A Study of Misinformation Games*

Constantinos Varsos[1,2], Giorgos Flouris[1], Marina Bitsaki[1], and Michail Fasoulakis[1]

[1] Institute of Computer Science, Foundation of Research and Technology Hellas, Vasilika Vouton 100, 70013, Greece
[2] Computer Science Department, University of Crete, Voutes University Campus, 70013, Greece
{varsosk,fgeo,mfasoul}@ics.forth.gr, bitsaki@tsl.gr

Abstract. A common assumption in game theory is that players have a common and correct (albeit not always complete) knowledge with regards to the abstract formulation of the game. However, in many real-world situations it could be the case that (some of) the players are misinformed with regards to the game that they play, essentially having an *incorrect* understanding of the setting, without being aware of it. This would invalidate the common knowledge assumption. In this paper, we present a new game-theoretic framework, called *misinformation games*, that provides the formal machinery necessary to study this phenomenon, and present some basic results regarding its properties.

Keywords: Misinformation in games · Natural misinformed equilibrium · Price of Misinformation · Normal-form games · Load balancing games

1 Introduction

A fundamental issue in interacting situations is the way decisions are made by the participants, a process captured by game theory, where typically is assumed that the rules of interaction (i.e., the game definition) are common knowledge among players (but see [19,5,11] for some exceptions). This may be unrealistic in circumstances where *misinformation* may cause players to have different and/or incorrect knowledge about the rules of interaction.

This can happen for various reasons. Specifically, such scenarios could occur on purpose (e.g., by deceptive agents communicating wrong information), due to random effects (e.g., noise in the communication channels, erroneous sensor readings), by design (e.g., by the game designer in order to enforce a socially-optimal behaviour), or due to environmental changes (e.g., the setting changes without players' knowledge). Misinformation could play a prominent role in the outcome of the game, without necessarily negative effects.

As a more concrete example, consider the classical Prisoner's Dilemma (PD) game, where two suspects (the players) are being interrogated, having the option

*For this work the fourth author was supported by the Stavros Niarchos-FORTH postdoc fellowship for the project ARCHERS.

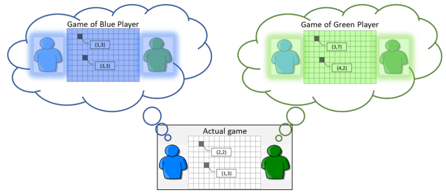

Fig. 1: Schematic representation of a misinformation game with 2 players.

to betray the other (B), or stay silent (S). Each of them will get a penalty reduction if he/she betrays the other, but if they both remain silent, the police can only convict them for lesser charge and not for the principal crime; if they both betray, they will get a reduced penalty for the principal crime. Using classical game theory, this situation is modelled by payoff matrix presented in Table 1a, where the only Nash equilibrium is for both players to betray.

Now suppose that the cogent evidence with regards to the lesser charge has been obtained in an illegal manner, and thus cannot be used in court. As a result, players' actual payoffs are as shown in Table 1b; however, this is not disclosed to the suspects, who still believe that they play under Table 1a. This would lead players to betray, although, had they known the truth (Table 1b), they also had other options (Nash equilibria), e.g., to both stay silent. We will refer to this game as the *misinformed Prisoner's Dilemma* (*mPD*) in the rest of this paper.

Table 1: Payoff matrices for the PD and mPD

	S	B
S	$(-1,-1)$	$(-3,-1/2)$
B	$(-1/2,-3)$	$(-2,-2)$

(a) Payoffs (in PD); also, players' view (in mPD).

	S	B
S	$(0,0)$	$(-3,-1/2)$
B	$(-1/2,-3)$	$(-2,-2)$

(b) Actual game (in mPD).

To study situations like mPD, we relax the classical assumption of game theory that agents know the correct information related to the abstract formulation of the game, and *admit the possibility that each player may have a different (and thus incorrect) perception about the game being played*, unknowingly to himself/herself or the other player(s). We call such games *misinformation games*. As shown in Figure 1, the main defining characteristic of misinformation games is that agents are unwitting of their misinformation, and will play the game under the misconceived game definition that they have. This essentially means that the assumption of common knowledge is dropped as well.

Obviously, in such a setting, game theory dictates the *actual player behaviour* in his/her own view, which may be different from the behaviour regarding the actual game. On the other hand, *the payoffs received* by the players are the ones provisioned by the actual game, which may differ from the ones they assume.

This paper's main objective is to *introduce the formal machinery necessary to study misinformation games*. Specifically, the contributions of this paper consisted of: i) defining misinformation games and recasting basic game-theoretic concepts without the assumption of common and correct knowledge (Subsection

3.1), ii) introducing a new metric, called the *Price of Misinformation (PoM)*, to quantify the effect of misinformation on the social welfare of players (Subsection 3.2), and iii) applying our ideas to load balancing games (Section 4).

2 Related work

Starting from the concept of games with misperceptions (see Chapter 12 in [19]) many studies model subjective knowledge of players with regards to game specifications, leading to the introduction of hypergames (HG) ([5,34,27,18,7,3] etc.) and games with unawareness (GwU) ([8,28,26,29,11] etc.), where players may be playing different games. Although we share motivation with these approaches, there are also some crucial distinctions. First, HG/GwU are behaviour-oriented (*what* the players will play), whereas misinformation games are outcome-oriented. Furthermore, HG focus on perceptional differences among players, and do not model the “actual game”, hence, HG lack grounding to the reality of the modelled situation. In misinformation games we close this gap, modelling also the environment, and allowing differences to also occur between each player and the environment. Moreover, in GwU, though the “actual game” is used as the basis of the models, the analysis based on consistency criteria and belief hierarchies. In misinformation games we do not make such assumptions.

In [15] authors define the notion of games with awareness based on an extensive-form game; they agglomerate descriptions of reality, changes in players’ awareness and players’ subjective views. Also, they define a generalized Nash equilibrium that is similar with our equilibrium concept. Nevertheless, their analysis is behaviour-oriented. The work in [11] incorporates game and unawareness as interrelated objects, whereas in [8] awareness architectures are provided to study players’ limited awareness of strategies. Further, in [33] authors focus on how unawareness affects incentives, whereas [30] provides a dynamic approach for extensive-form games with unawareness. Moreover, [23] proposed a model for games with uncertainty where players may have different awareness regarding a move of nature.

In [6,14] studied the case where one of the players knows the (mis)perceptions of the opponents. Also, in [32] the concept of subjective games is proposed, but without introducing any equilibrium concept. Another approach is given in [10] where an equilibrium concept is defined, but has a probabilistic dependence on the actual game specifications.

Initiated by Harsanyi [16] the concept of incomplete knowledge in games has attracted significant attention, mainly through the Bayesian games approach ([31,35,13,12], etc.), where a key assumption is that of common priors. Although this provides significant modeling advantages, it cannot address the situations considered by misinformation games, cases where knowledge is not common. Moreover, in Bayesian games, although agents are unsure as to their actual payoff, they are well-aware of that, and they do their best out of the uncertainty that they have. On the contrary, in misinformation games, the agents play according to their subjective game definition, without considering mitigation measures.

In particular, the mPD scenario (Table 1) cannot be captured by Bayesian games, as the players do not distinct the actual situation from the one provided to them. But even if a player suspects that the police are lying, has no clue as to what they are lying about. Therefore, he/she cannot form any probability distribution over an array of alternative plausible scenarios.

In [2,4,24] the case of uncommon priors was studied, but without addressing the scenario of private priors, which is the case considered in misinformation games. Additionally, the idea of agents understanding a different payoff matrix than the actual one has been considered in [20,1]. In these studies, the agents privately choose to modify their own objective payoffs (only), for personal reasons (i.e. bias). Here misinformation is restricted only to each agent's own payoffs, therefore our work can be viewed as a more general case of such settings.

3 Normal-form games

A *game in normal-form* is represented by a *payoff matrix* that defines the payoffs of all players for all possible combinations of pure strategies. Formally:

Definition 1. *A normal-form game G is a tuple $\langle N, S, P\rangle$, where:*

- *N is the set of the players,*
- *$S = S_1 \times \cdots \times S_{|N|}$, S_i is the set of pure strategies of player $i \in N$,*
- *$P = (P_1, \ldots, P_{|N|})$, $P_i \in \mathbb{R}^{|S_1| \times \ldots \times |S_{|N|}|}$ is the payoff matrix of player i.*

If player i randomly selects a pure strategy, then he/she plays a mixed strategy $\sigma_i = (\sigma_{i,1}, \ldots, \sigma_{i,|S_i|})$ which is a discrete probability distribution over S_i. Let the set of all possible mixed strategies σ_i be Σ_i. A strategy profile $\sigma = (\sigma_1, \ldots, \sigma_{|N|})$ is an $|N|$-tuple in $\Sigma = \Sigma_1 \times \ldots \times \Sigma_{|N|}$. We denote by σ_{-i} the $|N-1|$-tuple strategy profile of all other players except for player i in σ. The payoff function of player i is defined as: $f_i : \Sigma \to \mathbb{R}$, such that:

$$f_i(\sigma_i, \sigma_{-i}) = \sum\nolimits_{k \in S_1} \cdots \sum\nolimits_{j \in S_{|N|}} P_i(k, \ldots, j) \cdot \sigma_{1,k} \cdot \ldots \cdot \sigma_{|N|,j}, \tag{1}$$

where $P_i(k, \ldots, j)$ is the payoff of player i in the pure strategy profile $(k, \ldots, j)$. In other words, $f_i(\sigma_i, \sigma_{-i})$ represents player's i expected payoff as a function of σ. The *Nash equilibrium* in a normal-form game is defined as follows:

Definition 2. *A strategy profile $\sigma^* = (\sigma_1^*, \ldots, \sigma_{|N|}^*)$ is a Nash equilibrium, iff, for any i and for any $\hat{\sigma_i} \in \Sigma_i$, $f_i(\sigma_i^*, \sigma_{-i}^*) \geq f_i(\hat{\sigma}_i, \sigma_{-i}^*)$.*

3.1 Misinformation in normal-form games

Misinformation captures the concept that different players may have a specific, subjective, and thus different view of the game that they play.

Definition 3. *A* misinformation normal-form game *(or simply* misinformation game*) is a tuple $mG = \langle G^0, G^1, \ldots, G^{|N|}\rangle$, where all G^i are normal-form games and G^0 contains $|N|$ players.*

G^0 is called the *actual game* and represents the game that is actually being played, whereas G^i (for $i \in \{1, \ldots, |N|\}$) represents the game that player i thinks that is being played (called the *game of player i*). We make no assumptions as to the relation among G^0 and G^i, and allow all types of misinformation to occur. An interesting special class of misinformation games is the following:

Definition 4. *A misinformation game $mG = \langle G^0, G^1, \ldots, G^{|N|} \rangle$ is called* canonical *iff:*

- *For any i, G^0, G^i differ only in their payoffs.*
- *In any G^i, all players have an equal number of pure strategies.*

Although, non-canonical misinformation games may occur, e.g., when communication problems deprives a player from the option to use a viable strategy. However, we can transform any non-canonical misinformation game into an equivalent canonical game (in terms of strategic behaviour), using the process of *inflation* described as follows.

Let mG be a non-canonical misinformation game. To transform it into a canonical misinformation game with the same strategic behaviour, we compare G^0 with each G^i ($i > 0$). Then:

1. If G^i does not include a player of G^0, then we "inflate" G^i by adding this new player, with the same strategies as in G^0. We extend the elements of the payoff matrix of G^i to represent the payoffs of the new player, using any fixed constant value. Moreover, the current payoff matrix of G^i is increased by one dimension, by replicating the original payoff matrix as many times as needed (to accommodate the new player's strategies).
2. If G^i contains an imaginary player not included in G^0, then we add a new player in G^0, using the process described in #1 above. In addition, since Definition 3 requires that each player in G^0 is associated with a game in mG, we add a new game in mG, which is a replica of G^0.
3. If G^i does not contain a certain strategy which appears in G^0 (for a certain player), we add this new strategy, with payoffs small enough to be dominated by all other strategies.
4. If G^i contains an imaginary strategy that does not appear in G^0 (for a certain player), we inflate G^0 as in #3 above.

Repeating the above process a sufficient (finite) number of times, we derive a misinformation game that satisfies the first condition of Definition 4 and has the same strategic properties as the original. For the second condition, we inflate the games according to the largest dimension (number of strategies) of the largest game, as in #3 above. Therefore, we focus on canonical misinformation games.

The definition of misinformed strategies and strategy profiles is straightforward, once noticing that they refer to each player's own game:

Definition 5. *A* misinformed strategy*, $m\sigma_i$ of a player i is a strategy of i in the game G^i. We denote the set of all possible misinformed strategies of player i as Σ_i^i. A* misinformed strategy profile *of mG is an $|N|$-tuple of misinformed strategies $m\sigma = (m\sigma_1, \ldots, m\sigma_{|N|})$, where $m\sigma_i \in \Sigma_i^i$.*

As usual, we denote by $m\sigma_{-i}$ the $|N-1|$-tuple strategy profile of all other players except for player i in a misinformed strategy $m\sigma$. The payoff function f_i of player i under a given profile $m\sigma$ is determined by the payoff matrix of G^0, and is defined as $f_i : \Sigma_1^1 \times \cdots \times \Sigma_{|N|}^{|N|} \rightarrow \mathbb{R}$, such that:

$$f_i(m\sigma_i, m\sigma_{-i}) = \sum\nolimits_{k \in S_1^1} \cdots \sum\nolimits_{j \in S_{|N|}^{|N|}} P_i^0(k, \ldots, j) \cdot m\sigma_{1,k} \cdot \ldots \cdot m\sigma_{|N|,j},$$

where $P_i^0(k, \ldots, j)$ is the payoff of player i in the pure strategy profile $(k, \ldots, j)$ under the actual game G^0. Also, S_i^j denotes the set of pure strategies of player i in game G^j.

Observe that, although each player's strategic decisions are driven by the information in his/her own game (G^i), the received payoffs are totally dependent on the actual game G^0, that may differ than G^i. Further, the payoff function would be ill-defined if we consider non-canonical misinformation games.

Next, we define the solution concept of a misinformation game, where each player chooses a Nash strategy, neglecting what other players know or play:

Definition 6. *A misinformed strategy, $m\sigma_i$, of player i, is a* misinformed equilibrium strategy*, iff, it is a Nash equilibrium strategy for the game G^i. A misinformed strategy profile $m\sigma$ is called a* natural misinformed equilibrium *iff it consists of misinformed equilibrium strategies.*

In the following, we denote by nme_{mG} (or simply nme, when mG is obvious from the context) the set of natural misinformed equilibria of mG and by NE the set of Nash equilibria of G. Moreover, any natural misinformed equilibrium is consisted of Nash equilibrium strategy profiles, regarding G^is. The computation of a Nash equilibrium for each G^i is ***PPAD***-complete [9]. Thus, the same holds for a natural misinformed equilibrium.

3.2 Price of Misinformation

Inspired by the seminal work of [17] that introduced the *Price of Anarchy (PoA)* metric we define a metric, called the *Price of Misinformation (PoM)* to measure the effect of misinformation compared to the social optimum. For that, we consider a social welfare function $SW(\sigma) = \sum_i f_i(\sigma)$, and denote by *opt* the socially optimal strategy profile, i.e., $opt = \arg\max_\sigma SW(\sigma)$. PoM is defined as follows:

Definition 7. *Given a misinformation game mG, the* Price of Misinformation (PoM) *is defined as:*

$$PoM = \frac{SW(opt)}{\min_{\sigma \in nme} SW(\sigma)} \tag{2}$$

Using the definition of PoA [17] and (2) we derive the following formula:

$$\frac{PoM}{PoA} = \frac{\min_{\sigma \in NE} SW(\sigma)}{\min_{\sigma \in nme} SW(\sigma)} \tag{3}$$

Observe that, if $PoM < PoA$, then misinformation has a beneficial effect on social welfare, as the players are inclined (due to their misinformation) to choose socially better strategies. On the other hand, if $PoM > PoA$, then misinformation leads to a worse outcome from the perspective of social welfare.

Moreover, misinformation is a powerful tool for mechanism design as shown in the following proposition.

Proposition 1. *For any normal-form game G and strategy profile σ there is a misinformation game $mG = \langle G^0, G^1, \dots, G^{|N|} \rangle$ such that $G^0 = G$ and the only natural misinformed equilibrium of mG is σ.*

Proof. Let G' be a normal form game such that σ is the only Nash equilibrium (we can always construct such a game). Then $mG = \langle G, G', \dots, G' \rangle$ is the desired misinformation game. □

Corollary 1. *For every normal-form game G there is a misinformation game $mG = \langle G^0, G^1, \dots, G^{|N|} \rangle$ such that $G^0 = G$ and $PoM = 1$.*

The above results show that, given sufficient misinformation, anything is possible in terms of improving (or deteriorating) the social welfare.

4 Load balancing games

In this section, we apply our framework in load balancing games, as defined in [22], where tasks selfishly choose to be assigned to machines, so that no task has any incentive to deviate from its machine. Formally:

Definition 8. *A* load balancing game (lbg) *is a tuple $G = \langle k, m, s, w \rangle$, where $k = \{1, \dots, |k|\}$ is the set of tasks, each associated with a weight $w_j \geq 0$, and $m = \{1, \dots, |m|\}$ is the set of machines, each with speed $s_i > 0$.*

We consider the case where tasks play only pure strategies, thus, the *assignment* of tasks to machines is determined by a mapping $A : k \to m$ (note that each task is assigned to exactly one machine). The load of machine $i \in m$ under A is defined as $l_i = \sum_{j \in k: i = A(j)} w_j / s_i$. The cost of task j for choosing machine i is $c_j^i = l_i$. Furthermore, the social cost of assignment A is defined as $cost(A) = \max_{i \in m}(l_i)$, in other words the makespan under the assignment A. An assignment A^* is optimal if $cost(A^*) \leq cost(A)$ for all possible assignments A. An assignment A is a pure Nash equilibrium, if and only if, for any j and for any $\hat{i} \in m$, $c_j^{A(j)} \leq c_j^{\hat{i}}$, in other words for any alternative assignment of task j (say to machine $\hat{i}$) the cost is worse.

4.1 Misinformation in load balancing games

Introducing misinformation in lbgs follows similar patterns as in Section 3.1:

Definition 9. *A* misinformation lbg *is a tuple* $mG = \langle G^0, G^1, \dots, G^{|k|}\rangle$*, where all* G^j *are lbgs and* G^0 *contains* $|k|$ *tasks.*

Definition 10. *A misinformation lbg* $mG = \langle G^0, G^1, \dots, G^{|k|}\rangle$ *is called* canonical*, if and only if, for any* j*,* G^0, G^j *differ only with regards to the weights of the tasks and the speeds of the machines.*

Like in the standard case, a misinformed assignment nmA is a mapping of tasks to machines $nmA : k \to m$, where any task j chooses a machine according to its game G^j. Given a specific misinformed assignment nmA, the *actual load of a machine* i is $l_i^0 = \sum_{j \in k: i = nmA(j)} w_j^0 / s_i^0$, whereas the *perceived load of a machine* i for task h is $l_i^h = \sum_{j \in k: i = nmA(j)} w_j^h / s_i^h$. The *actual cost* of task j for choosing machine i is $c_j^{i,0} = l_i^0$, whereas the *perceived cost* is $c_j^{i,j} = l_i^j$. Similarly, the actual social cost of mA is $cost(nmA) = \max_{i \in m}(l_i^0)$.

As with normal-form games, the tasks choose the Nash equilibrium assignments in their own game without regards to what other tasks do. Formally:

Definition 11. *A misinformed task assignment* $nmA(j)$ *of task* j *is a* pure misinformed equilibrium task assignment*, if and only if it is a pure Nash equilibrium assignment for game* G^j*. A misinformed assignment* nmA *is called a* pure natural misinformed equilibrium assignment *if and only if it consists of pure misinformed equilibrium task assignments.*

As each G^j is an lbg, the existence of a pure Nash equilibrium assignment in every G^j is warranted by the results of [22,25,21], hence a natural misinformed equilibrium assignment in misinformation lbgs always exists. Moreover, using complexity results for standard lbgs [22], we can show the following:

Proposition 2. *Consider a misinformation lbg* mG *with* k *tasks, such that each* G^j *has* m *identical machines. Then, the computational complexity of computing a natural misinformed equilibrium assignment in* mG *is* $O(k^2 \log k)$*.*

Proof. On identical machines we can transform any assignment A into a pure Nash equilibrium in time $O(k \log k)$ [22]. To find a natural misinformed equilibrium, we repeat this once for each G^j $(j > 0)$, which requires $O(k^2 \log k)$ time.

4.2 Price of Misinformation in load balancing games

PoM in misinformation lbgs (that aim at minimizing cost instead of maximizing payoff) is defined as follows:

$$PoM = \frac{\max_{A \in nmA} cost(A)}{cost(A^*)}, \tag{4}$$

where $cost(A)$ is the worst cost among the pure natural misinformed equilibria assignments nmA and $cost(A^*)$ is the cost of the optimal assignment in the actual game. The following example is illustrative of the concepts presented in this section:

Example 1. Suppose that there are two identical machines with speed $s = 1$ and four tasks with $w_1 = w_2 = 1$ and $w_3 = w_4 = 2$. The optimal assignment maps a task of weight 1 and a task of weight 2 to each of the machines ($A^* = (1, 2, 1, 2)$). The worst pure Nash equilibrium assignment is $A = (1, 1, 2, 2)$ with $cost(A) = 4$, Figure 2-(b).

Now, consider the misinformation game mG in which tasks have different information on the weights. Let $w^1 = (w_1^1 = 6, w_2^1 = 1, w_3^1 = 2, w_4^1 = 2)$ be the weights in G^1 and $w^j = (w_1^j = 7, w_2^j = 1, w_3^j = 1, w_4^j = 1)$ in G^j, for $j = \{2, 3, 4\}$. The pure Nash equilibrium assignments in each game G^j are $A_1 = (1, 2, 2, 2)$ and $A_2 = (2, 1, 1, 1)$, thus the pure natural misinformed equilibrium assignments are all combinations aligned with i) task 1 is assigned to a different machine than tasks $\{2, 3, 4\}$ or ii) all tasks are assigned to the same machine. From the above, the worst natural misinformed equilibrium assignment is derived to be $nmA = (1, 1, 1, 1)$ (or $nmA = (2, 2, 2, 2)$) with $cost(nmA) = 6$, Figure 2-(c-d). It is interesting that in this example $PoA = 4/3$ and $PoM = 2$ implying that misinformation worsens the behaviour of the game. □

Fig. 2: (a) optimal assignment, (b) worst Nash equilibrium allocation, (c-d) worst natural misinformed equilibrium allocation.

In terms of mechanism design, misinformation is equally strong and flexible for lbgs as for normal-form games. In particular, for any lbg G and assignment A, we can construct a misinformation lbg $mG = \langle G^0, G^1, \ldots, G^{|N|} \rangle$ such that $G^0 = G$ and the only pure natural misinformed equilibrium assignment of mG is A, as well as a misinformation lbg $mG = \langle G^0, G^1, \ldots, G^{|N|} \rangle$ such that $G^0 = G$ and $PoM = 1$.

Due to the special form of lbgs, we can prove various bounds regarding their cost and PoM, based on the task weights and machine speeds. Propositions 3, 4, 5 show some such results:

Proposition 3. *Consider a canonical misinformation lbg* $mG = \langle G^0, G^1, \ldots, G^{|k|} \rangle$*, such that* $G^0 = \langle k, m, s, w \rangle$ *and* $s_i > 0$ *for all* i*. Then, for any assignment* nme*,* $cost(nme) \leq \sum_{j=1}^{k} w_j / \min_i s_i$*.*

Proof. The worst possible assignment nmA^* (from the social cost perspective) is to assign all tasks to the slowest machine, with $cost(nmA^*) = \sum_{j=1}^{k} w_j / \min_i s_i$. Misinformation can achieve this effect, so the result follows. □

Proposition 4. *Consider a misinformation lbg* $mG = \langle G^0, G^1, \ldots, G^{|k|} \rangle$*, such that* G^0 *has* m *identical machines and finite task weights. Then, the Price of Misinformation is* $PoM \leq m$*.*

Proof. An optimal assignment $cost(A^*)$ cannot be smaller than the average load over all machines (i.e., $(\sum_{j\in[k]} w_j)/m$). Also, the worst scenario is that all tasks are assigned into one machine, with cost $(\sum_{j\in[k]} w_j)$. Then, using equation (4), we conclude. □

Next, we consider the case of uniformly related machines, i.e., the case where the cost (processing time) of a job j of weight w_j on machine i with speed s_i is w_j/s_i. We can show the following:

Proposition 5. *Consider a misinformation lbg $mG = \langle G^0, G^1, \ldots, G^{|k|}\rangle$, such that $G^0 = \langle k, m, s, w\rangle$ with m uniformly related machines and finite task weights. Then, the Price of Misinformation is*

$$PoM \leq k \cdot \frac{S}{s} \cdot O\left(\frac{\log m}{\log\log m}\right), \tag{5}$$

where s is the slowest speed and S is the fastest speed.

Proof. Since there is the case that all tasks be assigned to the slowest machine we have that $cost(nmA) \leq \sum_{i=1}^{k} w_i/s \leq k \cdot M/s$, where M is the largest weight. Also, we have that $PoM = PoA \cdot \frac{\max_{A\in nmA} Cost(A)}{\max_{B\in NE} Cost(B)}$ with A be the worst natural misinformed equilibrium assignment, B the worst Nash equilibrium assignment and NE the set of Nash equilibria assignments. Furthermore, we have that $\max_{B\in NE} Cost(B) \geq M/S$. Finally, by chapter 20 of [22] we have that $PoA \leq O\left(\frac{\log m}{\log\log m}\right)$. □

5 Synopsis and future work

This paper is motivated by the idea that misinformation is a fact of life in most multi-player interactions, and thus having the formal machinery to analyse misinformation can help understand many real-world phenomena. Towards this aim, we introduce a novel game-theoretic framework, called *misinformation games*.

We argue that the concept of misinformation games has the potential to explain various phenomena, and raises several interesting problems to be studied from different perspectives. From the designer's perspective, we can consider questions like the sensitivity of the game against misinformation, or the identification of ways to exploit the misinformation as a means to improve social welfare (through PoM). From the players' perspective, one could study how the players will revise their game definition following the realisation that it is wrong, or the question of how to protect them from deceptive efforts. Finally, from the more general perspective, it makes sense to study various forms of equilibria, as well as the effect of different misinformation patterns on the game's outcome.

References

1. Acar, E., Meir, R.: Distance-based equilibria in normal-form games. In: The Thirty-Fourth AAAI Conference on Artificial Intelligence, AAAI'20 , The Thirty-Second

Innovative Applications of Artificial Intelligence Conference, IAAI'20, The Tenth AAAI Symposium on Educational Advances in Artificial Intelligence, EAAI'20. pp. 1750–1757. AAAI Press (2020)
2. Antos, D., Pfeffer, A.: Representing bayesian games without a common prior. In: 9th International Conference on Autonomous Agents and Multiagent Systems (AAMAS'10). pp. 1457–1458. IFAAMAS (2010)
3. Bakker, C., Bhattacharya, A., Chatterjee, S., Vrabie, D.L.: Metagames and hypergames for deception-robust control. ACM Trans. Cyber-Phys. Syst. **5**(3) (2021)
4. Banks, D., Gallego, V., Naveiro, R., Ríos Insua, D.: Adversarial risk analysis: An overview. WIREs Computational Statistics (2020)
5. Bennett, P.G.: Hypergames: Developing a model of conflict. Futures **12**(6), 489–507 (1980)
6. Chaib-Draa, B.: Hypergame analysis in multiagent environments. In: Proceedings of the AAAI Spring Symposium. pp. 147–149 (2001)
7. Cho, J.H., Zhu, M., Singh, M.: Modeling and Analysis of Deception Games Based on Hypergame Theory, pp. 49–74. Springer International Publishing (2019)
8. Copic, J., Galeotti, A.: Awareness as an equilibrium notion: Normal-form games (2006)
9. Daskalakis, C., Goldberg, P.W., Papadimitriou, C.H.: The complexity of computing a Nash equilibrium. SIAM Journal on Computing **39**(1), 195–259 (2009)
10. Esponda, I., Pouzo, D.: Berk–Nash equilibrium: A framework for modeling agents with misspecified models. Econometrica **84**(3), 1093–1130 (2016)
11. Feinberg, Y.: Games with unawareness. The B.E. Journal of Theoretical Economics (2020)
12. Gairing, M., Monien, B., Tiemann, K.: Selfish routing with incomplete information. Theory of Computing Systems **42**(1), 91–130 (2008)
13. Gao, J.: Uncertain bimatrix game with applications. Fuzzy Optimization and Decision Making **12**(1), 65–78 (2013)
14. Gharesifard, B., Cortés, J.: Stealthy strategies for deception in hypergames with asymmetric information. In: 2011 50th IEEE Conference on Decision and Control and European Control Conference. pp. 5762–5767 (2011)
15. Halpern, J.Y., Rêgo, L.C.: Extensive games with possibly unaware players. Math. Soc. Sci. **70**, 42–58 (2014)
16. Harsanyi, J.C.: Games with incomplete information played by "Bayesian" players, I–III Part I. The basic model. Management Science **14**(3), 159–182 (1967)
17. Koutsoupias, E., Papadimitriou, C.: Worst-case equilibria. In: Proceedings of the 16th annual conference on Theoretical aspects of Computer Science (STACS). pp. 404–413 (1999)
18. Kovach, N.S., Gibson, A.S., Lamont, G.B.: Hypergame theory: A model for conflict, misperception, and deception. Game Theory **2015** (2015)
19. Luce, R.D., Raiffa, H.: Games and Decisions: Introduction and Critical Survey. John Wiley and Sons, New York (1957)
20. Meir, R., Parkes, D.C.: Playing the wrong game: Smoothness bounds for congestion games with behavioral biases. SIGMETRICS Performance Evaluation Review **43**(3), 67–70 (2015)
21. Nash, J.F.: Non-cooperative games. The Annals of Mathematics **54**(2), 286–295 (1951)
22. Nisan, N., Roughgarden, T., Tardos, E., Vazirani, V.V.: Algorithmic Game Theory. Cambridge University Press (2007)

23. Ozbay, E.Y.: Unawareness and strategic announcements in games with uncertainty. In: Proceedings of the 11th Conference on Theoretical Aspects of Rationality and Knowledge. p. 231–238 (2007)
24. Roponen, J., Ríos Insua, D., Salo, A.: Adversarial risk analysis under partial information. European Journal of Operational Research **287**(1), 306–316 (2020)
25. Rosenthal, R.W.: A class of games possessing pure-strategy Nash equilibria. International Journal of Game Theory **2**, 65–67 (1973)
26. Sasaki, Y.: Generalized Nash equilibrium with stable belief hierarchies in static games with unawareness. Annals of Operations Research **256**(2), 271–284 (2017)
27. Sasaki, Y., Kijima, K.: Hypergames and bayesian games: A theoretical comparison of the models of games with incomplete information. Journal of Systems Science and Complexity **25**(4), 720–735 (2012)
28. Schipper, B.C.: Unawareness - A gentle introduction to both the literature and the special issue. Mathematical Social Sciences **70**, 1–9 (2014)
29. Schipper, B.C.: Self-confirming games: Unawareness, discovery, and equilibrium. In: TARK (2017)
30. Schipper, B.C.: Discovery and Equilibrium in Games with Unawareness. Mpra paper, University Library of Munich, Germany (2018)
31. Székely, G.J., Rizzo, M.L.: The uncertainty principle of game theory. The American Mathematical Monthly **114**(8), 688–702 (2007)
32. Teneketzis, D., Castañón, D.: Informational aspects of a class of subjective games of incomplete information: Static case. Journal of Optimization Theory and Applications **54**, 413–422 (1983)
33. Thadden, E.L., Zhao, X.: Multi-task agency with unawareness. Theory and Decision **77**(2), 197–222 (2014)
34. Vane, R., Lehner, P.: Using hypergames to increase planned payoff and reduce risk. Autonomous Agents and Multi-Agent Systems **5**(3), 365–380 (2002)
35. Zamir, S.: Bayesian Games: Games with Incomplete Information, pp. 426–441. Springer New York (2009)