STATISTICAL TIME-FREQUENCY DIFFERENTIATIONS OF ERP IN DYSLEXIA USING MATCHING PURSUIT ALGORITHM

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Abstract: The main purpose of this paper is to study energy differentiations of electroencephalogram (EEG) and event related potentials (ERP) of normal subjects and subjects with dyslexia. As ERP is considered to be nonstationary signal, traditional spectral analysis is not recommended. A most appropriate approach is time-frequency representation (TFR) which reveals temporal evolution of frequency components. In this study a non-orthogonal, iterative method for adaptive time-frequency approximation of signals called matching pursuit is used. This method decomposes the signal piece by piece using a dictionary of basis functions. At each step the best fitting analyzing function is adapted to an intrinsic structure of the signal, thus providing flexible signal representation. The major advantages of the matching pursuit are the good localization of transients, the robust universal estimate of the time-frequency energy density which is resistant to the existence of noise and the fact that the dictionary set of waveforms is not limited to a single basis. Time frequency statistics reveal statistical differences on energy distribution of specific time intervals and frequency components over time-frequency plane. Possible non-normal distributions of the energy values are taken into account and a normalization transform is used in order to be able to use robust parametric tests. According to this analysis, dyslexics appear to have statistically reduced energy compared to controls for frequency regions 5-20Hz and for time around the ERP component N100.

Introduction

In the present paper, the matching pursuit algorithm, which was first proposed by Mallat and Zhang [1] is applied in order to estimate the energy of EEG signals in time-frequency plane. The use of matching pursuit algorithm leads to a decomposition of each EEG signal into a linear expansion of Gabor atoms. The time-frequency estimation of EEG signal energy is the result of pseudo Wigner-Ville distribution of Hilbert transform of these Gabor atoms.

In previous works have been used some other methods in estimation of EEG signal energy in time-frequency plane like spectrogram [2], scalogram [3], bandpass filtering in overlapping bands [4].The approximation of a signal using nonorthogonal functions is a “nonpolynomial” problem (computational complexity grows exponentially with the dimension of signal). The matching pursuit algorithm is a sub-optimal solution but as the heuristic give relatively small error this can’t be considered as drawback. The major advantages of matching pursuit in contrary to the other time-frequency methods is that provides an optimal non-orthogonal selection of basis atoms and full parametrization of these atoms. The above features of matching pursuit is very important in time-frequency analysis of non-stationary signal like EEG/ERP, whose temporal changes in energy are significant as they are associated with functional brain activation.

No other methods possess these properties. For example, the Fourier analysis which used in spectrogram localization in time and frequency depends on epoch length (the STFT spectrogram divides the analyzed signal into overlapping epochs). Continuous wavelet transform or Cohen’s class transforms do not provide parametric description. Moreover Cohen’s class transforms are biased by cross-terms. Discrete wavelet transforms, which used in scalogram give parametric descriptions, but their time-frequency resolution is severely limited as it has high temporal resolution and low frequency resolution at high frequencies, and low temporal resolution and high frequency resolution at low frequencies [5].

According to earlier works the matching pursuit algorithm provides a unified parametrization of EEG, applicable in a variety of several standard research and clinical problems, encountered in analysis of evoked potentials [6], automatic detection and analysis of sleep spindles in overnight EEG recordings [7], ERD/ERS [6][8], pharmaco-EEG [9] and epileptic seizures [10][11].

Methods

Matching Pursuit Algorithm

As already mentioned, matching pursuit is an iterative procedure which provides a mathematical description (parametrization) of the signal.

First of all it needs to be generated a redundant set of time-frequency atoms which is called dictionary *D*. In this paper we construct the dictionary *D* from Gabor functions:

$$g_{\gamma}(t) = K(\gamma)e^{-\pi\left(\frac{t-u}{\sigma}\right)^2} \sin(2\pi f(t-u)+\phi) \tag{1}$$

K(γ) is a coefficient such that $||g_\gamma||=1$, $\gamma = \{u, f, \sigma, \varphi\}$ denotes parameters of the dictionary's functions where u is the translation in time, f the frequency, σ the spread in time, φ the phase.

Except of Gabor dictionaries, a wavepacket dictionary which was built with Daubechies 6 quadrature mirror filter has been also used. The functions of latter dictionary are not as well localized in time and frequency as Gabor functions and also do not include a phase parameter and thus cannot match signal components as well as Gabor functions. These drawbacks of wavepacket dictionary make Gabor dictionary the most appropriate for EEG signal decomposition.

Afterwards we compute a linear expansion of original signal s over a redundant set of atoms selected from the dictionary, in order to best match its inner structures. This is done by successive approximations of s with orthogonal projections on elements of the dictionary. Let $R^0 s = s$. We suppose that we have computed the n^{th} order residue $R^n s$ for $n \geq 0$. We choose an element $g_{\gamma_n} \in D$ from the dictionary D which best match the signal $R^n s$ which is the residual left after subtracting the results of previous iterations. The residue $R^n s$ can be also decomposed into:

$$\begin{cases} R^n s = \langle R^n s, g_{\gamma_n} \rangle g_{\gamma_n} + R^{n+1} s \\ g_{\gamma_n} = \arg\max\limits_{g_{\gamma i} \in D} \left| \langle R^n s, g_{\gamma i} \rangle \right| \end{cases} \tag{2}$$

where $argmax_{g_{\gamma i}\in D}$ means the $g_{\gamma i}$ giving the largest value of the product $<R^n s, g_{\gamma i}>$. The iterative procedure of decomposition stops either when the energy of residual signal is below a preset cut-off level ε or, alternative after a predetermined number of iterations, m.

After m iterations, a matching pursuit decomposes a signal s into:

$$s = \sum_{n=0}^{m-1} \langle R^n s, g_{\gamma_n} \rangle g_{\gamma_n} + R^m s \tag{3}$$

where $R^m s$ is the residual vector after m iterations and $\langle s,g \rangle = \int_{-\infty}^{\infty} s(t)\overline{g}(t)dt$ denotes inner product of functions s and g.

Because the orthogonality of $R^{n+1}s$ and to $g_{\gamma n}$ is valid in each step of the procedure, the form of energy conservation law becomes:

$$\|s\|^2 = \sum_{n=o}^{m} \left| \langle R^n s, g_{\gamma_n} \rangle \right|^2 + \|R^m s\|^2 \tag{4}$$

When the iterative procedure terminates the selection of Gabor atoms from dictionary is completed. After that we derive a time–frequency energy distribution by adding the pseudo Wigner-Ville distribution of the selected atoms that is free of cross terms.

The pseudo Wigner-Ville distribution is a Wigner-Ville distribution in which the infinite integrals have been replaced by finite integrals or, equivalently, a windowing function. More analytically, the pseudo Wigner-Ville distribution can be expressed as:

$$\begin{aligned} PW_x(t,f) = \int_{-\infty}^{\infty} h(\tau) \int_{-\infty}^{\infty} \kappa(u-t)... \\ ...x(u+\frac{\tau}{2})x^*(u-\frac{\tau}{2})e^{-j2\pi f\tau} du d\tau \end{aligned} \tag{5}$$

where with h and k Hanning windows corresponding to frequency and time smoothing respectively.

The representation of signal's spectral density which is constructed using equation (5) satisfies the time and frequency marginals. The use of matching pursuit is able to remove cross terms which might lead to misinterpretation and provides an accurate picture of the energy distribution in the time-frequency plane.

The definition of the time-frequency distribution of energy of the signal which described in (6) is:

$$E^{MP}(t,f) = \sum_{n=o}^{m} \left| \langle R^n s, g_{\gamma_n} \rangle \right|^2 PWg_{\gamma_n}(t,f) \tag{6}$$

In time-frequency signal analysis has almost universally used analytic signal which is a complex signal that contains both real and imaginary components. The imaginary part is obtained by Hilbert transform. The advantage of using the analytic signal is that in the frequency domain the amplitude of negative frequency components are zero. This satisfies mathematical completeness of the problem by accounting for all frequencies, yet does not limit the practical application since only positive frequency components have a practical interpretation. Moreover, in the Wigner distribution of discrete signal even when the sampling of the signal satisfies the Nyquist criterion, there are still aliasing components. A simple approach to avoid aliasing, as J. Ville proposed, is to use an analytic signal before computing the distribution. [12][13] and for this reason the Hilbert transform of selected atoms is calculated.

Time-frequency resolution of signal's representation depends on the uncertainty principle of Heisenberg [14] given by the equation

$$\Delta t \Delta f \geq \frac{1}{4\pi} \tag{7}$$

As the time-frequency plane must be calculated with a specific resolution, a discretization of time–frequency maps into resels (resolution elements) take place.

$$E^{MP}(i,j) = \int_{(i-1)\Delta t}^{i\Delta t} \int_{(i-1)\Delta f}^{i\Delta f} E^{MP}(t,f) dt df \tag{8}$$

Statistical Tests

In this section, a statistical framework is described in order to quantify statistical diferrentiations in the time-frequency plane. The null hypothesis is that there is no difference of the mean energies of two groups (controls, dyslectics) in a given resel. In order to use robust parametric statistical tests, possibly non-normal distributions of the energy values must be taken into account. So where data normality is not satisfied, the logarithmic transformation is applied that is found to be the best transformation for the absolute power [15]. The *t*-statistic is given by the equation

$$t(i,j) = \frac{tr\left(E^{MP}_{controls}(i,j)\right) - tr\left(E^{MP}_{dyslectics}(i,j)\right)}{s(i,j)} \qquad (9)$$

where *tr(E)* is the transformed energy (where is needed) and *s(i,j)* is the pooled variance of the two groups in the investigated resel. If the variances are statistically unequal, the degrees of freedom are adjusted according to Welch's correction.

As simultaneous tests for each resel (which is not independent to neighbor resels) take place the problem of multiple comparisons arises. To deal with this problem the false discovery rate (FDR) is applied. The FDR is tested for the significance level q=0.05. This methodology sorts the significancies p_i, $i=1,\dots,r$ (r: total number of resels) of all resels in an ascending order. Then the metric

$$k = \max\left\{ i : p_i \le \frac{i}{r\sum_{j=1}^{r} 1/j} q \right\} \qquad (10)$$

is calculated rejecting all hypotheses for which $p \le p_k$.

Subjects and Procedures for Signal Acquisition

Fifty seven (57) children participated in the experiment. Thirty eight (26 boys and 12 girls) of them were outpatient cases who had been diagnosed as suffering from learning disorders (dyslexia) according to the 10th edition of the International Classification of Diseases (ICD-10) and the rest 19 children (7 boys and 12 girls) were their healthy siblings. The mean ages for the dyslectic children and for the controls were 11.47±2.12 and 12.21±2.25 years, respectively.

The children's EEG signals were recorded at 15 electrodes (Fp1, F3, C5, C3, Fp2, F4, C6, C4, O1, O2, P4, P3, Pz, Cz, Fz) according to the 10–20 international system, referred to both earlobes. For more experimental details see Papageorgiou et al [16]. Raw EEG was sampled for 500msec with sampling frequency 1 kHz, thus oversatisfying the Shannon theorem. In order to remove the EEG noise, the total procedure consisted of 52 repetitions and the finally taken signal was the average of these repetitions.

Results

It has been observed that the N100 peak of dyslexics is reduced compared to controls in the majority of electrodes. These can be expressed as reduced energy in this time period. Characteristic waveforms can be seen in figure 1.

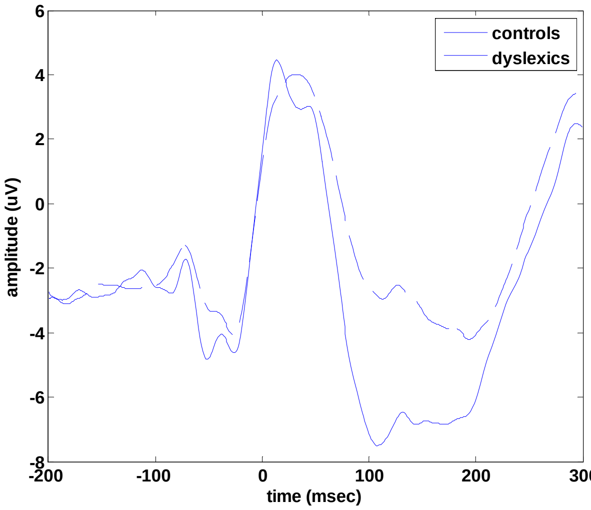

Figure 1: Mean waveforms of electrode Fz for controls (solid line) and dyslexics (dashed line). The time point 0 denotes the onset of the warning stimulus and ERP.

We want to represent these energies' differences and their evolution in both time and frequency domain. The matching pursuit decomposition algorithm and the pseudo Wigner-Ville Distribution were applied to the EEG/ERP signals of the participants in the experiment in order to extract time-frequency maps. The procedure was stopped when the decomposition had subtracted the 99% of the energy of the signal. The maps were discretized into resels of 0.02s x 4Hz in order to comply with Heisenberg theorem. When it was needed the logarithmic transformation was applied in order to achieve data normality.

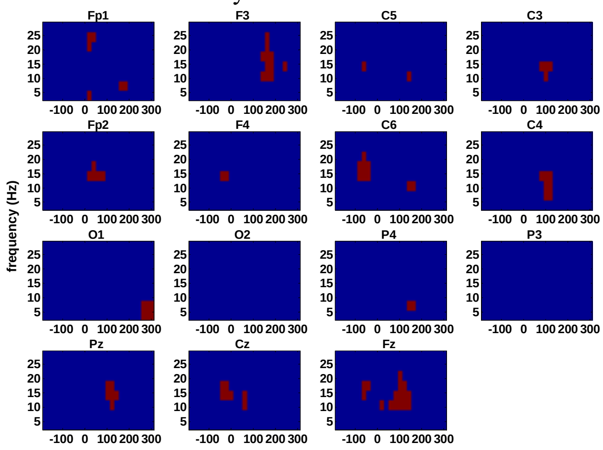

Figure 2: Time-frequency statistics for all 15 electrodes. Red color denotes rejection of null hypothesis (p<0.05)

Then, the statistical test described in section methods took place to find mean energy statistical differences between the two groups (controls, dyslexics) in each resel. The results were corrected for multiple comparisons with FDR whose significant level was set at 0.05. The statistical significances of the tests for all electrodes are shown in figure 2. It can be observed that differences lie mainly in the time period 50-200msec after the warning stimulus and contain frequencies 5-20Hz.

Discussion

The purpose of this work was the study of mean energy statistical differences between controls and dyslexics in time-frequency plane. A statistical framework was proposed in order to calculate statistical time-frequency differences facing the problem of multiple comparisons and normality of data.

It was shown that there are differences between the two examined groups mainly around N100 ERP component where a preliminary analysis showed that dyslexics have reduced mean peaks in absolute value. In all others ERP peaks, there were not abserved deviations between the two groups. The method appear to identify that most statistical differences lie in this time period. Also it appears that differences lie in specific frequencies (5-20Hz) which are the dominant frequenies in the N100 ERP component. What is important to note is that most of electrodes have similar behavior within the groups so differences appear in specific resels.

The differences are appeared in frontal and central areas of brain (electrodes Fp1, F3, C5, C3, C6, C4, Fz). On the other hand, minimal differences are appeared in parietal or occipital areas.

Conclusions

The proposed method can provide a quantitative means to assess statistical differences in averagy energies between two groups in time-frequency plane. It can provide insights to energy variations along time. This is of great importance mainly for non stationary signals like ERP where traditional spectral analysis (eg STFT) may be inaccurate.

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