SPECTRAL ENTROPY OF DYSLEXIC ERP SIGNAL BY MEANS OF ADAPTIVE OPTIMAL KERNEL

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ABSTRACT

In this paper, subband spectral entropy (SSE) and its relative form was used for the analysis of rest electroencephalogram (EEG) and Event Related Potentials (ERP). The recorded signals were taken from control children and children with dyslexia. Adaptive-Optimal-Kernel (AOK) time-frequency representation was used to produce high resolution spectrogram. Then, SSE and relative subband spectral entropy (RSSE) were calculated. The entropic patterns of both controls and dyslexics were investigated showing differences in specific electrode recordings.

Index Terms— EEG, ERP, spectral entropy, adaptive optimal kernel, time-frequency representation

1. INTRODUCTION

Electroencephalogram (EEG) is a well established tool for monitoring human brain function and understanding its mechanisms. It reflects thousands of simultaneously ongoing brain processes during cognitive functions. In many cases, useful conclusions can be drawn from the brain response to a certain stimulus or event of interest and this has lead to an increasing number of such studies in cognitive neuroscience.

Event-related potentials (ERP) are electrical signals reflecting cerebral activity which is associated with internal or external stimulus.

ERP present specific time-locked peaks that are characteristic indices of memory, attention and processing. Their oscillatory activity is increasingly thought to get synchronized during physiological or pathological brain states or during different mental states. These synchronizations can be divided in different frequency ranges (delta, theta, alpha, beta, gamma) and may occur simultaneously or may be partly or fully overlapping after the stimulus. This can affect the nature of signal and in some cases can provide useful information about brain dynamics.

A combination of both power and frequency characteristics that can be employed in such cases is entropy. Various entropy measures can be employed. In the time domain, measures like approximate entropy [1] or Shannon entropy [2] are available. In the frequency domain, spectral entropy [3] can be computed. A combination of time and frequency measures in the calculation of entropy can provide more complete information.

Spectral entropy is a measure of a signal’s complexity providing information about how widespread or narrow its spectrum is. A sinusoidal signal which contains in frequency domain only a single peak is characterized by a low entropy value, whereas a signal with many spectral components frequency is characterized by high entropy. Noise which contains almost equal spectral information in all frequencies appears to have the higher values of entropy. When the research is oriented in frequency bands, someone can use subband spectral entropy which investigates the distribution of subband energies in relation to total spectrum. Also when a signal contains significant temporal changes in its characteristics, a time evolution of entropy should be estimated.

Various techniques have been used in this direction. The conventional Fourier transform or the short time Fourier transform cannot reliably assess spectral entropy in cases of rapid changes of signal characteristics due to their low temporal resolution. To overcome these limitations, Quiroga introduced wavelet entropy (WE) [4], which is based on the time-frequency decomposition by means of the wavelet transform (WT). This technique has already applied in EEG/ERP signal analysis [5][6][7]. The problem with this approach is that results are strongly dependent on the selection of mother wavelet function. Different basis functions can produce different results making their interpretation sometimes ambiguous.

An alternative way is to produce high resolution spectrograms, which can accurately represent the time-frequency properties of a signal. The most important inherent problem in time-frequency representations is the existence of cross-terms, i.e. TF correlations of various components, making the interpretation of multi-component signals very difficult. To overcome this problem, Adaptive-Optimal-Kernel (AOK) time frequency representation [8], a
quite effective method in representing signals in the TF plane, can be employed. The main advantage of AOK is that it is an adaptive signal-dependent method where in each case the kernel is selected according to how well it is suited to signal’s characteristics. Therefore, high resolution TF analysis is achieved, while cross terms are attenuated. The method is adjusted by the choice of the kernel which involves a compromise between cross term reduction, loss of time frequency resolution and maintenance of certain properties of distribution [9][10].

In this study, spectral entropy was used to study the evolution of spectral distribution of ERP response of dyslexics, as compared with control children. Auditory stimuli have been demonstrated to generate multiple event-related oscillations in delta, theta, alpha, gamma frequency ranges [10]. In addition, it was investigated whether there are different entropy patterns between controls and dyslexics in the ERP during auditory stimulus processing and the spatio-temporal appearance of such patterns. Also, the study examined whether the ERP order/disorder is influenced by the frequency distribution of the background rest EEG.

2. METHODS

2.1. Adaptive Optimal Kernel Time-Frequency Representation

The Adaptive-Optimal-Kernel (AOK) Time-Frequency Representation (TFR) is a method that uses radially-Gaussian signal-dependent kernels which change shape to optimally smooth the distribution [12]. The TFR of a signal \( x(t) \) is given by

\[
P(t, \omega) = \int A(\theta, \tau) \cdot \Phi(\theta, \tau) e^{-j(\omega - j\omega) \tau} \, d\theta d\tau
\]

where \( A(\theta, \tau) \) is the ambiguity function (AF)

\[
A(\theta, \tau) = \int x(t + \frac{\tau}{2}) \cdot x^*(t - \frac{\tau}{2}) e^{j\theta} \, dt
\]

and \( \Phi(\theta, \tau) \) is the used kernel. Expressing equation (2) in polar coordinates, \( \psi = \arctan(\tau / \theta), r = \sqrt{\theta^2 + \tau^2} \),

\[
\Phi(r, \psi) = e^{-\frac{\sigma^2 r^2}{2}}
\]

where \( \sigma(\psi) \) is the spread function controlling the spread of the Gaussian at radial angle \( \psi \). The TFR is more accurate when the kernel can well match the components of the signal. The optimal kernel is provided by solving the following optimization problem,

\[
\max_{\phi} \int_{0}^{2\pi} \int_{0}^{\infty} |A(r, \psi) \cdot \Phi(r, \psi)|^2 r \, dr \, d\psi
\]

subject to constraint

\[
\frac{1}{4\pi} \int_{0}^{2\pi} \sigma^2(\psi) \, d\psi \leq \alpha
\]

where \( \alpha \) is the kernel volume.

AOK is quite effective in representing signals in the time-frequency domain, especially in representing complex signals that contain many components varying in time.

2.2. Spectral Entropy

Entropy is a measure derived from thermodynamics which describes the degree of disorder of a physical system. In signal analysis, spectral entropy describes how widespread or concentrated the spectrum is. In cases of dynamic signals it is important to investigate the temporal evolution of spectral distributions. The spectrogram \( S(t,f) \) provides the spectral density of an EEG signal \( x(t) \) at time point \( t \), \( t = t_1, ..., t_N \) and frequency levels \( f, f = f_1, ..., f_J \). The energy at each time sample \( t \) can be calculated by the equation

\[
E(t) = \sum_{f = f_1}^{f_j} |S(t, f)|^2
\]

and the total energy is given as

\[
E_{\text{total}} = \sum_{f = f_1}^{f_j} \sum_{t = t_1}^{t_N} |S(t, f)|^2 = \sum_{f = f_1}^{f_j} E_f
\]

The relative energy \( p_f \) which describes energy’s probability distribution in frequency levels, is given as

\[
p_f = \frac{E_f}{E_{\text{total}}}, \quad f = f_1, ..., f_j
\]

The distribution \( p_f \) is the spectral probability density function where \( \sum_{f = f_1}^{f_j} p_f = 1 \). The time-varying spectral entropy can be calculated by eq. (9)

\[
H(p_f) = -\sum_{f = f_1}^{f_j} p_f \cdot \log_2[p_f]
\]

The overall procedure is shown in Fig. 1.
2.3 Time-Varying Subband Spectral Entropy

When the temporal evolution of spectral entropy is needed, the analyzed signal can be divided in temporal windows of length \( l \). Each time window, denoted by index \( i \), can be produced by moving its starting point by a step of \( \Delta m \) samples. Sometimes it is important to investigate entropy in frequency subbands and not in single frequencies contained in the signal. The discretized energy in a specific time interval \( i \) and a frequency subband \( j \) is given by

\[
E(i, f) = \sum_{i=(i-1) \Delta m}^{(i+1) \Delta m} \sum_{f-j \Delta f}^{(f+j) \Delta f} E(t, f), \quad i = 1, \ldots, \left\lfloor \frac{N-1+\Delta m}{\Delta m} \right\rfloor
\]

where \( f_j \), \( \Delta f \) and \( N_f \) are the starting frequency, the length of the subband, and the number of subbands respectively. The total energy in each time interval is

\[
E(i, \text{total}) = \sum_{j=1}^{N_f} E(i, j)
\]

The relative energy is computed as

\[
p(i, j) = \frac{E(i, j)}{E(i, \text{total})}
\]

Finally, the spectral subband entropy for each time interval \( i \) is

\[
H^{(i)}(p) = -\frac{1}{N_f} \sum_{j=1}^{N_f} p(i, j) \cdot \log_2[p(i, j)]
\]

The obtained value is assigned to the central point of the time window.

2.4. Relative Subband Spectral Entropy and Subband Spectral Entropy Change

Relative Subband Spectral Entropy (RSSE) is another measure of order/disorder between two waveforms. It depicts how similar a probability distribution \( p_j \) is with respect to another probability distribution \( q_j \) taken as a reference. The probability distributions \( p_j \), \( q_j \) could represent two different signals or two different parts of the same signal. Considering that a chosen part of a signal, that is considered as reference, contains \( N_T \) time windows, the mean energy at level \( f_j \) is given as

\[
\overline{E}_j = \frac{1}{N_T} \sum_{i=1}^{N_T} E(i, j)
\]

The mean probability distribution \( \tilde{q}_j \) that describes the whole reference time interval is

\[
\tilde{q}_j = \frac{\overline{E}_j}{\sum_j \overline{E}_j}
\]
\[ H^{(i)}(p | q) = -\sum_{j=1}^{N_i} p(i, j) \log_2 \left( \frac{p(i, j)}{q_j} \right) \quad (16) \]

and represents the similarity degree of relative spectral entropy of a signal in relation to a reference period.

### 2.5. Subjects

In the experiment, 38 patients (twenty six boys and twelve girls with mean age and standard deviation 11.47±2.12 years) fulfilling the criteria of dyslexia as described in the 10th edition of the International Classification of Diseases (ICD-10), were studied. The patients were recruited from the Department of Psychiatry of the Eginition Hospital in Athens, where the EEG was recorded. The control group consisted of 19 children (7 boys and 12 girls with mean age and standard deviation 12.21±2.25 years). The statistical t-test for the mean ages of the two groups did not show any significant differences.

The local ethics committee approved the study. All control subjects and all caregivers of the demented patients gave their informed consent for participation in the current study. An EEG was recorded from all patients and controls. All of them had undergone assessment of educational attainment including reading, comprehension, spelling and arithmetic ability. Participants with hearing difficulties, history of head injury, attention deficit disorders or neurological disease were not included in the study.

### 2.6. Data recording and acquisition

The EEG and ERP were recorded from the 15 scalp loci of the international 10–20 system (channels Fp1, F3, C5, C3, Fp2, F4, C6, C4, O1, O2, P4, P3, Pz, Cz, Fz), with all electrodes referenced to the chin. An electrode placed on the subject's forehead served as ground. Data were recorded, with the subject in a relaxed state, awake and with closed eyes so as to minimize eye movements. Eye movements were recorded through electro-oculogram (EOG) and recordings with EOG higher than 75 µV were rejected. EEG data were first sampled at 1 kHz so that for signals in the frequency range 0–35 Hz the Shannon theorem was over satisfied. For each trial of the experiment, rest EEG signal was recorded for 500 msec before the stimulus and ERP was recorded for 1000 msec after the stimulus onset. The extraction of ERP signal was performed by averaging across the 52 trials of the experiment. Random brain activity was considered to be a random process with zero mean value. Averaging technique caused random brain activity or noise to be averaged out improving ERP’s SNR (ERP in relation to random brain activity and noise).

### 3. RESULTS

Entropy measures were estimated for 15 channels (Fp1, F3, C5, C3, Fp2, F4, C6, C4, O1, O2, P4, P3, Pz, Cz, Fz and T6) and for signals corresponding to high frequency warning stimulus. The AOK time-frequency representation was used in order to produce high resolution spectrogram. The spectrogram was cut off at 32 Hz, as in this frequency range more than 99% of total signal’s energy is concentrated, in all cases. Then a grid partitioning time and frequency axes was used. The time domain was divided into overlapping time windows of \( t = 128 \) msec length and step of \( \Delta m = 20 \) samples. It was shown that such window size is appropriate for the analysis of EEG signals, as the entropy does not increase significantly by increasing the window size. The frequency domain was divided in four subbands, delta (0-4Hz), theta (4-8Hz), alpha (9-16) and beta (16-32Hz).

Then, the relative subband spectral entropy was calculated. The reference period was defined as the time interval 0-500 msec, the interval before warning stimulus, corresponding to rest EEG recordings. The results were averaged within groups (controls, dyslexics) for each electrode. The mean waveforms of RSSE for controls and dyslexics are shown in Fig. 2.

![Fig. 2: Mean Relative Subband Spectral Entropy of 15 electrodes for controls (blue line) and dyslectics (red line) for high frequency stimulus.](image)

It can be observed that RSSE changes intensely at the stimulus onset. This makes clear that signal nature and consequently spectral entropy changes in the transition from rest EEG to ERP. In most cases, spectral entropy comes back to its rest EEG values after a short time period. On the other hand, some electrodes maintain the divergence from reference period for a long time, especially in control subjects.

Statistical analysis was performed in order to evaluate different entropic patterns between the two groups. The null hypothesis was that there is no difference
of the mean values of RSSE between the two groups (controls, dyslectics) in a given electrode at a specific time window. In order to use robust parametric statistical tests, possible non-normal distributions of the energy values must be taken into account.

When the normality of data was not satisfied, the logarithmic transformation [13] was applied. A picture of comparative statistics of RSSE is shown in Fig. 3. It can be observed that there are many electrodes (7 out of 15) that differentiate 2 groups in the first 500 msec. In electrodes C5, C3, O1, P4, RSSE of controls was bigger than that of dyslectics and in electrodes Fp1, F4 the opposite.

4. DISCUSSION

In this study, subband spectral entropy and relative subband spectral entropy were used in order to reveal differences in the time evolution of spectral band distribution in EEG/ERP signals between dyslectics and controls.

Analysis showed that there is an intense change of RSSE after the stimulus onset. This is ought to the fact that ERP, especially in its early components, changes its characteristics in relation to rest EEG. RSSE is a measure describing this dissimilarity. In most electrodes, RSSE return to rest EEG values, whereas in some specific electrodes the RSSE change is maintained for a long time period. Controls appear to have higher RSSE values than dyslectics. According to this, controls seem to react better to the external stimuli as it is reflected in their EEG recordings.

Considering the above results, it is reasonable to hypothesize that the dyslexia-associated differences observed here may be related to different strategies activated due to dyslexia-related functional brain organization as indicated from psychophysiological and neurobiological studies [14][15]. In corroborations to this notion, there appears to be consistent evidence that EEG and ERP patterns vary systematically with dyslexia [16].

Results show that the proposed method can describe the dynamic complexity of EEG and the transition from rest EEG to ERP. The spectral distribution can provide useful information about signals’ nature which can not be extracted easily by traditional statistical methods.

5. REFERENCES

