

# Provenance and Temporally Annotated Logic Programming

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**Abstract**— In this paper, we consider provenance and temporally annotated logic rules (pt-logic rules, for short), which are definite logic programming rules associated with the name of the source that they originate and the temporal interval during which they are valid. A collection of pt-logic rules form a provenance and temporally annotated logic program  $P$ , called pt-logic program, for short. We develop a model theory for  $P$  and define its maximally temporal entailments of the form  $A:\langle S, ti \rangle$ , indicating that atom  $A$  is derived from a set of sources  $S$  and holds at a maximal temporal interval  $ti$ , according to  $S$ . We define a consequence operator that derives exactly the maximally temporal entailments of  $P$  for a set of sources. We show that the complexity of the considered entailment is EXPTIME-complete.

**Keywords**- Annotated logic programming; provenance and temporal information; model theory; consequence operator.

## I. INTRODUCTION

Definite logic programming rules traditionally are not associated with the name of the source (provenance information) from which they originate and the temporal interval during which are valid. However, logic programming rules are usually derived from different sources that interact. Additionally logic programming rules may not always be valid, but be valid only for a specific temporal interval.

A temporal interval has the form  $[t, t']$ , where  $t, t'$  are time points and  $t \leq t'$ . In this paper, time points are years. However, this assumption can be generalized and we may assume any set of time points that can be mapped one-to-one to the set natural numbers. We consider definite logic programming rules associated with a source name and a validity temporal interval, called *provenance and temporally annotated logic rules*, or *pt-logic rules* for short.

We assume that *pt-logic* rules are applied similarly to definite logic programming rules but at each application new provenance and temporal information is derived for the derived atom  $A$ . In particular, derived atoms (*pt-atoms*) have the form  $A:\langle S, ti \rangle$ , where  $ti$  is the temporal interval at which  $A$  is valid, as derived from a set of sources  $S$ .

Obviously, if  $A:\langle S, ti \rangle$  is true then  $A:\langle S', ti \rangle$  is true, where  $S \subseteq S'$  and  $S'$  is a subset of a set of considered source names. Additionally, if  $A:\langle S, ti \rangle$  and  $A:\langle S', ti' \rangle$  are true, where temporal intervals  $ti, ti'$  are overlapping or consecutive then  $A:\langle S \cup S', ti'' \rangle$  is true, where  $ti''$  is the combination of  $ti$  and  $ti'$ .

A collection of *pt-logic* rules form a *provenance and temporally annotated logic program*, called *pt-logic program*, for short. The models of *pt-logic* programs  $P$  are defined, as well as, the simple entailments and temporally maximal entailments of  $P$ . We show that  $P$  has a minimal model containing exactly the temporally maximal entailments of  $P$ .

A set of three operators are defined which are applied on *pt-atoms* such that the closure of their composition derives the minimal model of  $P$ . We define a query language that consists of simple and composite queries, querying provenance and temporal information of derived *pt-atoms* based on certain conditions.

We show that the complexity of simple entailment of a *pt-atom* or temporally maximal entailment of a *pt-atom* from  $P$  is EXPTIME-complete.

The rest of the paper is organized as follows: In Section II, we define *pt-logic* rules, *pt-logic* programs, *pt-atoms*, and the instantiation of a *pt-logic* program. In Section III, we provide a model theory for *pt-logic* programs  $P$  and the minimal model of  $P$  is defined.

In Section IV, we define a consequence operator deriving the minimal model of  $P$ . In Section V, a query language for *pt-logic* programs is defined. Section VI contains related work. Finally, Section VII contains directions for future work.

## II. PROVENANCE AND TEMPORALLY ANNOTATED LOGIC PROGRAMS

In this Section, we define provenance and temporally annotated logic programs  $P$ . Additionally, we define the rules based on which the models of  $P$  are defined. We consider a set of variables  $Var$ , all preceded by the question mark symbol “?”.

**Definition 1.** A *provenance and temporally annotated logic rule*, called *pt-logic rule* for short, is a definite logic programming rule  $r$  without function symbols, associated with a source name  $nam$  and a temporal interval  $ti$ . In particular, it has the form  $\langle nam, ti \rangle: r$ .

**Definition 2.** A provenance and temporally annotated logic program  $P$ , called *pt-logic program* for short, is a set of *pt-logic* rules.

**Example 1.** The following is a *pt-logic* program  $P$ .

<Person, [1990,1994]>: *has\_job(Mary,Hairdresser)*.  
 <Person, [1995,2002]>: *has\_job(Mary,Secretaty)*.  
 <Person, [2006,2009]>: *has\_job(Mary,Hairdresser)*.  
 <Person,[1999,2002]>: *extra\_vacation(Mary)*.  
 <Person, [2001,2003]>: *has\_job(Peter, Garbage\_collector)*.  
 <Person, [2005,2008]>: *has\_job(Peter,Bulider)*.  
 <Job, [1980,1992]>: *heavy\_job(Hairdresser)*.  
 <Job, [1980, 2001]>: *heavy\_job(Garbage\_collector)*.  
 <Job, [2006, 2012]>: *heavy\_job(Builder)*.  
 <Vacation, [1988,2012]: *extra\_vacation(?x) ← has\_job(?x,?y), heavy\_job(?y)*.

Predicate *has\_job* indicates the job of a person. Predicate *heavy\_job* indicates that a job is heavy and unhealthy. Finally, predicate *extra\_vacation* indicates that a person gets for some reason extra vacation days.

*Convention:* In the following by  $P$ , we will denote a  $pt$ -logic program.

We define  $Names_P$  to be the set of source names appearing in  $P$ . Additionally, we define  $definite(P)$  to be the definite logic program derived from  $P$  after ignoring the provenance and temporal annotations of  $P$ . Further, we denote by  $t_P^{\min}$  the minimum temporal point appearing in  $P$  and by  $t_P^{\max}$  the maximum temporal point appearing in  $P$ .

**Definition 3.** A  $pt$ -atom of  $P$  is an atom  $A$  built using predicates and constants appearing in  $definite(P)$  and associated with (i) a set of source names that is a subset of  $Names_P$  or a provenance variable  $S$  and (ii) a temporal interval within  $[t_P^{\min}, t_P^{\max}]$  or a temporal variable  $ti$ . In particular, it has the form  $A: \langle S, ti \rangle$ .

Below, we define the set of rules  $[P]$ , built from ground  $pt$ -atoms of  $P$ , based on which the models of  $P$  are defined.

**Definition 4.** Let  $r \in P$  be a  $pt$ -rule  $\langle nam, ti \rangle: A_0 \leftarrow A_1, \dots, A_n$ . We define  $[r]_P$  to be the rule  $B_0: \langle S_0, ti_0 \rangle \leftarrow B_1: \langle S_1, ti_1 \rangle, \dots, B_n: \langle S_n, ti_n \rangle$ , where (i)  $B_0$  is derived from  $A_0$  by replacing all the variables in  $A_0$  by constants in  $definite(P)$ , (ii)  $S_i \subseteq Names_P$  s.t.  $S_0$  is the union of the sets  $S_1, \dots, S_n$  and  $\{nam\}$ , and (iii)  $ti_i$  is a temporal interval within  $[t_P^{\min}, t_P^{\max}]$  s.t.  $ti_0$  is the intersection of the temporal intervals  $ti_1, \dots, ti_n$  and  $ti$ . We define the *instantiation* of  $P$ , as follows:

$$[P] = \bigvee_{r \in P} [r]_P .$$

### III. MODEL THEORY OF PT-LOGIC PROGRAMS

In this Section, we present the model theory of  $pt$ -logic programs and entailment of  $pt$ -atoms.

First, we present three auxiliary definitions. We say that two temporal intervals  $ti$  and  $ti'$  are *overlapping* if they have at least one common time point. We say that two temporal intervals  $[t_1, t_2]$  and  $[t_3, t_4]$  are *consecutive* if  $t_3 = t_2 + 1$ . We say that a temporal interval  $[t_1, t_2]$  is *included* in a temporal interval  $[t_3, t_4]$  if  $t_3 \leq t_1$  and  $t_2 \leq t_4$ .

**Definition 4.** An *interpretation*  $I$  of  $P$  is a set of ground  $pt$ -atoms of  $P$  s.t. (i) if  $A: \langle S, ti \rangle, A': \langle S, ti' \rangle \in I$  then the

temporal intervals  $ti, ti'$  are neither overlapping nor consecutive and (ii) if  $A: \langle S, ti \rangle \in I$  then, for all  $S'$  s.t.  $S \subseteq S' \subseteq Names_P$ , there exists  $A': \langle S', ti' \rangle \in I$  s.t.  $ti$  is included in  $ti'$ .

Below, we define entailment of a  $pt$ -atom for an interpretation  $I$  of  $P$ .

**Definition 5.** Let  $I$  be an interpretation of  $P$  and let  $A: \langle S, ti \rangle$  be a ground  $pt$ -atom of  $P$ . We say that  $I$  (simply) *entails*  $A: \langle S, ti \rangle$ , denoted by  $I \models A: \langle S, ti \rangle$ , if there exists an  $A': \langle S, ti' \rangle \in I$  s.t.  $ti$  is included in  $ti'$ .

Additionally, we define an ordering on the interpretations of  $P$ . Let  $I, I'$  be interpretations of  $P$ . We say that  $I \leq I'$  iff for each  $A: \langle S, ti \rangle \in I$  there exists an  $A': \langle S, ti' \rangle \in I'$  s.t.  $ti$  is included in  $ti'$ . It is easy to see that if  $I \leq I'$  and  $I' \leq I$  then  $I = I'$ .

Below, we define the models if a  $pt$ -logic program  $P$ ,

**Definition 6.** Let  $M$  be an interpretation of  $P$ . We say that  $M$  is a *model* of  $P$  if for each  $r = A_0: \langle S_0, ti_0 \rangle \leftarrow A_1: \langle S_1, ti_1 \rangle, \dots, A_n: \langle S_n, ti_n \rangle \in [P]$ , it holds that: if  $M \models A_i: \langle S_i, ti_i \rangle$ , for all  $i = 1, \dots, n$ , then  $M \models A_0: \langle S_0, ti_0 \rangle$ . We denote the set of models of  $P$  by  $\mathcal{M}_P$ .

Let  $ti = [t, t']$  be a temporal interval. We define  $start(ti) = t$  and  $end(ti) = t'$ .

Below, we define simple entailment of a ground  $pt$ -atom from a  $pt$ -logic program  $P$ , as well as maximally temporal entailment.

**Definition 7.** We say that  $P$  (simply) *entails* a ground  $pt$ -atom  $A: \langle S, ti \rangle$ , denoted by  $P \models A: \langle S, ti \rangle$ , iff for each  $M \in \mathcal{M}_P, M \models A: \langle S, ti \rangle$ . We say that  $P$  *maximally temporally entails* a ground  $pt$ -atom  $A: \langle S, ti \rangle$ , denoted by  $P \models^{\max} A: \langle S, ti \rangle$  iff (i)  $P \models A: \langle S, ti \rangle$ , (ii) there exists  $M \in \mathcal{M}_P$  s.t.  $M$  does not entail  $A: \langle S, [end(ti)+1, end(ti)+1] \rangle$ , and (iii) there exists  $M \in \mathcal{M}_P$  s.t.  $M$  does not entail  $A: \langle S, [start(ti)-1, start(ti)-1] \rangle$ .

Below, we define the minimal model of a  $pt$ -logic program  $P$  w.r.t.  $\leq$ .

**Definition 8.** Let  $M_1, \dots, M_n$  be all the models of  $P$ , we define the interpretation  $M_{min} = \{ A: \langle S, ti \rangle \mid \text{there exist } A: \langle S, ti_i \rangle \in M_i, \text{ for all } i = 1, \dots, n, \text{ and } ti \text{ is the intersection of } ti_1, \dots, ti_n \}$ .

Below, we show that  $M_{min}$  is an interpretation of  $P$ .

**Proposition 1.**  $M_{min}$  is an interpretation of  $P$ .

**Proof:** Obviously,  $M_{min}$  is a set of  $pt$ -atoms of  $P$ . Let  $M_1, \dots, M_n$  be all the models of  $P$ . If  $A: \langle S, ti \rangle \in M_{min}$  then there exists  $A': \langle S, ti_i \rangle \in M_i$ , for all  $i = 1, \dots, n$ , and  $ti$  is the intersection of  $ti_1, \dots, ti_n$ . Since  $M_1, \dots, M_n$  interpretations of  $P$ , it holds that if  $A: \langle S, ti \rangle \in M_{min}$  then, for all  $S'$  s.t.  $S \subseteq S' \subseteq Names_P$ , there exists  $A': \langle S', ti' \rangle \in M_{min}$  s.t.  $ti$  is included in  $ti'$ . Additionally, since  $M_1, \dots, M_n$  interpretations of  $P$ , there are no  $A: \langle S, ti \rangle, A': \langle S, ti' \rangle \in M_{min}$  s.t. temporal intervals  $ti, ti'$  are overlapping or consecutive.

Obviously,  $M_{min} \leq M$ , for all  $M \in \mathcal{M}_P$ .

Below, we show that  $M_{min}$  is a minimal model of  $P$ .

**Proposition 2.**  $M_{min}$  is a minimal model of  $P$ .

**Proof:** Let  $M_1, \dots, M_k$  be all the models of  $P$  and let  $r = A_0: \langle S_0, ti_0 \rangle \leftarrow A_1: \langle S_1, ti_1 \rangle, \dots, A_n: \langle S_n, ti_n \rangle \in [P]$ . Assume that  $M_{min} \models A_i: \langle S_i, ti_i \rangle$ , for all  $i=1, \dots, n$ . Then,  $M_j \models A_i: \langle S_i, ti_i \rangle$ , for all  $j=1, \dots, k$  and  $i=1, \dots, n$ . Thus,  $M_j \models A_0: \langle S_0, ti_0 \rangle$ , for all  $j=1, \dots, k$ . Therefore,  $M_{min} \models A_0: \langle S_0, ti_0 \rangle$ .

In Proposition 3, below, we show that  $P$  and  $M_{min}$  have the same simple entailments.

**Proposition 3.** Let  $A: \langle S, ti \rangle$  be a ground  $pt$ -atom. It holds that:  $P \models A: \langle S, ti \rangle$  iff  $M_{min} \models A: \langle S, ti \rangle$ .

**Proof:**

$\Rightarrow$ ) Let  $P \models A: \langle S, ti \rangle$ . Then, for each  $M \in \mathcal{M}_P$ ,  $M \models A: \langle S, ti \rangle$ . This means that for each  $M \in \mathcal{M}_P$ , there exists  $A: \langle S, ti' \rangle \in M$  s.t.  $ti$  is included in  $ti'$ . Thus, by the definition of  $M_{min}$ , there exists  $A: \langle S, ti'' \rangle \in M_{min}$  s.t.  $ti$  is included in  $ti''$ . Thus,  $M_{min} \models A: \langle S, ti \rangle$ .

$\Leftarrow$ ) Assume that  $M_{min} \models A: \langle S, ti \rangle$ . Then, there exists  $A: \langle S, ti' \rangle \in M_{min}$  s.t.  $ti$  is included in  $ti'$ . Thus, by the definition of  $M_{min}$ , for all  $M \in \mathcal{M}_P$ , there is  $A: \langle S, ti'' \rangle \in M$  s.t.  $ti$  is included in  $ti''$ . Therefore,  $P \models A: \langle S, ti \rangle$ .

In Proposition 4, below, we show that  $M_{min}$  contains exactly the maximally temporal entailments of  $P$ .

**Proposition 4.** Let  $A: \langle S, ti \rangle$  be a ground  $pt$ -atom. It holds that:  $P \models^{\max} A: \langle S, ti \rangle$  iff  $A: \langle S, ti \rangle \in M_{min}$ .

**Proof:**

$\Rightarrow$ ) Let  $P \models^{\max} A: \langle S, ti \rangle$ . This means that for each  $M \in \mathcal{M}_P$ , there exists  $A: \langle S, ti' \rangle \in M$  s.t.  $ti$  is included in  $ti'$ . Additionally, there exists  $M \in \mathcal{M}_P$ , that does not contain  $A: \langle S, ti' \rangle \in M$  s.t. time point  $end(ti)+1$  is contained in  $ti'$  and there exists  $M \in \mathcal{M}_P$ , that does not contain  $A: \langle S, ti' \rangle \in M$  s.t. time point  $start(ti)-1$  is contained in  $ti'$ . Therefore,  $A: \langle S, ti \rangle \in M_{min}$ .

$\Leftarrow$ ) Assume that  $A: \langle S, ti \rangle \in M_{min}$ . This means that for each  $M \in \mathcal{M}_P$ , there exists  $A: \langle S, ti' \rangle \in M$  s.t.  $ti$  is included in  $ti'$ . Additionally, there exists  $M \in \mathcal{M}_P$ , that does not contain  $A: \langle S, ti' \rangle \in M$  s.t. time point  $end(ti)+1$  is contained in  $ti'$  and there exists  $M \in \mathcal{M}_P$ , that does not contain  $A: \langle S, ti' \rangle \in M$  s.t. time point  $start(ti)-1$  is contained in  $ti'$ . Therefore,  $P \models^{\max} A: \langle S, ti \rangle$ .

#### IV. COMPUTING THE MINIMAL MODEL OF A PT-PROGRAM

In Section, we provide three operators s.t. the closure of their composition provides the minimal model of a  $pt$ -logic program  $P$ .

We denote by  $Q_P$  the ground  $pt$ -atoms of  $P$ . We define the operator  $W_P$  from the powerset of  $Q_P$  to the powerset  $Q_P$ , as follows:

$$W_P(Q) = \{ A: \langle S, ti \rangle \mid \text{it exists } A: \langle S, ti \rangle \leftarrow A_1: \langle S_1, ti_1 \rangle, \dots, A_n: \langle S_n, ti_n \rangle \in [P] \text{ s.t. } \langle S_i, ti_i \rangle \in Q, \text{ for all } i=1, \dots, n \}$$

We now define the operator  $Z_P$  from the powerset of  $Q_P$  to the powerset of  $Q_P$ , as follows:

$$Z_P(Q) = \{ A: \langle S, ti \rangle \mid \text{it exists } A: \langle S', ti' \rangle \in Q \text{ and } S' \subseteq S \subseteq \text{Names}_P \}$$

Before, we define the third operator, we provide a few definitions. Let  $Q \subseteq Q_P$ . We define  $intervals(Q, S, A) = \{ ti \mid \text{it exists } A: \langle S, ti \rangle \in Q \}$ . Let  $TI$  be a set of temporal intervals. The *maximal subset* of  $TI$  w.r.t. a temporal interval  $ti \in TI$  is a set  $B$  that (i) contains  $ti$  and (ii) if  $ti' \in B$  and there exists a  $ti'' \in TI$  s.t.  $ti'$  and  $ti''$  are overlapping or consecutive then  $ti'' \in B$ . The *maximal interval* of  $TI$  w.r.t. a temporal interval  $ti \in TI$ , denoted by  $max\_interval(TI, ti)$ , is the temporal interval  $[t, t']$  formed by the minimum time point  $t$  and maximum time point  $t'$  appearing in a maximal subset of  $TI$  w.r.t.  $ti$ .

We are now ready to define the operator  $R_P$  from the powerset of  $Q_P$  to the powerset of  $Q_P$  as follows:

$$R_P(Q) = \{ A: \langle S, ti \rangle \mid \text{it exists } A: \langle S, ti' \rangle \in Q \text{ and } ti = max\_interval(intervals(Q, S, A), ti') \}$$

Finally, we define the consequence operator  $T_P$  from the powerset of  $Q_P$  to the powerset of  $Q_P$  as the composition of  $W_P$ ,  $Z_P$ , and  $R_P$ . In particular,  $T_P(Q) = R_P(Z_P(W_P(Q)))$ , for  $Q \subseteq Q_P$ .

It can be easily seen that the consequence operator  $T_P$  is monotonic with respect to  $\subseteq$ . That is, if  $Q, Q' \subseteq Q_P$  s.t.  $Q \subseteq Q'$  then  $T_P(Q) \subseteq T_P(Q')$ . We will show that the closure of operator  $T_P$  coincides with  $M_{min}$ .

**Proposition 4.** Let  $M$  be an interpretation of  $P$ . It holds that  $M$  is a model of  $P$  iff  $T_P(M) \leq M$ .

**Proof:**

$\Rightarrow$ ) Let  $M$  be a model of  $P$ . Consider the rules  $A: \langle S, ti \rangle \leftarrow A_1: \langle S_1, ti_1 \rangle, \dots, A_n: \langle S_n, ti_n \rangle \in [P]$  s.t.  $A_i: \langle S_i, ti_i \rangle \in M$ , for all  $i=1, \dots, n$ . Then,  $M \models A: \langle S, ti \rangle$ . Since  $M$  is an interpretation of  $P$ ,  $M$  satisfies each  $pt$ -atom in  $T_P(M)$ . Thus,  $T_P(M) \leq M$ .

$\Leftarrow$ ) Assume that  $T_P(M) \leq M$ . We need to show that if  $A: \langle S, ti \rangle \leftarrow A_1: \langle S_1, ti_1 \rangle, \dots, A_n: \langle S_n, ti_n \rangle \in [P]$  s.t.  $M \models A_i: \langle S_i, ti_i \rangle$ , for all  $i=1, \dots, n$ , then  $M \models A: \langle S, ti \rangle$ . Note that there exists  $A: \langle S, ti' \rangle \leftarrow A_1: \langle S_1, ti'_1 \rangle, \dots, A_n: \langle S_n, ti'_n \rangle \in [P]$  s.t.  $A_i: \langle S_i, ti'_i \rangle \in M$ , for all  $i=1, \dots, n$ . Note that  $T_P(M)$  is an interpretation of  $P$  and that  $T_P(M) \models A: \langle S, ti' \rangle$ . Since  $T_P(M) \leq M$ , it follows that  $M \models A: \langle S, ti' \rangle$ . Now, since  $ti$  is included in  $ti'$ , it follows that  $M \models A: \langle S, ti \rangle$ . Thus,  $M$  is a model of  $P$ .

Below, we show that  $M_{min}$  can be computed as the closure of the  $T_P$  operator.

**Proposition 5.**  $M_{min} = T_P^{\uparrow \omega}(\{\})$ .

**Proof:** From Proposition 4, it follows that  $M_{min} = minimal_{\leq} \{ M \mid M \text{ is an interpretation of } P \text{ and } T_P(M) \leq M \}$ . Since  $M_{min}$  is a model of  $P$ , it follows from Proposition 4 that  $T_P(M_{min}) \leq M_{min}$ . Further note that since the operator  $T_P$  is monotonic, it follows that  $T_P(T_P(M_{min})) \leq T_P(M_{min})$ . Additionally,  $T_P(M_{min})$  is an interpretation of  $P$ . Thus,  $M_{min} \leq T_P(M_{min})$ . Therefore,  $T_P(M_{min}) = M_{min}$ . Thus,  $M_{min} = T_P^{\uparrow \omega}(\{\})$ . Note that  $T_P^{\uparrow \omega} \subseteq Q_P$ .

**Proposition 6.** Simple and maximally temporal entailment of a ground  $pt$ -atom from a  $pt$ -logic program  $P$  is EXPTIME-complete w.r.t. the size of  $P$ .

**Proof: Membership)** Note that the computation of  $M_{min}=T_P^{1\omega}(\{\})$ , involves the application of at most an exponential number of rules applied at most exponential number of times. Obviously, the computation of  $max\_interval(intervals(Q,S,A),ti')$  is polynomial w.r.t. the size of  $Q$ . Therefore, the computation of  $M_{min}$  is in EXPTIME.

**Hardness)** It follows directly by the fact that datalog is program-complete for EXPTIME [1].

## V. A QUERY LANGUAGE FOR PT-LOGIC PROGRAMS

Below, we define the queries that can be applied to a *pt*-logic program  $P$ .

A simple *pt*-query of type 1 has the form  $SQ=A:<?S, ti>$ , where  $A:<?S, ti>$  is a *pt*-atom of  $P$ ,  $ti$  is a temporal interval, and  $?S$  is a provenance variable. The answers of  $SQ$  w.r.t.  $P$ , denoted by  $Ans_P(SQ)$ , is the set of mappings  $v$  from the variables of  $A$  to the constants of  $definite(P)$  and from  $?S$  to a subset of  $Names_P$  s.t.  $P \models v(A:<?S, ti>)$ .

A simple *pt*-query of type 2 has the form  $SQ=A:<?S, ?ti>$ , where  $A:<?S, ?ti >$  is a *pt*-atom of  $P$  (note that  $?S$  is a provenance variable and  $?ti$  is a temporal variable). The answers of  $SQ$  w.r.t.  $P$ , denoted by  $Ans_P(SQ)$ , is the set of mappings  $v$  from the variables of  $A$  to the constants of  $definite(P)$ , from  $?S$  to a subset of  $Names_P$ , and from  $?ti$  to a temporal interval s.t.  $P \models^{max} v(A:<?S, ?ti>)$ .

A simple *pt*-query of type 3 has the form  $SQ=A_1:<?S_1, ?ti> \wedge \dots \wedge A_n:<?S_n, ?ti>$  where each  $A_i:<?S_i, ?ti >$  is a *pt*-atom of  $P$ . The answers of  $SQ$  w.r.t.  $P$ , denoted by  $Ans_P(SQ)$ , is the set of mappings  $v$  s.t. if  $u_i \in Ans_P(A_i:<?S_i, ?ti>)$  s.t.  $u_1, \dots, u_n$  coincide on the common variables of  $A_i$  and  $?S_i$  then  $v$  coincides with  $u_i$  on the variables of  $A_i$  and  $?S_i$  and  $v(?ti)$  is the intersection of  $u_1(?ti), \dots, u_n(?ti)$ , if such intersection exists.

A simple *pt*-query of type 4 has the form  $SQ=A_1:<?S_1, ?ti> \wedge \dots \wedge A_n:<?S_n, ?ti> \wedge included(?ti, [t,t'])$ , where each  $A_i:<?S_i, ?ti >$  is a *pt*-atom of  $P$  and  $[t,t']$  is a temporal interval. The answers of  $SQ$  w.r.t.  $P$ , denoted by  $Ans_P(SQ)$ , is the set of mappings  $v$  s.t. if  $u_i \in Ans_P(A_i:<?S_i, ?ti>)$  s.t.  $u_1, \dots, u_n$  coincide on the common variables of  $A_i$  and  $?S_i$  then  $v$  coincides with  $u_i$  on the variables of  $A_i$  and  $?S_i$  and  $v(?ti)$  is the intersection of  $u_1(?ti), \dots, u_n(?ti)$  and  $[t,t']$ , if such intersection exists.

A complex *pt*-query has the form  $CQ=SQ_1 \wedge \dots \wedge SQ_n \wedge filter$ , where  $SQ_i$  are simple *pt*-queries, each having a different temporal variable, and  $filter$  is an expression of the following EBNF grammar:

```
term:=duration(?ti) | start(?ti) | end(?ti) | c, where ?ti is a
temporal variable and c is a decimal.
complex_term:= term | complex_term (+|-|*|/) complex_term
temp_comparison:= complex_term (<|>|=|≤|≥|≠)
                    complex_term
prov_comparison:= (?S (⊂|⊃|⊆|⊇|=) ?S') / (?S (⊂|⊃|
                    ⊆|⊇|=) S''), where ?S, ?S' are
                    provenance variables and S'' ⊆ Names_P.
filter:= temp_comparison / prov_comparison | filter (∧|
                    ∨) filter,
```

such that each provenance and temporal variable appearing in  $filter$  appears in  $SQ_1 \wedge \dots \wedge SQ_n$ .

The answers of  $CQ$  w.r.t.  $P$ , denoted by  $Ans_P(CQ)$ , is the set of mappings  $v$  s.t. (i) if  $u_i \in Ans_P(SQ_i)$  s.t.  $u_1, \dots, u_n$  coincide on the common variables of  $SQ_i$  then  $v$  coincides with  $u_i$  on the variables of  $SQ_i$  and (ii)  $v(filter)$  holds.

Consider a simple or complex query  $CQ$ . Let  $v, u \in Ans_P(CQ)$ . We say that answer  $v$  is *more informative* than answer  $u$ , denoted by  $v \leq u$ , if (i)  $v, u$  coincide on their no provenance variable mappings, and (ii) for each provenance variable  $?S \in domain(v)$ , it holds that  $v(?S) \subseteq u(?S)$ . We define  $Answer_P(CQ)=minimal_{\leq}(Ans_P(CQ))$  and we consider that these are the *desired answers*. This is because if  $P \models A:<S, ti>$  then  $P \models A:<S', ti>$ , for each  $S \subseteq S' \subseteq Names_P$ .

**Example 2.** Consider the simple query  $SQ = extra\_vacation(Peter):<?S, ?ti>$ . Then, the answers to this query is (i) the mapping  $v$ , where  $v(?S)=\{Person, Job, Vacation\}$  and  $v(?ti)= [2001,2001]$ , and (ii) the mapping  $u$ , where  $u(?S)=\{Person, Job, Vacation\}$  and  $u(?ti)=[2006,2008]$ .

Consider now the simple query  $SQ = extra\_vacation(Mary):<?S, ?ti>$ . Then, the answers to this query is (i) the mapping  $v$ , where  $v(?S)=\{Person\}$  and  $v(?ti)= [1999,2002]$ , and (ii) the mapping  $u$ , where  $u(?S)=\{Person, Job, Vacation\}$  and  $u(?ti)=[1900,1992]$ .

Consider now the simple query  $SQ = extra\_vacation(Mary):<?S, ?ti> \wedge extra\_vacation(Peter):<?S', ?ti>$ , requesting the common temporal intervals that Mary and Peter get extra vacation days, independently of the set of sources that this information is derived. Then, the answer to this query is the mapping  $v$ , where  $v(?ti)=[2001,2001]$ ,  $v(?S)=\{Person\}$  and  $v(?S')=\{Person, Job, Vacation\}$ .

Consider now the simple query  $SQ = extra\_vacation(Mary):<?S, ?ti> \wedge extra\_vacation(Peter):<?S, ?ti>$ , requesting the common temporal intervals that Mary and Peter get extra vacation days, as derived from the same set of sources. Then, the answer to this query is the mapping  $v$ , where  $v(?ti)=[2001,2001]$  and  $v(?S)=\{Person, Job, Vacation\}$ .

Consider now the complex query  $CQ= has\_job(Mary, ?x):<?S, ?ti> \wedge included(?ti, [1993,2008]) \wedge ?S=\{Person\}$ , requesting the temporal intervals that Mary has a job from 1993 to 2008, as derived from the source *Person*. The answers to this query is (i) the mapping  $v$ , where  $v(?ti)=[1993, 2002]$  and  $v(?S)=\{Person\}$  and (ii) the mapping  $u$ , where  $u(?ti)=[2006,2008]$  and  $u(?S)=\{Person\}$ .

Consider now the complex query  $CQ = extra\_vacation(Mary):<?S, ?ti> \wedge extra\_vacation(Peter):<?S, ?ti> \wedge start(?ti) \leq end(?ti') \wedge start(?ti') \leq end(?ti)$ , requesting the temporal intervals that Mary and Peter get extra vacation days, as long as these overlap, and as derived from the same set of sources.

Then, the answer to this query is the mapping  $v$ , where  $v(?ti)=[2001,2001]$ ,  $v(?ti')=[1999,2002]$ , and  $v(?S)=\{Person, Job, Vacation\}$ .

## VI. RELATED WORK

In this Section, we discuss related work.

Flouris et al. [2] add provenance information to RDF theory [3], [4]. In particular, they extend RDF triples to RDF quadruples, where the fourth element is the set of graph names that participated in the derivation of the RDF triple through a limited subset of the RDFS entailment rules. They also discuss atomic update operations (i.e. inserts and deletes) of the RDF quadruples. Comparing to this paper, [2] does not present a model theory and does not consider the temporal domain. Note that our approach can also be applied to RDFS, as RDFS inference rules can be expressed through definite logic programming rules [5].

In [6], we extend extended logic programming rules [6] with their validity temporal intervals. We consider derivations based on temporal time points and Answer Set Programming [7], and provide an algorithm that returns the maximal temporal intervals that a literal is true. Present work has a different model theory and implementation than [6], as we consider only definite logic programming rules and on the other hand we also consider the provenance domain. Yet, the query language presented here is an extension of the query language presented in [6].

In [8], the authors present a general framework for representing, reasoning, and querying with RDFS annotated data on the Semantic Web. They show that their formalism can be instantiated on the temporal, fuzzy, and provenance domain. The authors can associate RDF triples with their validity temporal intervals and supportive sources and apply the RDFS inference rules (which are always valid). Yet, [8] does not support simple queries of type 4. Moreover, our query answering is more efficient, since during query answering, we directly work on maximal temporal intervals. In [8], all entailed temporal intervals returned by the query are considered and then the maximal ones are returned. Further, our semantics is different than [8]. For example, consider the annotated RDF triples  $p(a,b) : <n, [1990, 2000]>$  and  $q(c,d) : <n, [1995, 2010]>$ . Then, according to [8], the answer to query  $p(a,b) : <?S, ?ti> \wedge q(c,d) : <?S, ?ti> \wedge end(?ti) < start(?ti)$  will provide the mapping  $v$  s.t.  $v(?ti)=[1990,1994]$  and  $v(?ti)=[1995, 2010]$ . In our case, we will provide no answers, since  $2000 > 1995$ .

In [9], the authors present a framework to incorporate temporal reasoning into RDFS. The authors associate RDF triples with their validity temporal interval and apply the RDFS inference rules (which are always valid). Unlike our work, their semantics is based on time points and not on temporal intervals. Additionally, [9] does not consider the provenance domain. Further, it does not support simple queries of type 3 and type 4 and the filter condition is limited.

In [10], the authors extend RDF graphs with temporal information, by associating RDF triples with their validity interval. They consider any entailment regime that can be expressed through definite rules  $A_0 \leftarrow A_1, \dots, A_n$ , where  $A_i$  is an RDF triple. Each such rule is replaced by the temporal rule  $A_0 : [max(t_1, \dots, t_n), min(t'_1, \dots, t'_n)] \leftarrow A_1 : [t_1, t'_1], \dots, A_n : [t_n, t'_n]$ . These rules are applied recursively, until a fixpoint is reach.

Then, maximal validity temporal intervals for each derived RDF triple are produced. Yet, this work does not present a model theory based on temporal intervals and does not consider the provenance domain. Additionally, it does not support simple temporal queries of type 4 and the filter condition is left unspecified.

Work in [11] provides a framework to support spatial and temporal analysis over RDFS data. With respect to the temporal component, [11] is similar to [10], as it also computes the maximal validity temporal intervals of derived RDF triples, using the RDFS entailment rules. Yet, [11] does not consider the provenance domain.

Finally, we would like to note that our theory cannot be considered as a special case of annotated logic programming [12], as the model theory and the operational semantics are different there.

## VII. CONCLUSION

In this paper, we have presented a model theory and the operational semantics of a *pt-logic* program  $P$ , that is a set of definite logic programming rules, annotated with the source that have been derived and their validity temporal interval. We have defined the simple and maximally temporal entailments of  $P$ , showing that there exists a minimal model  $M_{min}$  that contains exactly these maximal entailments. Additionally, we defined a consequence operator whose closure coincides with  $M_{min}$ . Further, we showed that simple and maximally temporal entailment from a *pt-logic* program  $P$  is EXPTIME-complete w.r.t. the size of  $P$ . A query language for our framework is proposed.

As future work, we plan to extend our theory to extended logic programs. Further, we plan to add additional parameters to definite logic programming rules such that space and trust.

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