Compressed sensing and applications in positioning, audio coding, and video compression

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Overview

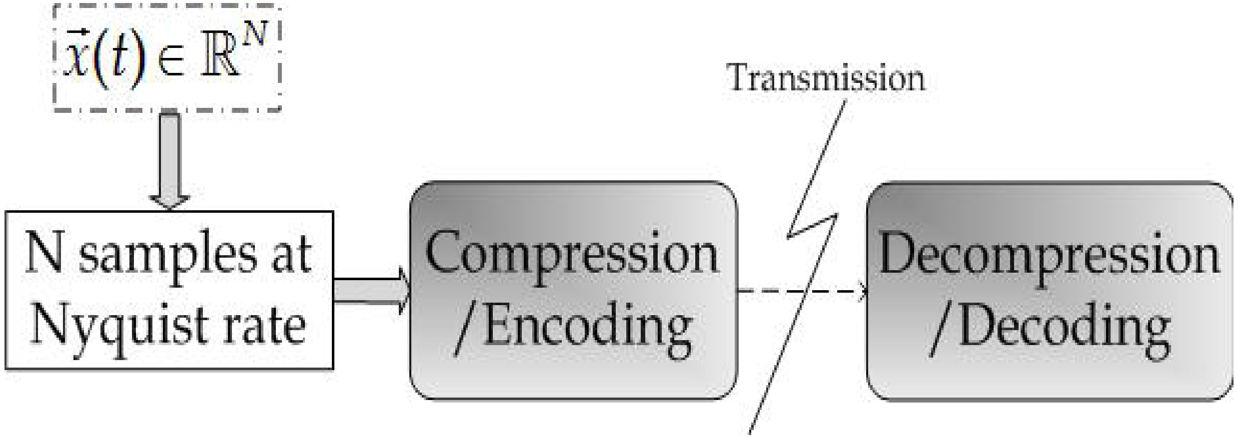
- CS Fundamentals
- Three Applications:
  - Localization via Compressive sensing (CS)
  - Multichannel Audio Coding
  - Compressed Video Sensing

Sensing by Sampling

- Story starts @ 1950
- Long-term trend for digital data acquisition: uniformly sample data at *Nyquist-Shannon rate.*
- Increased capabilities of modern digital devices: too much data!
- Effective and precise description of the information content of a given signal or an ensemble of signals for *storage*, *processing* or *transmission.*
- *e.g.,* 6.1 Mp digital camera senses 6.1e+6 samples JPEG compression to reduce size.

Sensing by Sampling

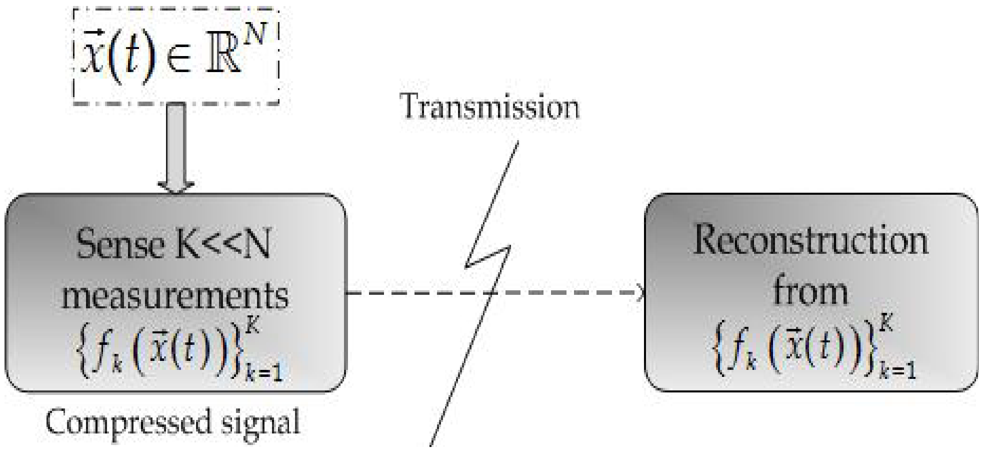
- Traditional approach:



- can be taxing on the sensor
  - high-speed A/Ds may be required
  - must know the compression
  - can be computationally intensive
- if x(t) is a sparse signal (K<<N), then this is wasteful and expensive (computations, storage)

Sensing by Sampling

- Alternative approach:



- What functions $\{f_k(x(t))\}^K$ to use? Linear projections are appealing due to their simplicity.
- How many measurements to take? - Depends on the complexity (degrees of freedom) of ~$x(t)$.
- How can we reconstruct $x(t)$ from the measured values? - Depends on the model we assume on $x(t)$.

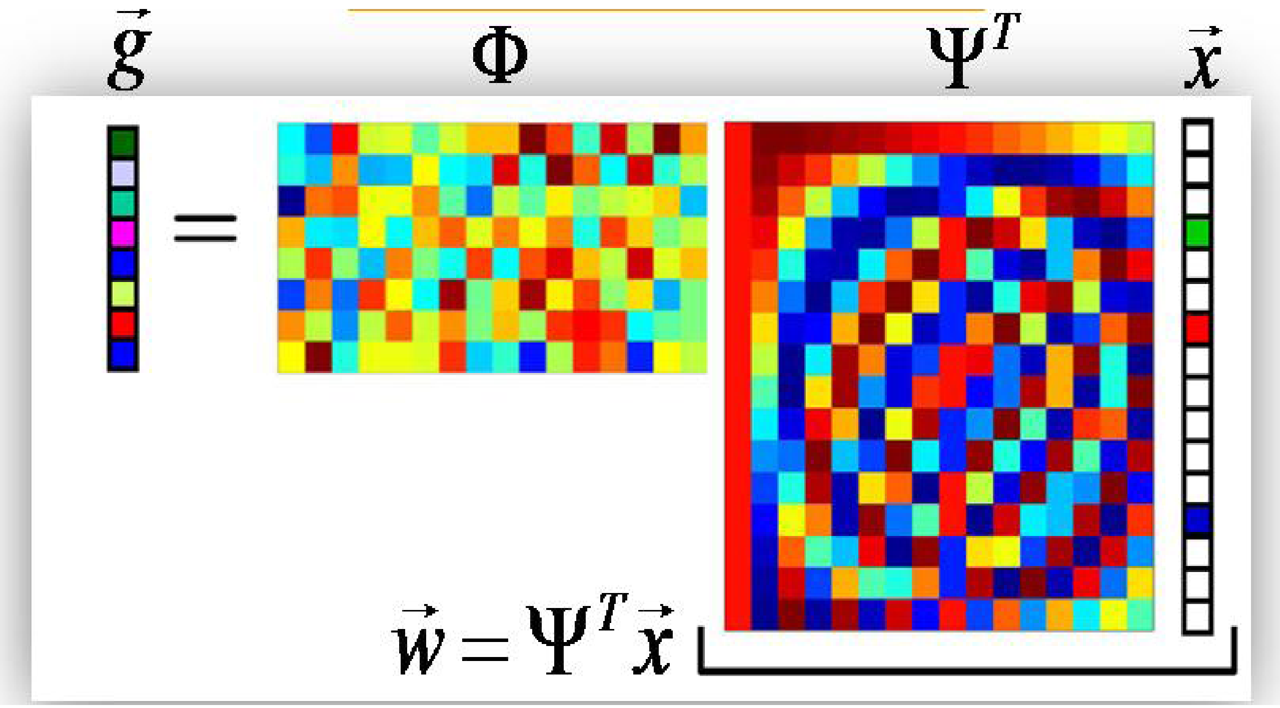
Compressed Sensing [Donoho06][Tao06]

- **Heart of CS:** specific choice of a model for our signals based on *sparse and redundant representations* with respect to appropriate dictionaries.
- Performance depends on the *degree of sparsity* of the (discrete-time) signal $\boldsymbol{x} \sim R^N$
- Transform-domain representation:
  - $\boldsymbol{x} = \boldsymbol{\Psi}\,\boldsymbol{w}$
- $\boldsymbol{x}$ is called ***L-sparse*** in basis $\boldsymbol{\Psi}$ ($L \ll N$) if it has only L non-zero components.
- In real-world, $\boldsymbol{x}$ is ***L-compressible*** in basis $\boldsymbol{\Psi}$
  - $|\boldsymbol{w}_{(l)}| < O(l^{-\alpha}),\ \alpha > 1$

1. D.Donoho "Compressive sensing," *IEEE Trans. Inform. Theory*, vol, no.52, no.4, pp. 1289-1306, April 2006.
2. E.Candes, J. Romberg and T.Tao, "Robust uncertainty principles: Exact signal reconstruction from higly incomplete frequency information". *IEEE Trans. Inform. Theory*. Feb. 2006.

Sensing/Encoding Part

- Consider also an $M \times N$ measurement matrix $\boldsymbol{\Phi}$ (rows of $\boldsymbol{\Phi}$ are *incoherent* with the columns of $\boldsymbol{\Psi}$.
- If $\boldsymbol{x}$ is L-compressible in $\boldsymbol{\Psi}$, perform directly a *compressed set of measurements* $\boldsymbol{g}$ ~> **simplified sensing system**.
    - $\mathbf{g} = \boldsymbol{\Phi}\,\boldsymbol{\Psi}^T\boldsymbol{x} = \boldsymbol{\Phi}\,\boldsymbol{w}$



Sensing/Encoding Part - Universality

Universality property

Let $\mathbf{\Phi}$ contain i.i.d. random entries. Then, incoherence with any fixed transform matrix $\mathbf{\Psi}$ is guaranteed with *high probability*

Appropriate families of matrices are the following:

- *Gaussian matrices*: zero-mean Gaussian distribution with variance $1/M$. Exact reconstruction of $\vec{w}$ (equivalently of $\vec{x}$) is achieved with probability $1 - \mathcal{O}(e^{-\gamma N})$ ($\gamma > 0$) if

$$L \leq c \cdot \frac{M}{\log\left(\frac{N}{M}\right)} \tag{5}$$

- *Binary matrices*: samples from the symmetric Bernoulli distribution, $\mathrm{P}\{[\mathbf{\Phi}]_{mn} = \pm 1/\sqrt{M}\} = 1/2$

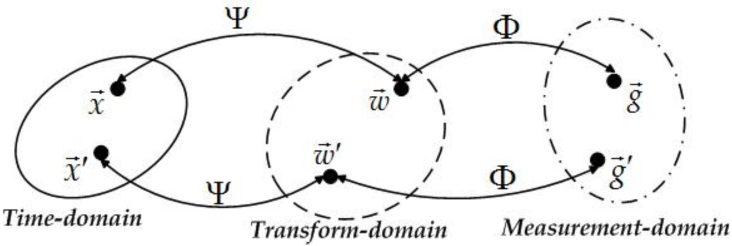
Restricted Isometry Property (RIP)

RIP

$\mathbf{\Phi}$ satisfies the RIP of order L whenever

$$(1-\epsilon_L)\frac{M}{N}\|\vec{w}\|_2^2 \leq \|\mathbf{\Phi}\vec{w}\|_2^2 \leq (1+\epsilon_L)\frac{M}{N}\|\vec{w}\|_2^2 \quad (6)$$

holds simultaneously for all L-sparse vectors $\vec{w} \in \mathbb{R}^N$ for sufficiently small values of ϵ_L.



Reconstruction/Decoding Part

- Core concept of CS theory: use the *a posteriori computing power* to reduce the potential a priori sampling complexity
- Considerations: RIP is satisfied, reconstruction in the (sparse) transform-domain ($\vec{x}, \vec{w}$ are equivalent)
- *Noiseless case*: solve an optimization problem to recover $\vec{w}$ accurately (NP-hard):

$$\vec{w}_{opt} = \arg \min_{\vec{w}' \in \mathbb{R}^N} \|\vec{w}'\|_0 \ , \quad \text{subject to} \quad \vec{g} = \mathbf{\Phi} \vec{w}' \tag{7}$$

- Relaxation (ℓ_1*-minimization problem*):

$$\vec{w}_{opt} = \arg \min_{\vec{w}' \in \mathbb{R}^N} \|\vec{w}'\|_1 \ , \quad \text{subject to} \quad \vec{g} = \mathbf{\Phi} \vec{w}' \tag{8}$$

Reconstruction/Decoding Part

- *Noisy case*:

$$\vec{g} = \mathbf{\Phi}\vec{w} + \vec{n} \tag{9}$$

($\vec{n} \in \mathbb{R}^M$ is a stochastic or deterministic error term (measurement noise) with bounded energy $\|\vec{n}\|_2 \leq \varepsilon$)

- ℓ_1*-minimization problem with relaxed inequality constraints* (convex problem):

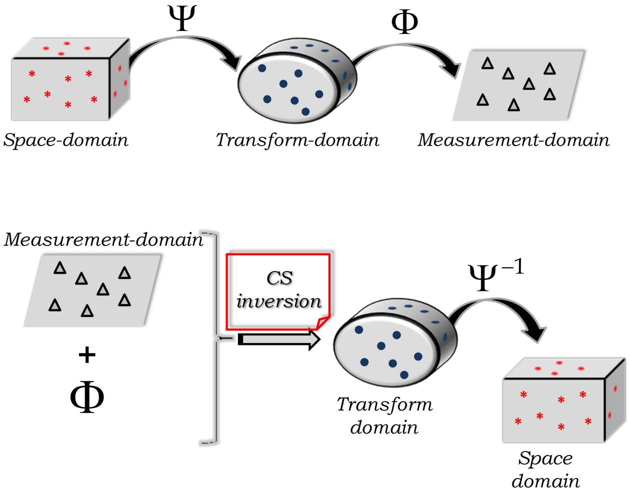
$$\vec{w}_{opt} = \arg\min_{\vec{w}' \in \mathbb{R}^N} \|\vec{w}'\|_1 \ , \quad \text{subject to} \quad \|\vec{g} - \mathbf{\Phi}\vec{w}'\|_2 \leq \varepsilon \tag{10}$$

- Reconstruction methods (zero-mean Gaussian noise with bounded variance):
  - Dantzig selector:

$$\vec{w}_{opt} = \arg\min_{\vec{w}' \in \mathbb{R}^N} \|\vec{w}'\|_1 \ , \quad \text{subject to} \quad \|\mathbf{\Phi}^T(\vec{g} - \mathbf{\Phi}\vec{w}')\|_\infty \leq \kappa_1 \tag{11}$$

  - Penalized least squares minimization:

$$\vec{w}_{opt} = \arg\min_{\vec{w}' \in \mathbb{R}^N} \left\{ \|\vec{g} - \mathbf{\Phi}\vec{w}'\|_2^2 + \kappa_2 \|\vec{w}'\|_0 \right\} \tag{12}$$


Ψ
Φ
Space-domain
Transform-domain
Measurement-domain
Measurement-domain
+
Φ
CS inversion
Ψ^{-1}
Transform domain
Space domain

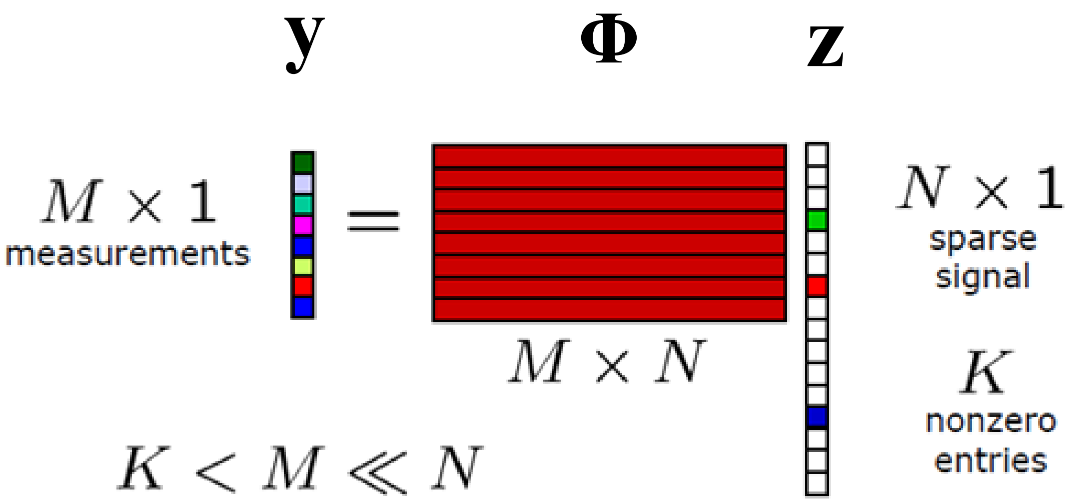
Compressive Sensing

Core concept of CS theory: ***<u>use the a posteriori computing power to reduce the a priori sampling complexity.</u>***

- Two basic Principles
  - Sparsity: small number of non-zero values in a certain domain
  - Incoherence: sampling waveforms have an extremely dense representation in the basis

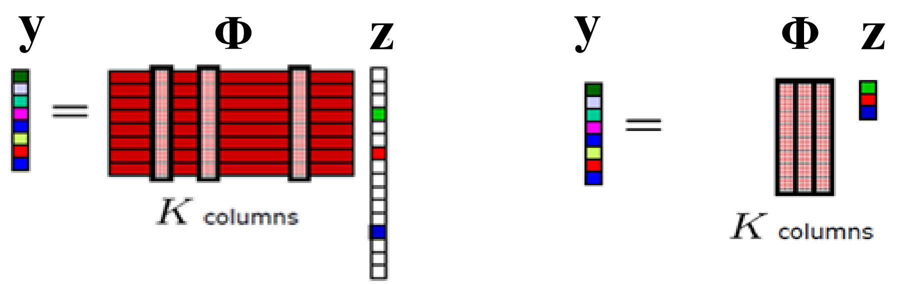
Compressive Sensing

- Want to construct a measurement matrix **Φ** : we can recover a K sparse signal of dimension N from M measurements



Compressive Sensing

- Sparsity assumption



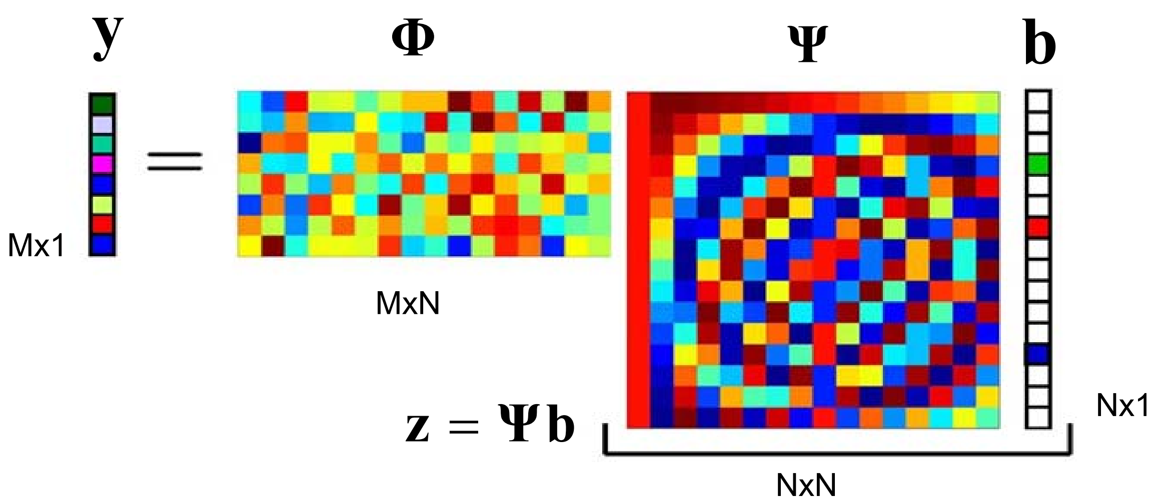
- Restricted Isometry Property (RIP): **Φ** preserves the Euclidean length of K-sparse signals

- Random matrices satisfy RIP property

- By choosing **Φ** randomly, RIP holds with high probability if $M \geq cK\log\left(N/K\right) \ll N$, c:small constant

Compressive Sensing

- Random measurements can be used for signals sparse in any basis ⟶ universality



- **Φ** and **Ψ** must be incoherent
  - Vectors $\boldsymbol{\varphi}_i$ cannot be sparsely represented in terms of the bases **Ψ**

Compressive Sensing - Reconstruction

- **b** can be reconstructed accurately by solving the following optimization problem

$$\hat{\mathbf{b}} = \arg\min \|\mathbf{b}\|_1 \quad s.t. \quad \mathbf{y} = \mathbf{\Theta b}$$

  - $\mathbf{y} = \mathbf{\Phi z} \overset{\mathbf{z}=\mathbf{\Psi b}}{=} \mathbf{\Phi \Psi z} \overset{\mathbf{\Theta}=\mathbf{\Phi\Psi}}{=} \mathbf{\Theta b}$
  - b is sparsely represented
  - $M \geq cK \log\left(N/K\right) \ll N$,c:small constant

Compressive Sensing - Reconstruction

- *Basis Pursuit (BP)* [Saunders98]*:*

$$\hat{\mathbf{b}} = \arg\min \|\mathbf{b}\|_1 \quad s.t. \quad \mathbf{y} = \mathbf{\Theta b} \qquad (1)$$

- Interior point method
- Simultaneously decides on all of the component of **b**
- Computational complexity: $O\left(N^3\right)$,N:size of basis matrix
- Interesting observation: In many cases it successfully finds the sparsest representation

S. Chen, D. Donoho, and M. Saunders. Atomic decomposition by basis pursuit. In SIAM Journal on Scientific Computing, 1998

Compressive Sensing - Reconstruction

- *Orthogonal Matching Pursuit (OMP)* [Gilbert07]

Idea

- Greedy algorithm: Identify the basis vectors that "match" the signal best
- Approximate **b** by choosing the columns of basis vector that contribute most to **y**
- Repeat the process by removing the contribution of the component

- Computational complexity: $O(KMN)$
  - K:sparce vector, M: number of measurements, N:size of basis matrix

J. A. Tropp and A. C. Gilbert. Signal recovery from random measurements via orthogonal matching pursuit. In IEEE Trans. on Information Theory, December 2007

Compressive Sensing - Reconstruction

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J. A. Tropp and A. C. Gilbert. Signal recovery from random measurements via orthogonal matching pursuit. In IEEE Trans. on Information Theory, December 2007

Compressive Sensing – Bayesian Reconstruction

- Design of a BCS algorithm based on Gaussian Scale Mixtures (GSM).
- Extension to reconstruct a single signal based on a set of multiple observation vectors .
- Go against the common tenet of using a Gaussian assumption (often inaccurate) by proposing a CS method for reconstructing highly impulsive signals in the presence of heavy-tailed noise, exploiting the power of alpha-Stable distributions in representing highly impulsive and thus sparse phenomena.
- Introduce a new SαS measurement matrix, which is adapted to the statistical characteristics of the sparse signal.
- Propose a novel Lagrangian function for the estimation of a sparse vector by solving a constrained optimization problem using the duality theory and the method of subgradients, by exploiting Fractional Lower-Order Moments.

George Tzagkarakis PhD Thesis, Univercity of Crete, November 2009.

Compressive Sensing – Conclusions

- CS is good when:
  - **Signals are sparse in some (known) basis.**
  - **Measurements or computations at sensor end are expensive, but**
  - **Computations at the receiver end are cheap, such as in:**

- Imaging
- Sensor Networks
- MRI
- Astronomy
- Biosensing
- Radar
- and many others…

Compressive Sensing – References

- Rice CS Resources:
  - http://dsp.rice.edu/cs
- Nuit Blanche blogspot:
  - http://nuit-blanche.blogspot.com/search/label/CS
- Prof. Emmanuel Candes website:
  - http://www-stat.stanford.edu/~candes/
- Prof. Terence Tao website:
  - http://www.math.ucla.edu/~tao/
- Prof. David Donoho website:
  - http://www-stat.stanford.edu/~donoho/

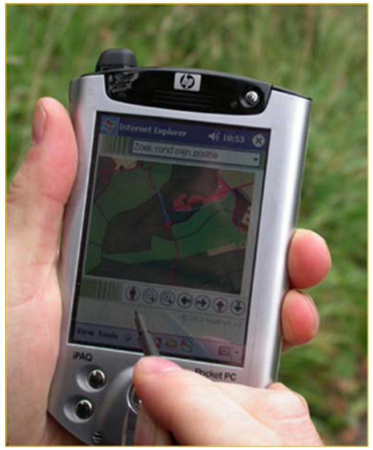
- Our work at FORTH-ICS:
  - http://www.ics.forth.gr/~tsakalid/

Indoor Positioning in Wireless LANs Using Compressive Sensing Signal-Strength Fingerprints

Sofia Nikitaki &
Panagiotis Tsakalides

Motivation

- Emergence of location-based services in
  - ✓ emergency situations
  - ✓ transportation
  - ✓ entertainment



- Location-sensing, **critical** for supporting location-based services

Classification of Localization Systems [Borriello01]

- Use of Hardware
  - Infrastructure based (RF-based, Bluetooth, RFID)
  - Cooperative localization (Ad-hoc networks)
- Local vs. Remote computation
  - Client-based approach
  - Infrastructure based approach
- Position Description
  - Absolute localization
  - Relative localization
- Localization technique
  - Map-based
  - Distance-prediction
- Accuracy - Precision

Hightower J., Borriello G..”A Survey and Taxonomy of Location Systems for Ubiquitous Computing”, Technical Report, University of Washington, Department of Computer Science and Engineering , Seattle, WA, 2001.

Infrastructure Requirements

Client-based approach

- Need of special software/driver/application in the mobile device
  - Difficult to support all operation systems and wireless chipset
  - Pre-installation of the software is needed
  - Processing Power requirements

Infrastructure-based approach

- No need of extra specific/ proprietary hardware
- No client changes required
- Adequate power
- Emergency situations

Distance Based Systems[Liu07]

- Geometric properties of triangles to estimate the location

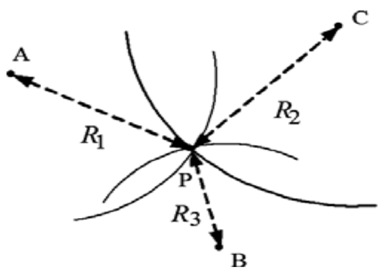
- Lateration
  - Position estimation by measuring its distances from multiple reference points (RP)
  - Time of flight - Signal attenuation

- Angulation
  - Position estimation by computing angles relative to multiple reference points

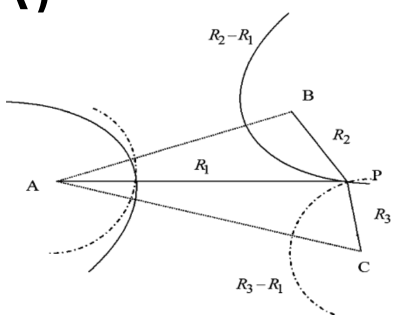
H. Liu, H. Darabi, P. Banerjee, J. Liu, "Survey of Wireless Indoor Positioning Techniques and Systems". In IEEE Transactions on Systems, man and cybernetics, 2007

Lateration – Time of flight [Win09]

- Time of Arrival (TOA)
  - Distance is proportional to the propagation time
  - 2-D requires 3 RPs
  - Compute the intersection points , NLS method



- Time Difference of Arrival (TDOA)
  - Solve a hyperbolic equation
  - 2-D requires 2 measurement units



Issues:

- Synchronization of measurement units
- Additional measurement unit hardware

D. Dardari, A.Conti, U.J. Ferner, A.Giorgetti and M.Z. Win. Ranging with ultrawide bandwidth signals in multipath environments. In IEEE, vol. 97, no. 2, Feb 2009.

Lateration – Received Signal Strength (RSS)[Chen09]

- Signal attenuation methods calculate the path loss due to propagation
- Empirical & theoretical models translate the received signal into distance
- Path loss model:[Rappaport01]

Distance between receiver&transmitter

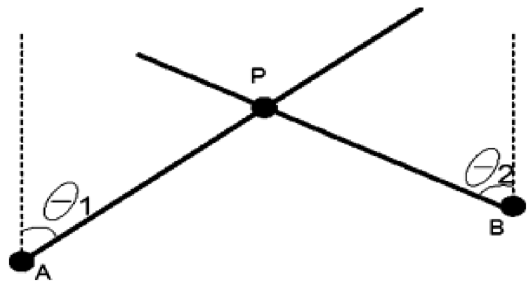
$$P_r = P_t - PL(d_0) - 10a\log(\|n - p\|_2) - X_\sigma$$

- Issues: Multipath fading and shadowing in indoor environments.

1. J. Yang and Y. Chen. "Indoor localization using improved rss-based lateration methods". In IEEE Globecom Wireless Communications Symposium, Hololulu, 2009.
2. Rappaport, T.S., Rappapart, T. Wireless Communications: Priciples and Practice 2nd edn. Prentice-Hall, Englewod Cliffs 2001.

Angulation

- Estimated position: Intersection of several pair of angle direction lines
- Accomplished with directional antennae – array of antennae



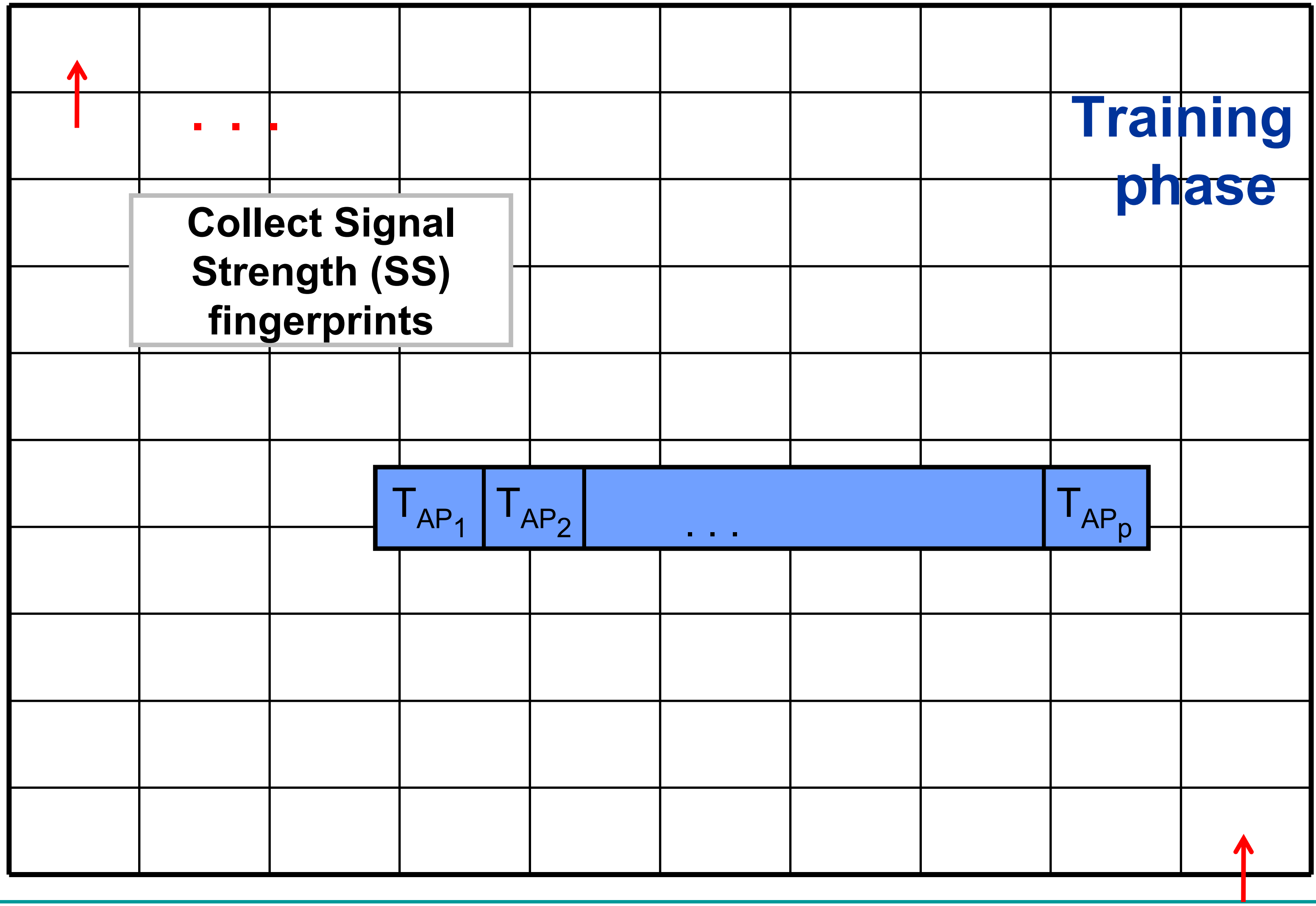
Advantages

- 2-D requires 2 measurement units
- No time synchronization

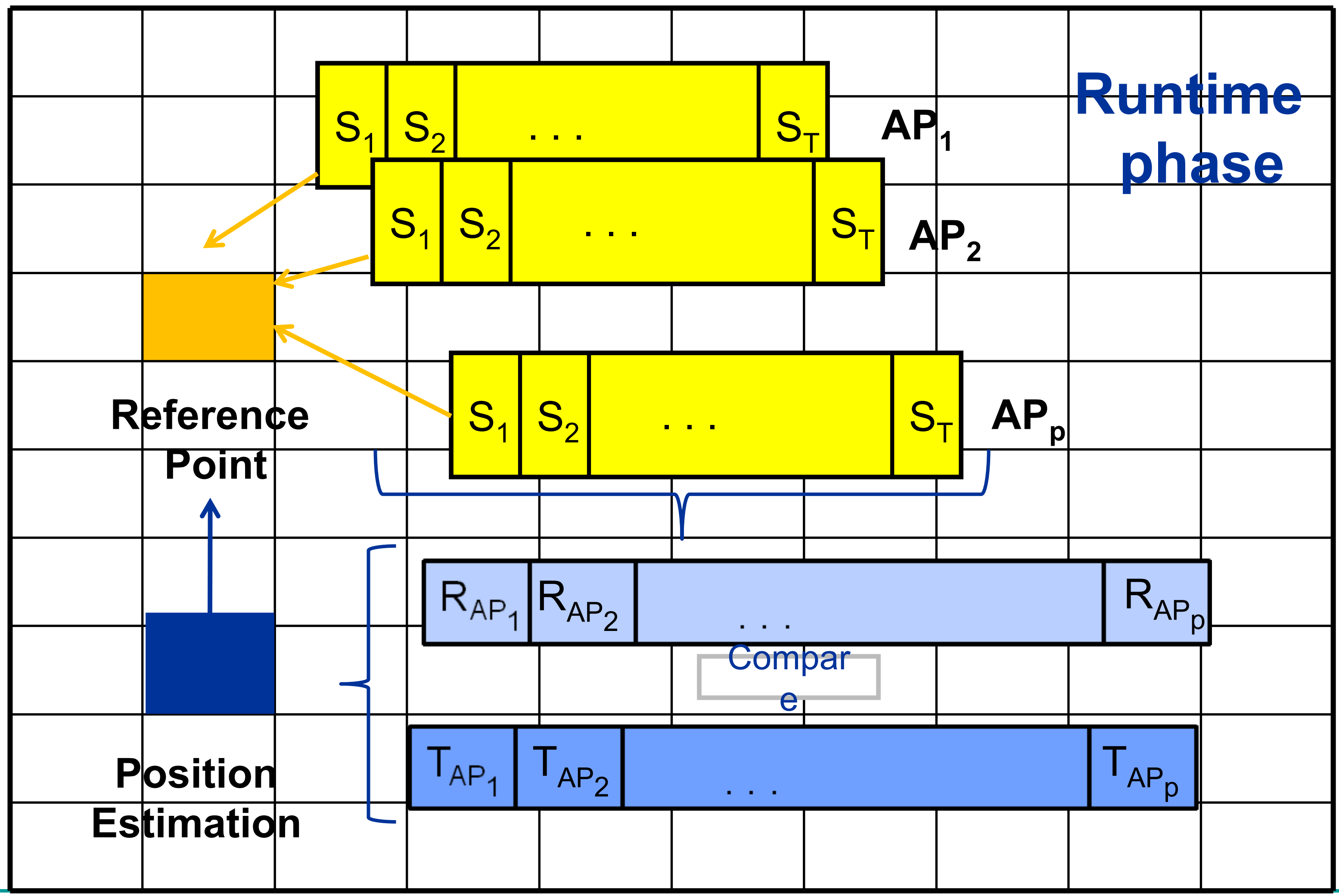
Disadvantages

- Large & complex hardware
- Difficult to acquire accurate angle measurements

Map-based Systems



Map-based Systems



Confidence Interval - Percentile [Papadopouli 07]

- Confidence Interval
  - Training phase: each AP creates a confidence interval from each cell
  - Runtime phase: A weight is computed for each cell to indicate the likelihood to be the estimated position
    Each AP assigns a vote for each cell
  Estimated position: Cell with the maximum weight

- Percentiles
  - Similar to confidence interval
  - Detailed information from the SS Distribution

K. Vandikas, L. Kriara, T. Papakonstantinou, A. Katranidou, H. Baltzakis and M. Papadopouli. Empirical-based analysis of a cooperative location-sensing system. In ACM Autonomics'07, Rome, Italy, October 28-30, 2007

K-nearest neighbour method [Radar00],[Rizos06]

- Training phase: Signatures from average SS of each AP
- Runtime phase
  - Computation of the generalized distance between runtime signature and training
  - Estimated position $\hat{c}$:

K=1

$$\hat{c} = \arg\min \left(\sum_{i=1}^{D} |y_r - y_c|^q \right)^{1/q}$$

 # Possible position

Runtime measurement vector

Training measurement vector

- q=1:Manhantan distance
- q=2: Euclidian distance
- Best performance with K=3 and K=4

1. P. V. Bahl and V. Padmanabhan. Radar: An in-building rf-based user location and tracking system. In IEEE Conference on Computer Communications (Infocom), Tel Aviv, Israel, Mar. 2000

2. B.Li,J.Salter,A.G.Dempster and C.Rizos. "Indoor positioning techniques based on wireless" lan.In Auswirelss Conference,2006.

Probabilistic approach [Wallach02],[Shankar03]

- Large variation of SS at each cell ⟹ use SS distribution to address the noisy wireless channel

- Training phase: for each fingerprint the frequency of each SS value is used to generate a probability distribution represented by a histogram
  - APs are independent
  - All positions are equally likely

- Runtime phase: MLE
- Estimated position: $\boxed{\hat{c} = \arg\max p(\mathbf{y} | c_i)}$
  - Area of confidence, set of cells c_i such that $\sum_i p(c_i) > P_{th}$.

1. A. M. Ladd, K. E. Bekris, G. Marceau, A. Rudys, L. E. Kavraki, and D. S. Wallach "Robotics-Based Location Sensing using Wireless Ethernet". In MOBICOM 2002
2. Youssef M, Agrawala A, Shankar A U. "WLAN location determination via clustering and probability distributions". Proc first IEEE Int'l Conf on Pervasive Computing and Communications. 2003

Multivariate Gaussian model [Papadopouli10]

- Information about the geometry/topology of the environment
  - Interdependencies among signal measurements from various APs - Multivariate Gaussian distribution: to construct a statistical signature ∀ cell

- Training phase: SS collection
- Runtime phase
  - Signature ∀cell from active APs
  - KLD distance estimation of the training and runtime signatures
- Estimated position: smallest KLD distance

D. Milioris, L. Kriara, A. Papakonstantinou, G. Tzagkarakis, P. Tsakalides and M. Papadopouli. "Empirical evaluation of siganl-strength fingerprint positioning in wireless LANs." In MSWiM 2010.

Related Work

- Systems based on IEEE 802.11

System	Method	Reference Points	# Access	Area (m^2)	Accuracy (m)
MvGs	Multivariate Gaussian model	108	13	84	1.60
Shankar03	Probabilistic Approach	110	4 per cell	1700	2.1
CLS	Percentile	108	13	84	2.65
	Confidence Interval	108	13	84	2.82
Radar	KNN	N/A	3	980	3
Chen09	Lateration technique	101	5	3400	~4

Localization Systems

Distance-Based Systems

- ☺ Easy adaptation
- ☹ Less accurate
- ☹ Path-loss models does not always hold
- ☹ Computationally inefficient

Map-based Model

- ☺ Fairly accurate
- ☺ Better capture of the physical space
- ☹ Training is time consuming
- ☹ Re-calibration to adapt environmental changes

Why bother with CS?

- Already existing methods
  - High computational complexity
  - Cost ineffective
- Reduce the number of measurements
- Improve the accuracy of the system
- Benefit of the sparsity of the problem

Proposed Framework

Proposed System[Nikitaki,Tsakalides10]

Key idea : Spatial Sparsity

Reconstruction of a sparse signal from a set of RSSI measurements by Compressive Sensing

- Grid-based representation of the physical space
- One central unit samples the reference signal

- No specific/ proprietary hardware, application to the mobile device $\Rightarrow$ Localization performed in the central unit

S. Nikitaki and P.Tsakalides, "Localization in Wireless Networks via Spatial Sparsity", Proc. Forty-Fourth Annual Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, November 2010.

Problem Setup

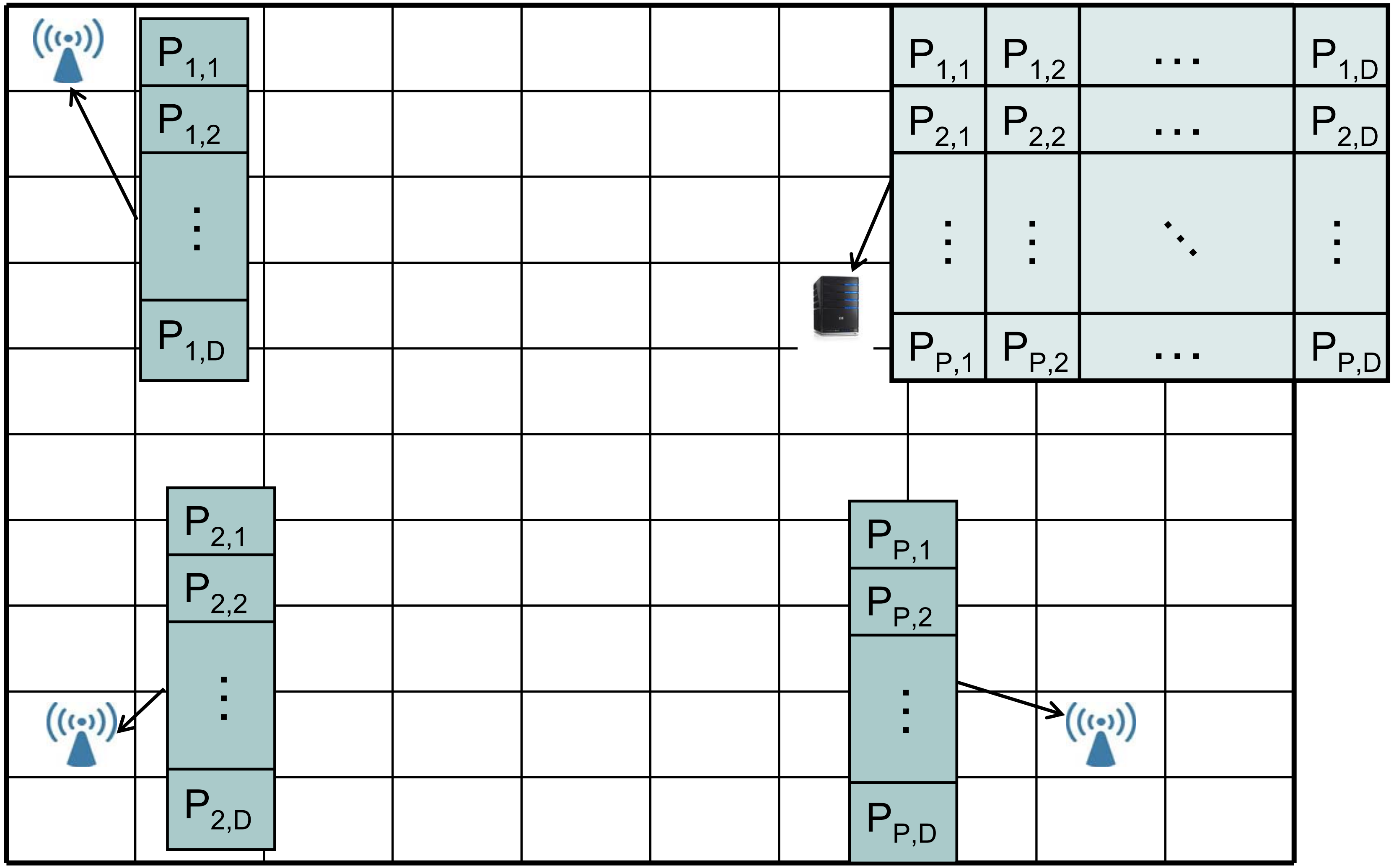
- Assume
  - 1 mobile node with unknown positions $n = [x, y]^T$
  - P APs with known positions
  - D reference points

- Set of possible positions $\Im = \{p_1, p_2, \ldots, p_D\}$
- Sparse vector $\mathbf{b} \in \mathbb{R}^D$, $\mathbf{b} = [0,1,0,\ldots,0]^T$

S. Nikitaki and P.Tsakalides, "Localization in Wireless Networks via Spatial Sparsity", Proc. Forty-Fourth Annual Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, November 2010.

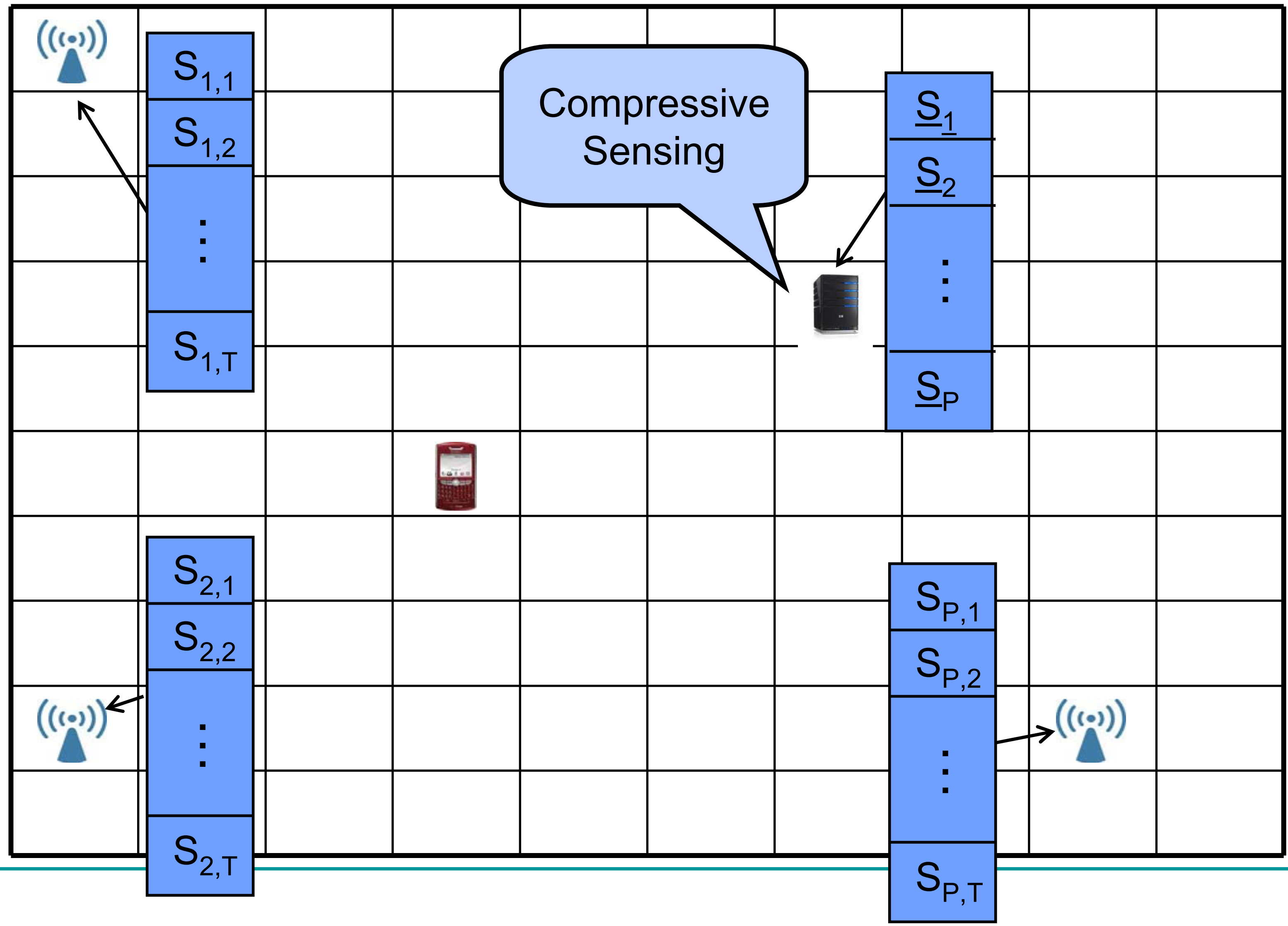
Algorithm

Training phase



Algorithm

Runtime phase



Problem Setup

i-th AP:

- Signature map of the
  - $\boldsymbol{\psi}_i = \left[P_{i,1}, P_{i,2}, \ldots, P_{i,D} \right]^T_{(1xD)}$
  - $P_{i,j}$: average RSSI value the AP receives from location j

- Average runtime value receives is:

$$z_i = \boldsymbol{\psi}_i \mathbf{b}$$

S. Nikitaki and P.Tsakalides, "Localization in Wireless Networks via Spatial Sparsity", Proc. Forty-Fourth Annual Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, November 2010.

Position estimation

Central unit

$$\boxed{\mathbf{z} = \boldsymbol{\Psi} \quad \mathbf{b}}$$

- Signal ensemble:

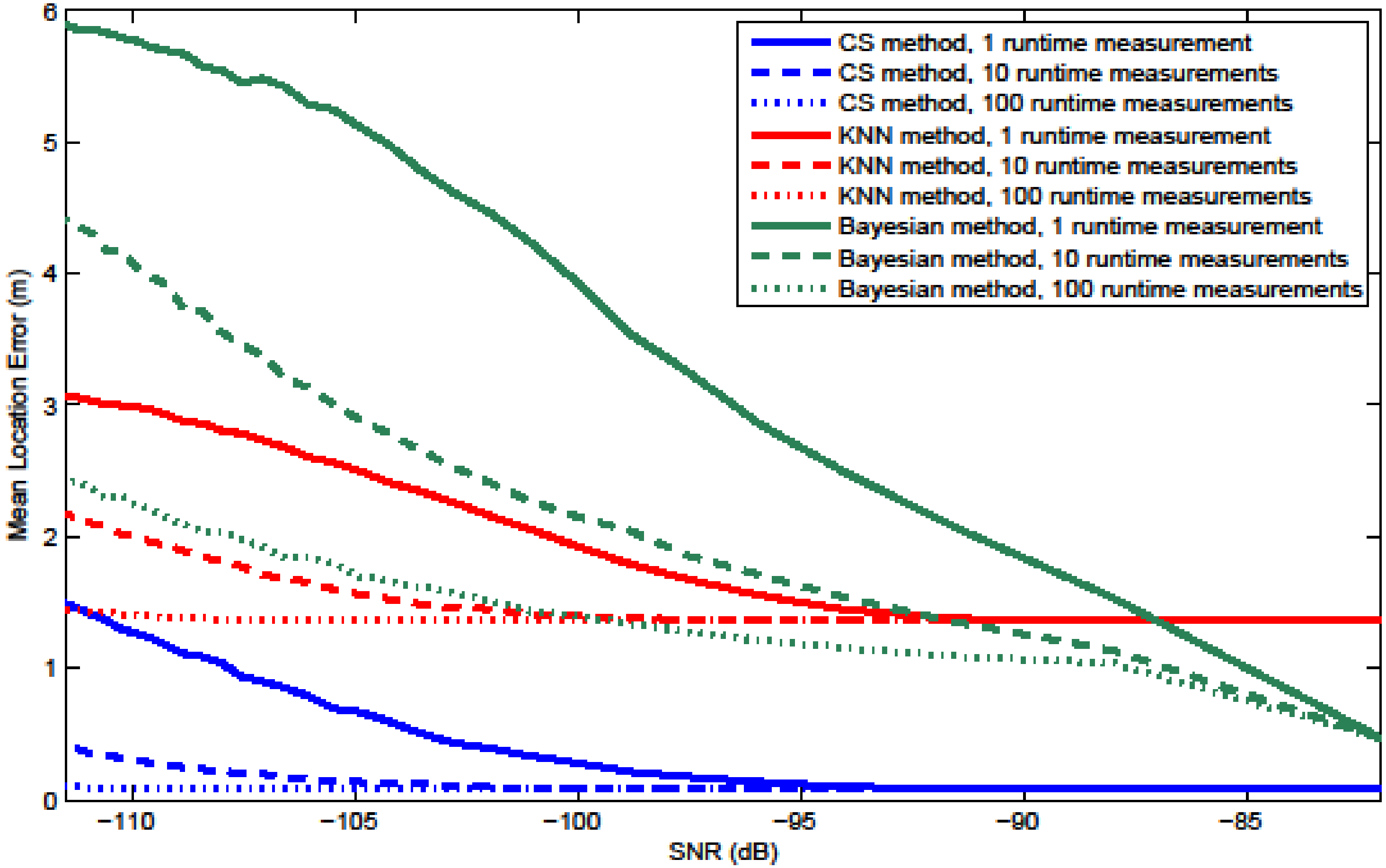
$$\begin{bmatrix} z_1 \\ \vdots \\ z_P \end{bmatrix}_{(Px1)} = \begin{bmatrix} \Psi_1 \\ \vdots \\ \Psi_P \end{bmatrix}_{(PxD)} \cdot \mathbf{b} \qquad (2)$$

- Reconstruction of sparse vector

$$\boxed{\hat{\mathbf{b}} = \arg\min \|\mathbf{b}\|_1 \quad s.t. \quad \mathbf{z} = \boldsymbol{\Psi}\mathbf{b}}$$

S. Nikitaki and P.Tsakalides, "Localization in Wireless Networks via Spatial Sparsity", Proc. Forty-Fourth Annual Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, November 2010.

Results(1)

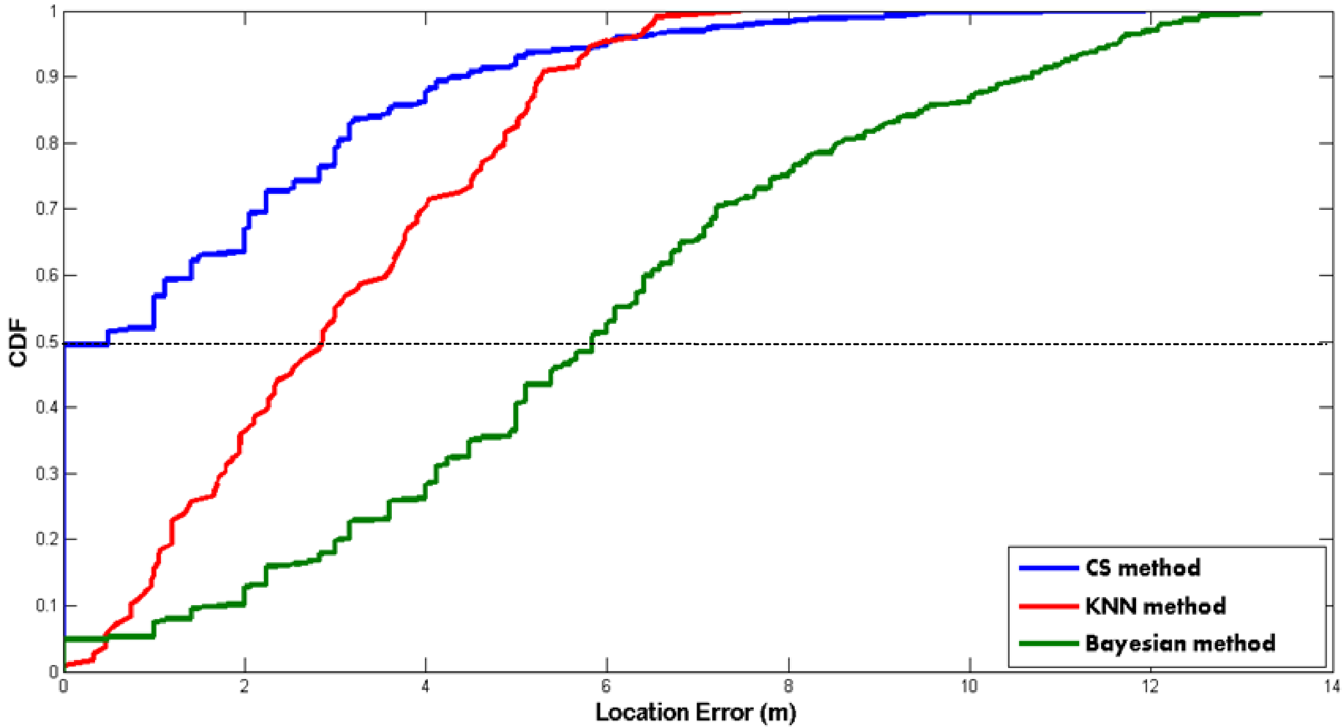


Simulate area of $84m^2$
grid size: 0.5m x 0.5m

CS method 50%(1.54m) improvement over KNN
CS method 74%(4.3m) improvement over Bayesian

S. Nikitaki and P.Tsakalides, "Localization in Wireless Networks via Spatial Sparsity", Proc. Forty-Fourth Annual Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, November 2010.

Results (2)



CDF curves (P|X|≤ x) for the three methods for SNR = -110 dB and 1 runtime RSSI measurement.

The location error of the CS based method is less than 0.5 m 50% of the time

Localization based on Jointly Compressed Sensing

Motivation

- Exploit the intra- & inter- signal correlation structures
- Reduce the amount of signal strength data needed for accurate positioning

Key idea

- Jointly sparsity of the signal ensemble – Distributed Compressed Sensing (DCS) theory

Distributed Compressed Sensing Theory

[Baron09]

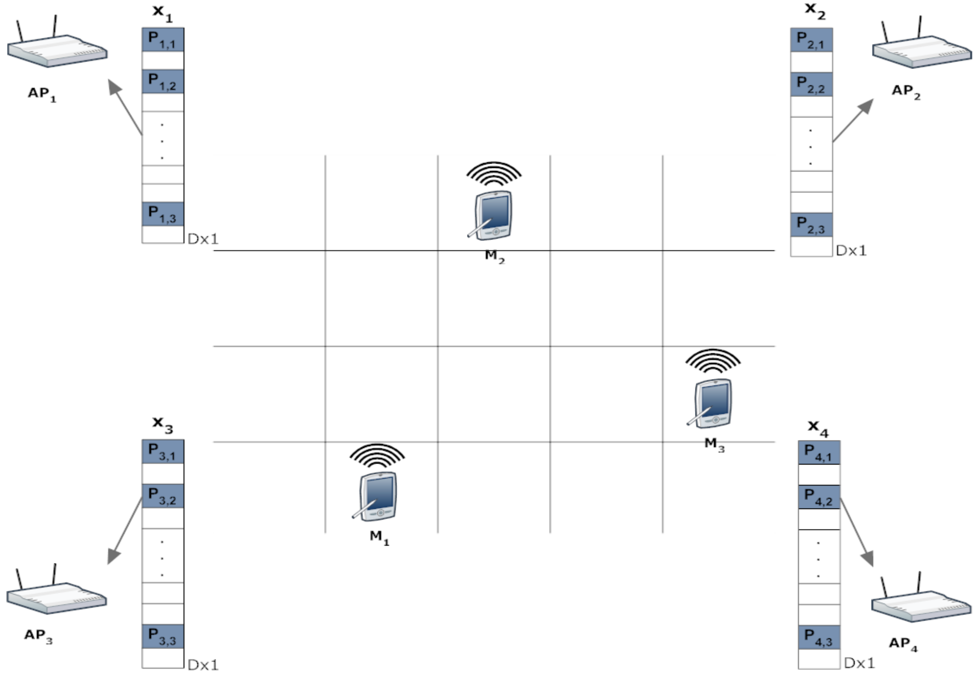
- Multiple collection points usually capture related phenomena

- A joint structure is expected for the signal ensemble + intra signal correlation between the individual measurements

- Jointly sparsity models
  1. Sparse common component + innovations
  2. Common sparse supports (indoor positioning case)
  3. Non sparse common component + sparse innovations

D. Baron, M.B. Wakin, M.F. Duarte, S. Sarvotham and R.B. Baraniuk, "Distributed Compressed Sensing," Computing Research Repository, 2009.

Indoor Localization
Common sparse support set



- The joint structure of the signal ensembles makes the DCS applicable for indoor positioning in WLANs
- Correlated signals among APs

Training Phase

- Each AP creates a signature map of the spatial space

$$\Psi_j = \begin{pmatrix} \boxed{\begin{matrix} P_{1,1,j} \\ P_{2,1,j} \\ \vdots \\ P_{N,1,j} \end{matrix}} & \begin{matrix} P_{1,2,j} \\ P_{2,2,j} \\ \vdots \\ P_{N,2,j} \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \ddots \\ \cdots \end{matrix} & \begin{matrix} P_{1,D,j} \\ P_{2,D,j} \\ \vdots \\ P_{N,D,j} \end{matrix} \end{pmatrix}$$

N received RSSI signals the j-th AP
receives from a node each potential position of the user

Runtime Phase(1)

- Each AP constructs a random measurement matrix $\mathbf{\Phi}_j \in \mathbb{R}^{M\times N}$.
- Measurement matrix: contains i.i.d. random variables from a Gaussian p.d.f with mean zero and variance 1/D (D is the length of the sparse vector). Universality property
- Runtime measurements:

$$\mathbf{y}_j = \mathbf{\Phi}_j \mathbf{x}_j = \mathbf{\Phi}_j \mathbf{\Psi}_j \mathbf{b} \overset{\mathbf{\Theta}_j = \mathbf{\Phi}_j \mathbf{\Psi}_j}{=} \mathbf{\Theta}_j \boxed{\mathbf{b}}$$

Common Support Set

for all APs

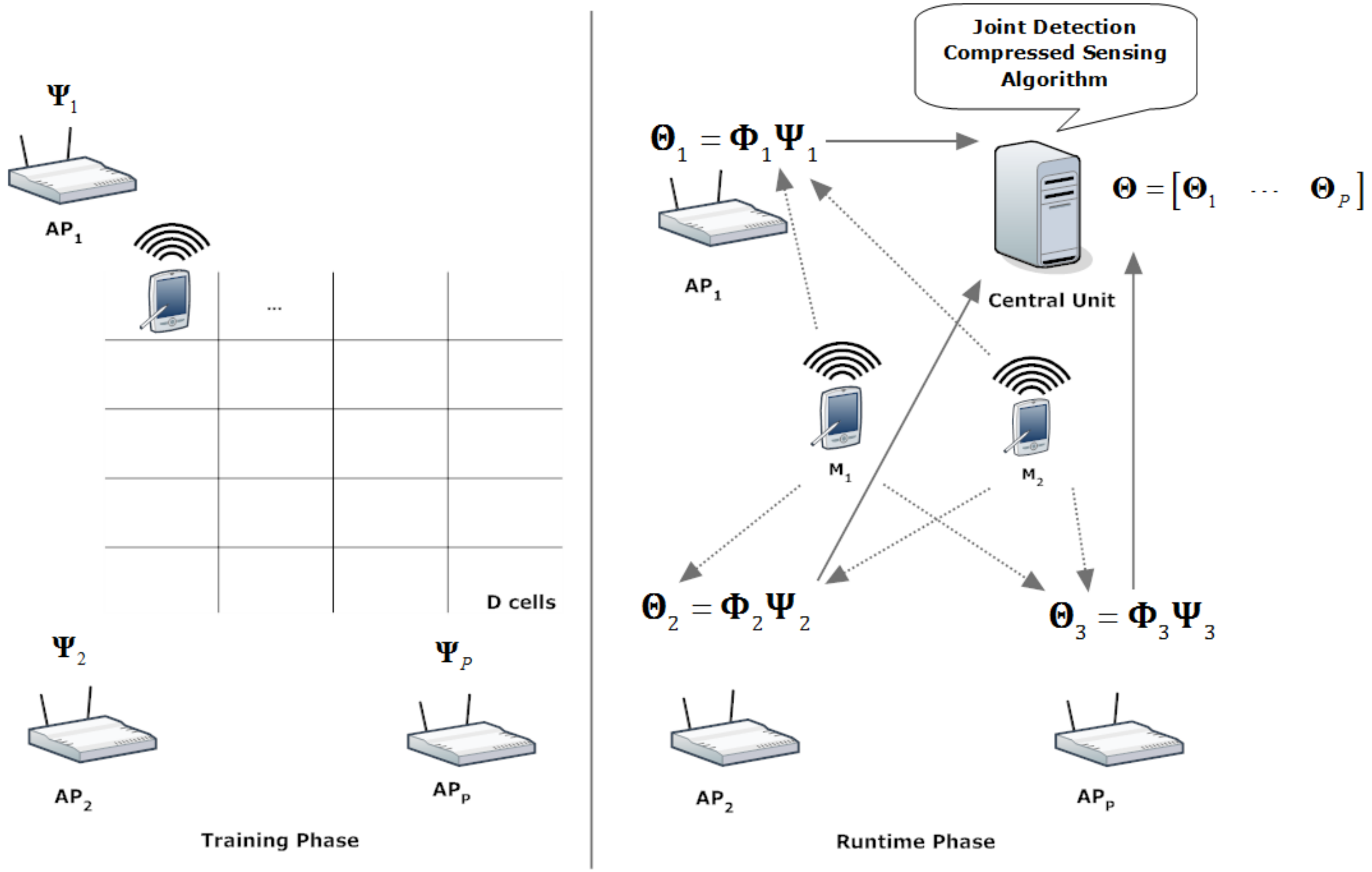
Runtime Phase
Central Unit

- Joint detection of position of the mobile user based on the common support set among the J APs

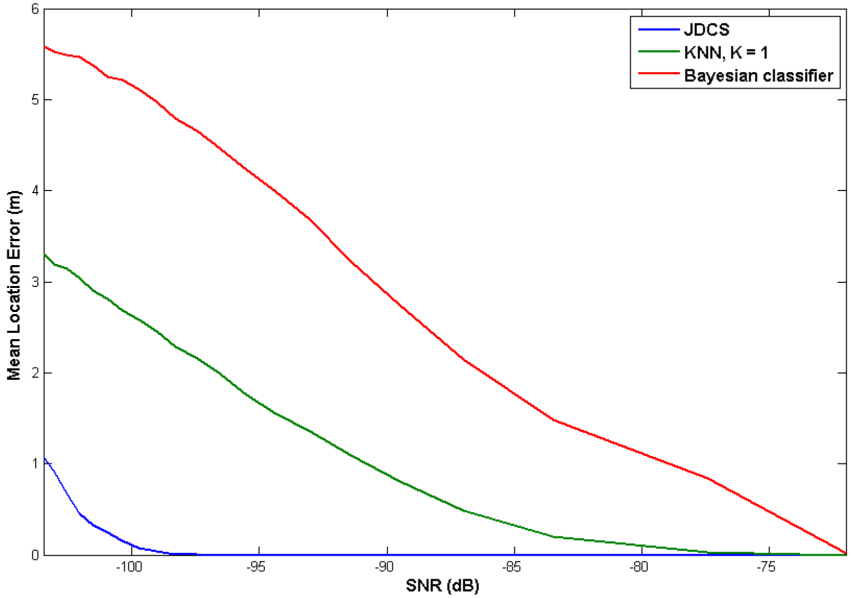
JDCS(Joint Detection Compressed Sensing)
Input: $\mathbf{\Theta}_j$, RSSI measurements $\mathbf{y}_j$ for all J APs
Outputs: Estimated Position

1. Estimate the sparse coefficient vector $\mathbf{b}_j$ for each AP
2. Individual coefficients vectors from each AP are added together
3. Detection of the position of the mobile user: largest coefficient of vector **b**.
4. Return the estimated position

Centralized CS-based Indoor Localization



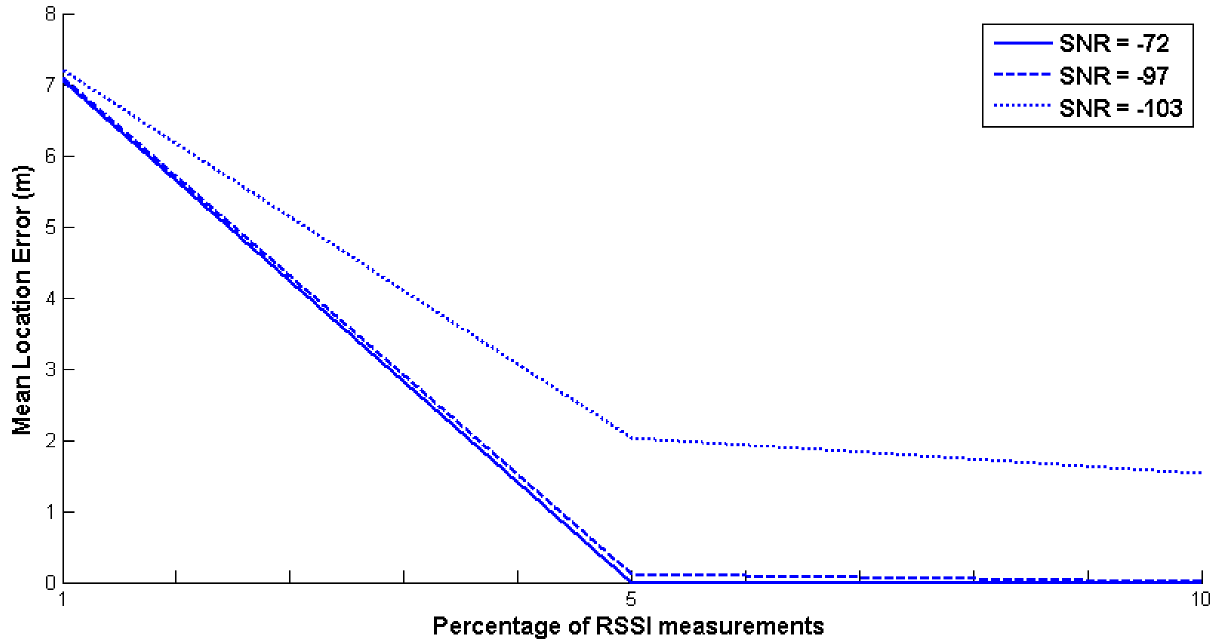
Experiments (1)



- Laboratory area of 7m x 12 m.
- Grid-based structure 0.5m x 0.5m.
- 109 different cells

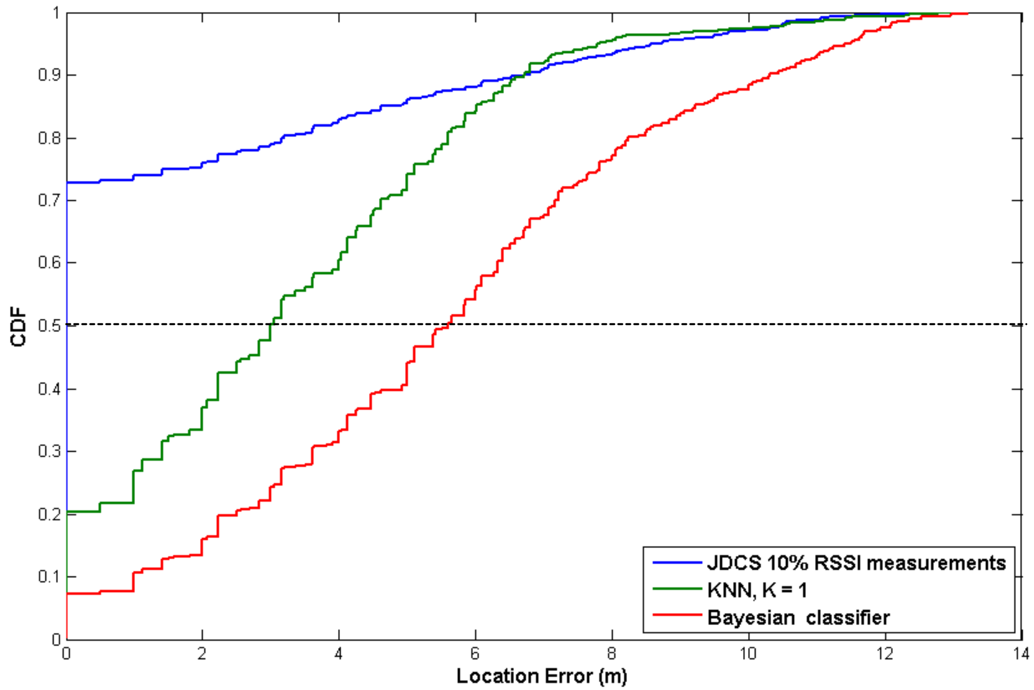
Mean location error vs. SNR for the KNN, Bayesian and the JDCS methods.
The JDCS algorithm has better performance in all cases.

Experiments (2)



- Mean location error vs. percentage of RSSI runtime measurements for the Joint Detection Compressed Sensing localization for different SNR values.
- Desired accuracy with only 5% to 10% of the runtime measurements.

Experiments (3)



- CDF curves (P|X|≤x) for the three methods for low SNR
- JDCS considers only 10% of the available runtime RSSI measurements
- Location error of the JDCS method is less than 0.1m, 73% of the time

Decentralized CS-based Indoor Localization

- Centralized scheme - single point failure
  - If central unit fails the system becomes inoperative & location sensing ineffective

- Decentralized scheme
  - Each AP independently computes the position of the mobile user
  - Spreading information through the network to converge in a global estimation via average consensus (Gossip Algorithms)

Key Idea

- Fully decentralized cooperative CS localization approach to distribute perform localization based on correlated data sampled by APs

Decentralized Localization via CS

- Training phase
  - Each AP j collects RSSI fingerprints in order to capture the physical space $\mathbf{\Psi}_j$
  - Each AP j creates a measurement matrix $\mathbf{\Phi}_j$
- Runtime phase
  - Each AP samples locally RSSI measurements to collect $\mathbf{y}_j$ and computes the correlations among the runtime measurements and the matrix $\mathbf{\Theta}_j$
  - Update the estimations of the APs via average consensus
  - Upon convergence, each AP obtains the global estimation of the mobile user's location

Multichannel Audio Coding using Sinusoidal Modelling and Compressed Sensing

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Overview

- Compressed Sensing (CS)
- Sinusoidal Modelling (SM)
- Multichannel Audio Coding using SM and CS
- Results – Bitrates, Listening Tests
- Conclusions

Sinusoidal Modelling

- Sinusoids plus Noise Model

$$\boldsymbol{x} = \boldsymbol{s} + \boldsymbol{n}$$

where $\boldsymbol{x}$ is the current frame of the audio signal
$\boldsymbol{s}$ is the sinusoidal part
$\boldsymbol{n}$ is the noise part (residual)

Sinusoidal Modelling

- Sinusoidal Part
  - consists of K sinusoids
  - each one characterised by

$$\{f_k, \alpha_k, \theta_k\}$$

where f_k is the frequency of the k-th sinusoid
α_k is its amplitude
θ_k is its phase

Sinusoidal Audio Coding Using Compressed Sensing

- Sinusoidal Model is currently coded parametrically

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  - Novel idea
  - “Shocking” idea

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Sinusoidal Audio Coding Using Compressed Sensing

- Sinusoidal part is sparse in frequency
- Force only FFT bin selection
  - use random samples as measurements
  - satisfies the incoherency requirement
  - less computationally complex than Gaussian measurements

CS Reconstruction

- Reconstruction done with a modified smoothed ℓ_0 algorithm
- Very efficient when combined with the FFT basis and random sampling

CS Reconstruction Issues

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CS Reconstruction Issues

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  - Reconstruction errors can be very bad in audio.
- Solution:
  - Detect frame reconstruction errors
  - Cyclic redundancy check (CRC)
  - What do you do when you detect a frame reconstruction error? $\rightarrow$ Operating Modes

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- Retransmission
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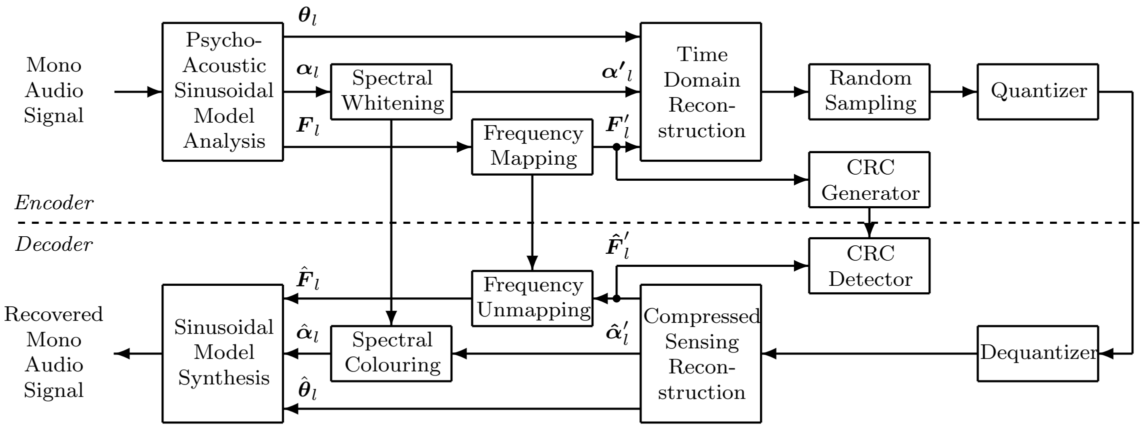
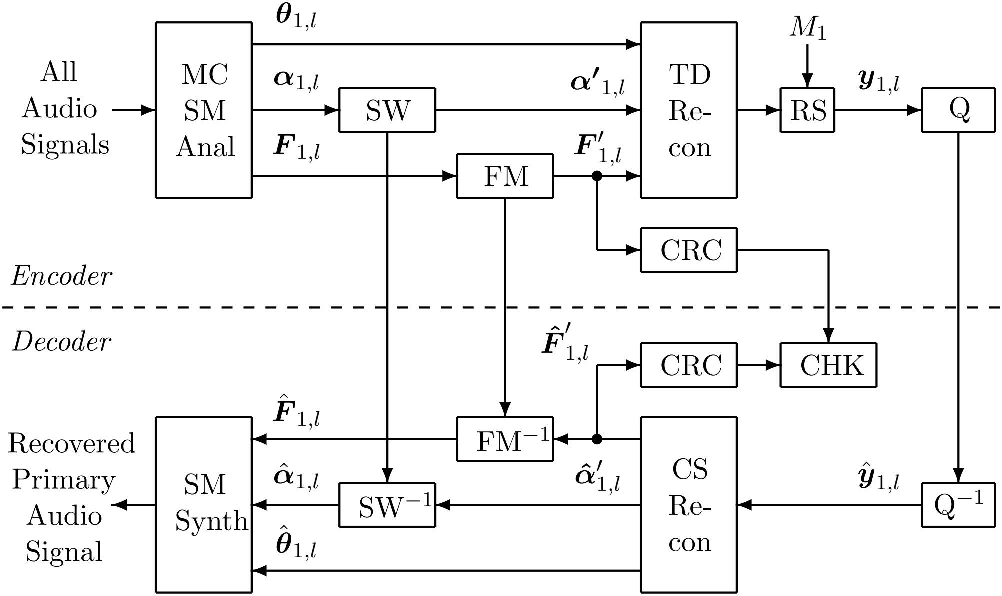
- Error-free
  - Reconstruct the frame *in the encoder*, to ensure that the reconstruction is error-free.

Single Channel Model

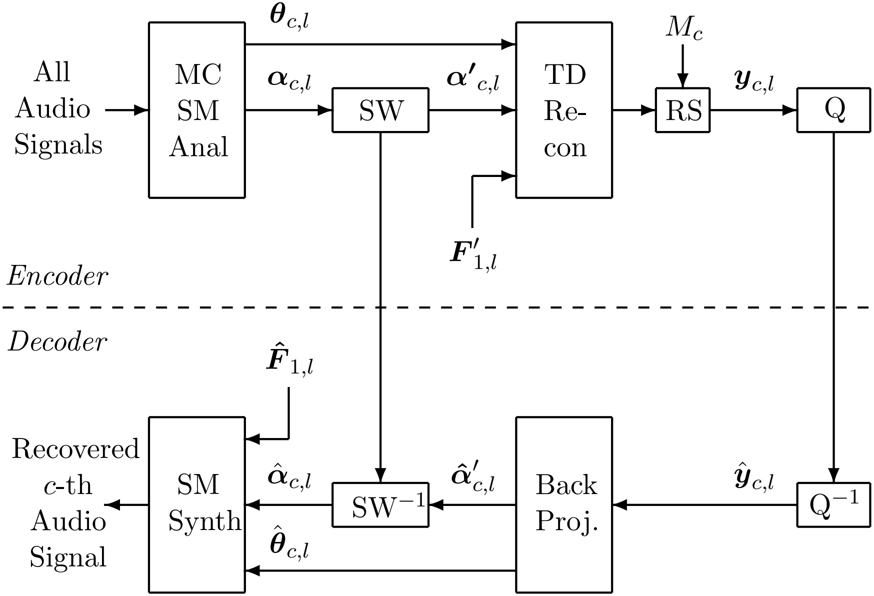


Multi Channel Model



(a) Primary Audio Channel

Multi Channel Model



(b) c-th Audio Channel

Listening Tests

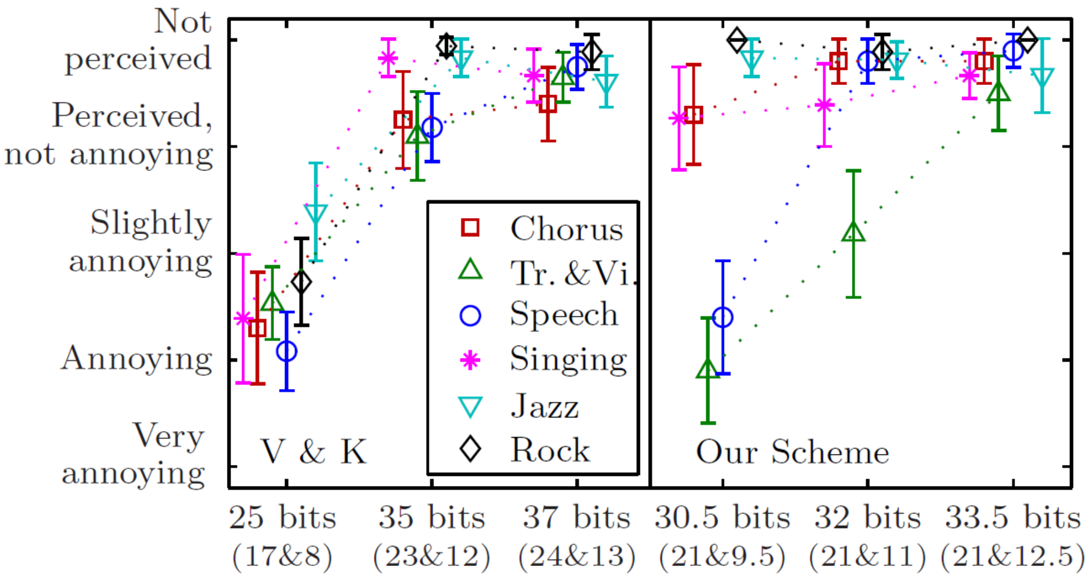
- All audio sampled at 22kHz
- 20ms frames with 50% overlapping
- 2048-point FFT
- 80 sinusoids with no residual
- 4-bit quantization of the random samples
- Audio signals are available at

  `http://www.ics.forth.gr/~mouchtar/cs4sm/`

Bitrates

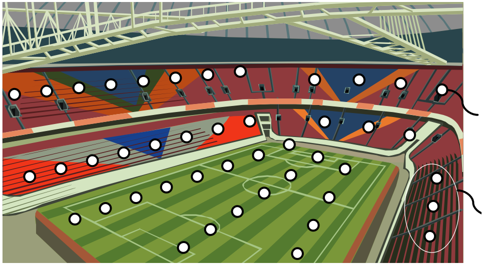
		raw	overhead			final	per
chan	M	bitrate	CRC	FM	SW	bitrate	sine
1	240	960	8	406	320	1694	21.2
2	210	840	0	0	160	1000	12.5
2	180	720	0	0	160	880	11.0
2	150	600	0	0	160	760	9.5

Listening Test Results for Stereo Signals

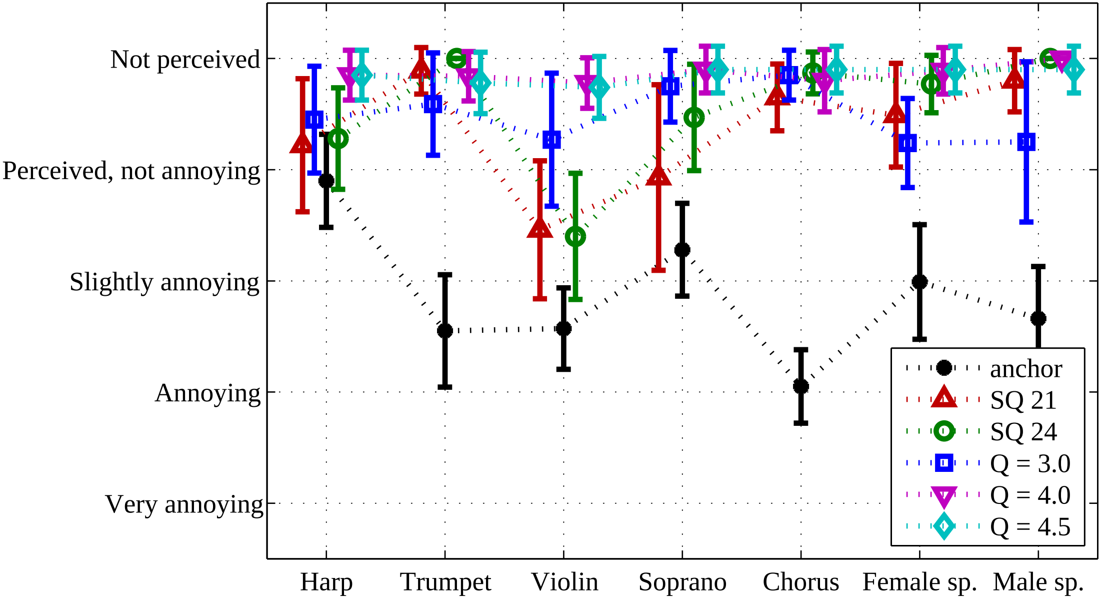


Conclusions

- Novel method of encoding a multichannel audio
- Performs very similarly to current state-of-the-art
- Relatively low-complexity reconstruction
  - suitable for capturing audio scenes with a WSN



Listening Test Results for Mono Signals ($K = 25$)



Listening Test Results for Stereo Signals ($K = 25$)