zeta} = \boldsymbol{\Psi}\boldsymbol{b}. \tag{9}$$

The index of the non-zero element of vector $\boldsymbol{b}$ indicates the position of the mobile node.

IV. Experimental Results

In this Section, we study the performance of the proposed scheme in terms of location error, under different RSSI variation characteristics. The location error is defined as the Euclidean distance between the estimated position of the mobile node and the true one. The purpose of the experiment is to evaluate the performance of the CS localization method compared to traditional localization techniques using real data measurements. Specifically, our experiment was performed in a laboratory area of $7m \times 12m$. For this area, a grid-based structure was considered with cells of size $50cm \times 50cm$. The experiment involved a total of 13 APs. The RSSI observations from the mobile device were recorded for a period of 100 seconds (one reception per second) over 109 cells during the training phase. For the implementation of the CS method we used the MATLAB code included in the l_1 magic package [13].

Figure 2 compares the effectiveness of the proposed localization framework with two well-known localization algorithms, the KNN (K=3) and the Bayesian classification method. In order to estimate the performance under different signal-to-noise-ratio (SNR) values, we added white Gaussian noise to the runtime measurement vectors. We computed the SNR by averaging all RSSI measurements (dBm) and subtracting the logarithm of the variance of the added white Gaussian noise. For each possible position in the testbed, we performed 100 Monte Carlo simulations for different SNR values in order to calculate the mean location error. For each algorithm, the average runtime measurement was computed by considering 1, 10 and 100 RSSI samples.

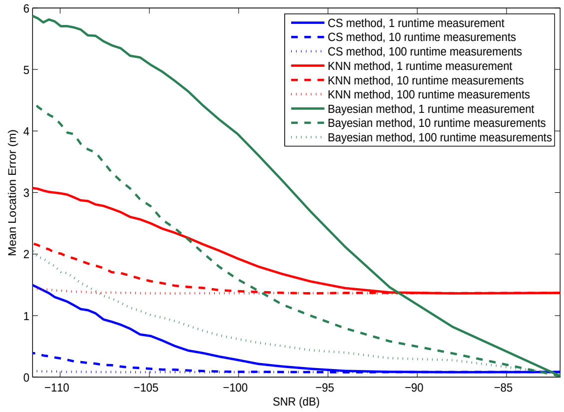

Fig. 2. Mean location error vs. SNR for the KNN, Bayesian and the proposed CS-based localization methods. We used 1, 10 and 100 runtime measurements. The CS-based algorithm has better performance in all cases, especially for low SNR values.

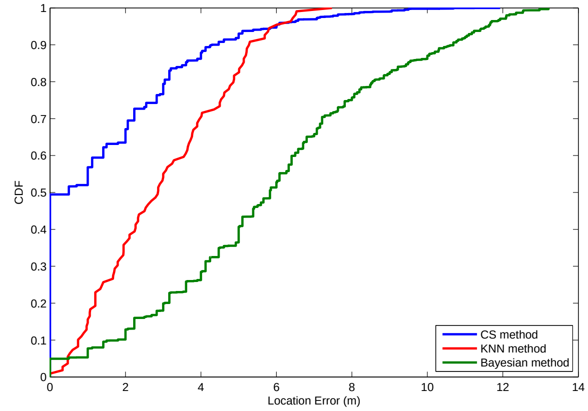

Fig. 3. CDF curves ($P(|X| \leq x)$) for the three methods for SNR = -110 dB and 1 runtime RSSI measurement. Observe that the location error of the CS-based method is less than 0.5 m 50% of the time.

Figure 2 shows that as the number of measurements increases, the accuracy of all three methods is improved, as expected. But for a certain number of measurements and a certain SNR, the proposed CS localization scheme achieves a significant reduction in terms of the localization error over the KNN and the Bayesian classification methods. Particularly, in the case where the noise is high (SNR = -110 dB), we notice that the proposed algorithm leads to improvements in terms of mean localization error in the order of 50% (1.54 m) and 74% (4.3 m) over the KNN and the Bayesian algorithms, respectively.

Figure 3 illustrates the empirical CDF curve ($P(|X| \leq x)$) of the localization error for the three methods in the case of low SNR = -110 db and when one RSSI sample is considered in runtime phase. We observe that the median location error (*i.e.*, the value bellow which 50% of the location errors fall) is 0.5 m for the proposed CS-based approach vs. 2.8 m and 5.8 m for the KNN and the Bayesian methods, respectively.

V. Conclusions

In this paper, we have proposed a novel localization protocol that uses the compressive sensing theory to reformulate the localization problem in wireless networks. The key idea is that the mobile location is sparse over the ground plane. The proposed RSSI-based method is implemented at the APs in order to reduce the computational overhead at the mobile device. The results indicate that the CS localization scheme increases the accuracy compared with other localization techniques.

Future work will investigate the performance of the algorithm with additional real runtime measurements in various operational environments. Another issue of future research is to apply different reconstruction algorithms in order to solve the ℓ_1 minimization problem described in (9). We will focus on the impact of the various reconstructions algorithms in terms of the accuracy of the localization algorithm and the computational time. An exhaustive comparison will reveal the tradeoff between the desired accuracy of the location estimator and the associated complexity of the algorithm.

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