Camera Self-Calibration Using the Singular Value Decomposition of the Fundamental Matrix

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ABSTRACT

This paper deals with a fundamental problem in motion and stereo analysis, namely that of determining the camera intrinsic calibration parameters. A novel method is proposed that follows the autocalibration paradigm, according to which calibration is achieved not with the aid of a calibration pattern but by observing a number of image features in a set of successive images. The proposed method relies upon the Singular Value Decomposition of the fundamental matrix, which leads to a particularly simple form of the Kruppa equations. In contrast to the traditional formulation that supplies an overdetermined system of constraints, the derivation proposed here provides a straightforward answer to the problem of determining which constraints to employ among the set of available ones. Moreover, the derivation is a purely algebraic one, without a need for resorting to the somewhat non-intuitive geometric concept of the absolute conic. Apart from the fundamental matrix, no other quantities that can be extracted from it (e.g., the epipoles) are needed for the derivation. Experimental results demonstrate the effectiveness of the proposed method in accurately estimating the intrinsic calibration matrices for several image sequences.

Keywords: Image Sequence Analysis, Stereo and Motion, Self-Calibration, Kruppa Equations.

1 Introduction

It is well-known that a pinhole camera acts like a projective transformation device [4]. Numerous vision tasks, however, ranging from visual navigation and 3D reconstruction to novel view synthesis and augmented reality, require the computation of metric (i.e. Euclidean) quantities from images. In order to facilitate this, the problem of determining the intrinsic calibration parameters of the camera needs to be solved [5]. Early approaches for coping with this problem rely upon the presence of an artificial calibration object in the set of captured images [17]. Knowledge of the 3D shape of the calibration object supplies the 3D coordinates of a set of reference points in a coordinate system attached to the calibration object. Thus, the transformation relating the 3D coordinates to their associated image projections can be recovered through an optimization process. The major drawback with such approaches is that they are suitable for off-line calibration only. In other words, they are insufficient in cases where the intrinsic parameters undergo constant changes due to focusing or zooming.

In a seminal paper, Maybank and Faugeras [14] have demonstrated that the calibration problem can be solved without resorting to a calibration object. By tracking a set of points among images of a rigid scene, captured while the camera is pursuing unknown, unrestricted motion, the calibration parameters can be computed by determining the image of the absolute conic. The absolute conic is a special conic lying at the plane at infinity, having the property that its projection depends on the intrinsic parameters only. This fact is expressed mathematically by the so-called Kruppa equations. In following years, several researchers have investigated the application of the Kruppa equations to the self-calibration problem. For example, Zeller [19, 20] and Heyden and ASTRON [9] propose variants of the basic approach. Pollefeys and Van Gool [15] describe a stratified approach to calibration, which starts from projective calibration, augments it with the homography of the plane at infinity to yield affine calibration and finally upgrades to Euclidean calibration. Luong and Faugeras [12] use the Kruppa equations to derive systems of polynomial equations. These equations are of degree four in five unknowns (i.e. the camera intrinsic parameters) and are solved with the use of numerical continuation methods.

In this work, we propose a novel simplification of the Kruppa equations and show how it can be employed for self-calibration. The simplification is derived in a purely algebraic manner and is based solely on the fundamental matrix. Estimates of the epipoles, which are known to be difficult to compute accurately\(^1\), are not needed. Therefore, compared to existing self-calibration methods, the proposed one has the potential of being more stable and robust with respect to measurement noise. The rest of the paper is organized as follows. Section 2 reviews some background material and introduces the notation that is used in the remainder of the paper. Section 3 derives the classical Kruppa equations using a purely algebraic scheme. The simplified Kruppa equations are derived in section 4. Section 5 describes in detail the proposed self-calibration method and discusses some implementation issues. Experimental results from a prototype implementation are presented in section 6. The paper concludes with a brief discussion in section 7.

2 Notation and Background

The projection model assumed for the camera is the projective one. This model projects a 3D point \(\mathbf{M} = [x, y, z]^T\) to a 2D image point \(\mathbf{m} = [u, v]^T\) through a \(3 \times 4\) projection matrix \(\mathbf{P}\), as \(s\mathbf{m} = \mathbf{PM}\), where \(s\) is a nonzero scale factor and the notation \(\mathbf{p}\) is such that if \(\mathbf{p} = [x, y, \ldots, 1]^T\) then \(\mathbf{p} [x, y, \ldots, 1]^T\).

In the case of a binocular stereo system, every physical point \(\mathbf{M}\) in space yields a pair of 2D projections \(\mathbf{m}_1\) and \(\mathbf{m}_2\) on the two images. Those projections are defined by the following relations:

\[s_1\mathbf{m}_1 = \mathbf{P}_1\mathbf{M} , \quad s_2\mathbf{m}_2 = \mathbf{P}_2\mathbf{M}\]  \hspace{1cm} (1)

Assuming that the two cameras are identical and that the world coordinate system is associated with the first camera, the two projection matrices are given by:

\[\mathbf{P}_1 = [A|0] , \quad \mathbf{P}_2 = [AR|At]\]  \hspace{1cm} (2)

where \(R\) and \(t\) represent respectively the rotation matrix and the translation vector defining the rigid displacement between the camera centers.

\(^1\)This is particularly true in the case that the epipoles lie at infinity.
By employing Laguerre’s formula, the angle between the two cameras. Matrix $A$ is the $3 \times 3$ intrinsic parameters matrix, having the following well-known form [4, 5]:

$$
A = 
\begin{bmatrix}
\alpha_u & -\alpha_v \cos \theta & \nu_u \\
0 & \alpha_v \sin \theta & \nu_v \\
0 & 0 & 1
\end{bmatrix}
$$

The parameters $\alpha_u$ and $\alpha_v$ correspond to the focal distances in pixels along the axes of the image, $\theta$ is the angle between the two image axes and $(\nu_u, \nu_v)$ are the coordinates of the image principal point. In practice, $\theta$ is very close to $\pi/2$ for real cameras [4].

Let $K$ denote the symmetric matrix $AA^t$. By eliminating the scalars $s_1$ and $s_2$ associated with the projection equations (1), the following equation relating the pair of projections of the same 3D point is obtained:

$$
\mathbf{m}_2^T \mathbf{f} \mathbf{m}_1 = 0
$$

In this equation, matrix $F$ is the fundamental matrix, given by

$$
F = A^* [t], R A^{-1} \tag{4}
$$

where $A^* = (A^{-1})^t$ is the adjoint matrix of $A$ and $[x]_\times$ denotes the antisymmetric matrix of vector $x$ that is associated with the cross product. This matrix has the property $[x]_\times y = x \times y$ for each vector $y$ and has the following analytic form:

$$
[x]_\times = 
\begin{bmatrix}
0 & -x_3 & x_2 \\
x_3 & 0 & -x_1 \\
-x_2 & x_1 & 0
\end{bmatrix}
$$

The fundamental matrix $F$ describes the epipolar geometry between the pair of views considered. It is the equivalent of the essential matrix $E = [t], R$ in the uncalibrated case, as dictated by (see also Eq.(4))

$$
F = A^* E A^{-1} \tag{5}
$$

Due to the above relation, $E$ can be written as a function of $F$ as follows:

$$
E = A^* F A \tag{6}
$$

As pointed out by Trivedi [16], the symmetric matrix $E E^t$ is independent of the rotation $R$ since

$$
E E^t = [t]_\times R R^t [t]_\times = [t]_\times ([t]_\times)^t \tag{7}
$$

Substitution of Eq.(6) into the above equation yields

$$
F F^t = A^* [t]_\times ([t]_\times)^t A^{-1} \tag{8}
$$

This equation will be employed in subsequent sections for algebraically deriving the Kruppa equations.

Knowledge of the calibration matrix $A$ enables certain 3D Euclidean measurements to be made directly from the images. More specifically, the angle between two 3D line segments $L_1$ and $L_2$ can be computed as follows [19]: Let $l_1, l_1$, and $l_2$ be the projections of $L_1$ and $L_2$ in the two images. By employing Laguerre’s formula, the angle between $L_1$ and $L_2$ is shown to be given by

$$
\cos(L_1, L_2) = \frac{|S(v_1, v_2)|}{\sqrt{S(v_1, v_1) S(v_2, v_2)}} \tag{9}
$$

where $v_1$ and $v_2$ are the projections in the first image of the intersections of $L_1$ and $L_2$ with the plane at infinity and

$$
S(m, n) = m^T A^{-1} A^{-1} n \tag{10}
$$

Points $v_1$ and $v_2$ are determined by

$$
v_1 = l_1 \times H_{\infty} l_1', \quad v_2 = l_2 \times H_{\infty} l_2',
$$

where $H_{\infty}$ is the homography of the plane at infinity, corresponding to the rotational component of motion and being defined by

$$
H_{\infty} = A R A^{-1} \tag{11}
$$

3 Deriving the Classical Kruppa Equations

In this section, the well-known Kruppa equations are derived in a simple and purely algebraic manner, i.e. without making use of the absolute conic [14, 5] or the plane at infinity [19]. Part of this derivation will be later employed for obtaining the simplified equations proposed in this paper.

We start by using Eq.(4) to compute the epipole $e_i$ in the second image. Given that $F e_i = 0$, $e_i$ must satisfy

$$
A^{-1} R^t ([t]_\times)^t A^{-1} e_i = 0 \tag{12}
$$

Owing to the fact that $( [t]_\times)^t t = 0$, $e_i$ is obtained as $e_i = \lambda \tilde{a}$, where $\lambda$ is a nonzero scalar. This equation also supplies the following expression for $t$:

$$
t = \lambda' A^{-1} e_i, \tag{13}
$$

where $\lambda' = 1/\lambda$. Equation (13) yields matrix $[t]_\times^2$ as $[t]_\times = \lambda' det(A^{-1}) A [e_i]_\times A$. Substitution of this last relation into Eq.(8), yields directly the Kruppa equations:

$$
F F^t [e_i]_\times K([e_i]_\times)^t = \frac{F F^t}{12} = \frac{F F^t}{13} = \frac{F F^t}{22} = \frac{F F^t}{23} = \frac{F F^t}{33} \tag{14}
$$

These equations, however, are linearly dependent since

$$
(F F^t - \beta [e_i]_\times K([e_i]_\times)^t) e_i = 0 \tag{16}
$$

As shown in [19], there are only two independent equations among the set of the six equations given by Eq.(15). These equations are second order polynomials in the coefficients of $K$, and therefore of order four in the coefficients of $A$. Thus, in the case of autocalibration, it is common to start by estimating $K$ and then use Cholesky decomposition [6] to obtain $A$.

At this point, it should be noted that the question of deciding which two equations out of the total six to use, remains open. Up to now, this problem has been resolved either by employing a parameterization of the epipolar geometry as in [14, 12], or by randomly selecting one equation for estimating the scale factor and then substituting the result into two others that are arbitrarily chosen among the remaining five ones [1]. The alternative of taking into account all equations simultaneously can also be considered, although numerical methods usually fail to produce a solution in the case of high order, overdetermined polynomial systems such as this. In the following section, a simple answer to the above question is provided by an approach which directly leads to three linearly dependent equations, out of which two are linearly independent.

4 The Simplified Kruppa Equations

This section develops a simpler variant of the Kruppa equations. The principal motivation is twofold: First, to directly derive less equations than the six of the original formulation, so that the task of choosing the ones to employ for self-calibration becomes simpler. Second, to avoid employing the epipole $e_i$, since its accurate estimation is difficult in...
the presence of noise and/or degenerate motions. Towards this end, the Singular Value Decomposition (SVD) [6] of the matrix $F$ is employed:

$$F = UDV^t$$  \hspace{1cm} (17)

Recalling that $F$ is of rank 2, the diagonal matrix $D$ has the following form:

$$D = \begin{bmatrix} r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $r$ and $s$ are the eigenvalues of the matrix $FF^t$, whereas $U$ and $V$ are two orthogonal matrices. By making use of this relation, the epipole in the second image $e'$ can be deduced very simply. Specifically,

$$F'e' = VD^tU'e' = 0$$  \hspace{1cm} (18)

Since $D$ is a diagonal matrix with a last element equal to zero, the following direct solution for $e'$ is obtained:

$$e' = \gamma Um, \quad \gamma \neq 0$$  \hspace{1cm} (19)

with $m = [0, 0, 1]^t$. Therefore, the matrix $[e']_\times$ is equal to

$$[e']_\times = \mu UMU^t$$  \hspace{1cm} (20)

where $\mu$ is a nonzero scale factor and $M = [m]_\times$ is given by:

$$M = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

By substituting Eq.(20) into Eq.(8), a new expression for the Kruppa equations is obtained:

$$FKF' = \mu UMU'^tKUM'^tU'$$  \hspace{1cm} (21)

Since $U$ is an orthogonal matrix, left and right multiplication of Eq.(21) by $U'^t$ and $U$ respectively, yields the following notably simple expression for the Kruppa equations:

$$DV'^tKVD' = \mu MU'^tKUM'^t$$  \hspace{1cm} (22)

Because of the simple forms of the matrices $D$ and $M$, relation (22) corresponds to three linearly dependent equations. Indeed, denoting by $u_1, u_2, u_3$ the column vectors of $U$ and by $v_1, v_2, v_3$ the column vectors of $V$, the matrix equation (22) is equivalent to

$$DV'^tKVD' = \begin{bmatrix} r^2v_1^tKv_1 & rs^t v_1^tKv_2 & 0 \\ rs^t v_2^tKv_1 & s^2v_2^tKv_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$MU'^tKUM'^t = \begin{bmatrix} u_2^tKu_2 & -u_2^tKu_1 & 0 \\ -u_1^tKu_2 & u_1^tKu_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The above expressions finally yield the following three linearly dependent equations for the matrix $K$:

$$r^2v_1^tKv_1 = rs^t v_1^tKv_2 = s^2v_2^tKv_2$$  \hspace{1cm} (23)

Only two of these three equations are linearly independent. They are the simplified Kruppa equations, derived in a particularly straightforward manner. Moreover, the use of the SVD has enabled us to deduce automatically which three out of the six equations present in the original formulation should be taken into account. It is worth noting that these equations are closely related to the generalization of the Kruppa equations that has been proposed by Luong [13]. Luong has used these equations for demonstrating the equivalence between the constraints of Trivedi [16] and those of Huang and Faugeras [10]. The same ideas can also be found at the origin of the recent article by Hartley [8], who directly derives the Kruppa equations from the fundamental matrix. Both Luong and Hartley base their developments on changes of rather astute coordinate reference frames, which amount to generalizing the Kruppa equations by considering that the absolute conic can have two different images in each retina. In this work, a different approach is taken which remarkably simplifies the task of self-calibration. Using an algebraic method, a simpler variant of the Kruppa equations is derived without making use of the absolute conic. Experimental results from the calibration of real image sequences using the proposed method are also reported. To the best of our knowledge, this is the first time that experimental results obtained from the application of the simplified Kruppa equations to real imagery are being reported.

5 Self-Calibration Using the Simplified Kruppa Equations

In this section, the application of the simplified Kruppa equations to the problem of self-calibration is presented and related implementation issues are clarified. Following the approach of [19], the equations derived in section 4 are embedded in a non-linear optimization framework and solved iteratively. We begin with a discussion regarding the choice of an appropriate initial solution that forms the starting point for the optimization stage. We then formulate the optimization problem and explain how it is solved to obtain the intrinsic calibration parameters.

5.1 Finding an Initial Solution

Let $Sp = [r, s, u_1^t, u_2^t, u_3^t, v_1^t, v_2^t, v_3^t]^t$ be the $20 \times 1$ vector formed by the parameters of the SVD of $F$. Let also $\rho_i(Sp, K)$, $i = 1 \ldots 3$ be the three ratios defined by Eq.(23). Each pair of images defines a fundamental matrix, which in turn yields the following two equations regarding the elements of $K$:

$$\rho_1 (Sp, K) \phi_1 (Sp, K) - \phi_2 (Sp, K) = 0$$

$$\rho_1 (Sp, K) \phi_0 (Sp, K) - \phi_1 (Sp, K) = 0$$

(24)

The above system of equations is of degree two in five unknowns defining the $K$ matrix. A good initial approximation regarding the position of the principal point in the image, is to assume that it coincides with the image center. Additionally, if the skew angle $\theta$ is assumed to be equal to $\pi$, the number of unknowns in Eq.(24) is reduced to two, namely elements $K_{11}$ and $K_{22}$ of the $K$ matrix. Therefore, the system of equations (24) becomes of degree two in two unknowns and thus can be solved analytically. The system can have at most $2^2 = 4$ solutions, some of which might be meaningless. More specifically, every solution for $K_{11}$ and $K_{22}$ which is such that the associated $K$ matrix is not positive definite, is discarded. Solutions are also discarded in the case that the related aspect ratio, defined as the ratio $\frac{K_{11}}{K_{22}}$, is very far from being equal to 1. Assuming the availability of $M$ images that have been acquired with constant camera intrinsic parameters, a total of $N \leq \frac{M(M-1)}{2}$ fundamental matrices can be defined. These matrices give rise to $N$ second order systems of the form of Eq.(24) that have at most $4N$ solutions for the two focal lengths $\alpha_u$ and $\alpha_v$. The following strategies for choosing among the available initial solutions have been examined:

- Use one of the solutions in random.
- Use the average of the available solutions.
- Use the median of the available solutions.

Although the above strategies can produce considerably different starting points, our experiments have indicated that the choice of an initialization strategy is not crucial for the convergence of the non-linear optimization algorithm. In other words, the latter has generated very similar final results, starting from different starting points.
5.2 Non-Linear Optimization

Let \( \pi_{ij}(S_F, K) \) denote the difference \( \phi_i(S_F, K) - \phi_j(S_F, K) \) and let \( \sigma^2_{\pi_{ij}}(S_F, K) \) be its variance. According to [4], this variance can be computed as\(^3\)

\[
\sigma^2_{\pi_{ij}}(S_F, K) = \frac{\partial \pi_{ij}(S_F, K)}{\partial S_F} \Lambda_{S_F} \frac{\partial \pi_{ij}(S_F, K)}{\partial S_F}^T,
\]

where \( \Lambda_{S_F} \) is the 20 \( \times \) 20 covariance matrix associated with \( S_F \). Matrix \( \Lambda_{S_F} \) is computed in a similar manner from the 9 \( \times \) 9 covariance matrix \( \Lambda_F \) of the fundamental matrix \( F \) [11], which is supplied as a by-product of the procedure for estimating \( F \) [3]. As will soon be clear, the variances \( \sigma^2_{\pi_{ij}}(S_F, K) \) are used to automatically weight the residuals \( \pi_{ij}(S_F, K) \) according to their uncertainty.

Matrix \( K \) is computed as the solution of the non-linear least squares problem, namely

\[
K = \text{argmin}_K \sum_{i=1}^{N} \frac{\pi_{12}(S_F, K)}{\sigma^2_{\pi_{12}}(S_F, K)} + \frac{\pi_{13}(S_F, K)}{\sigma^2_{\pi_{13}}(S_F, K)} + \frac{\pi_{23}(S_F, K)}{\sigma^2_{\pi_{23}}(S_F, K)} (26)
\]

Recalling that each fundamental matrix yields two independent equations and \( K \) consists of five unknowns, the minimum number of required displacements (i.e. \( N \)) is in the general case equal to three. Additional constraints provided by more than three fundamental matrices can improve the accuracy of the solution. The reason for including the third simplified Kruppa equation (i.e. \( \pi_{ij}(S_F, K) \)) in Eq.(26), although it is dependent on the other two, is that it further constrains the solution, providing slightly better results [12]. The minimization of Eq.(26) is done using a classical Levenberg-Marquardt algorithm, using the initial solution computed in the previous stage. Apart from \( K \) itself, the minimization in Eq.(26) can also provide its associated covariance matrix. In the case that a priori information in the form of angles or ratios of line segments in the scene is available, it can be incorporated in Eq.(26) as described in [19]. The matrix \( A \) is extracted from \( K \) in three steps. First, \( A^{-1} \) is computed by employing the Cholesky decomposition of \( K^{-1} \), then it is transposed and inverted to yield \( A \).

6 Experimental Results

The proposed calibration method has been extensively validated with the aid of both synthetic and real image sequences. Representative results from several of these experiments are given in this section. To quantify the accuracy of the recovered calibration, the estimated \( A \) matrices along with Eq.(9) have been employed to measure 3D angles from corresponding image line segments. This computation, in addition to the calibration matrix \( A \), requires the homography of the plane at infinity \( \mathbf{H}_\infty \) to be known. To calculate \( \mathbf{H}_\infty \), the essential matrix is first computed from the fundamental matrix using Eq.(6). Then, the essential matrix is decomposed into a rotation matrix \( R \) and a translation vector \( t \), such that \( \mathbf{E} = [t]_3 \times R \). More details concerning this decomposition can be found for example in [18, 7]. Finally, \( \mathbf{H}_\infty \) is computed from \( A \) and \( R \) using Eq.(11).

6.1 Synthetic Experiments

To study the effect of increasing amounts of noise on the recovered intrinsic calibration parameters, a set of experiments using simulated data has been carried out. More specifically, a simulator has been constructed, which given appropriate values for the camera intrinsic parameters and the camera translational and rotational motion, projects a set of randomly chosen 3D points on the simulated retinas. Zero mean Gaussian noise is then added to the coordinates of the resulting retinal points, to account for the fact that in practice, feature extraction algorithms introduce some error when locating image points (i.e. corners). The experimental procedure and the related parameter values for the particular experiments reported here were as follows: The simulated retina is 640 \( \times \) 480 pixels, the principal point is at (310, 270), the angle between the retinal axes is \( \pi/2 \) and the focal lengths are 840 and 770 in horizontal and vertical pixel units respectively. Three rigid displacements of the camera have been simulated and 800 random 3D points have been projected to the corresponding four positions of the simulated retina. The standard deviation of the Gaussian noise added to retinal points was increased from 0 to 1.5 pixels. A non-linear method [21] has then been employed to estimate from the noisy retinal points the six fundamental matrices defined by the three displacements. Following this, the proposed self-calibration method was applied to the estimates of the fundamental matrices to recover the intrinsic calibration matrix. The minimization of Eq.(26) has been performed using four unknowns, i.e. the skew parameter has been assumed to be known. The obtained estimates are shown in table 1. The leftmost column of this table indicates the standard deviation of the Gaussian noise added to retinal points, while the second left column consists of the estimated calibration matrices. To give an indication regarding the suitability of the recovered calibration matrices for metric measurements, they have been used to recover from the retinal projections a set of angles formed by the simulated 3D points. Using these 3D points, 100 random angles were formed. The angles were then measured using the known 3D coordinates of the points defining them. Subsequently, the recovered calibration matrices were combined with the 2D projections of the points defining the angles to reestimate the latter using Eq.(9). The mean and standard deviation of the relative error between the actual angle values and the estimated ones are summarized in the third column of table 1. The values in the parentheses are the error statistics computed when angles were estimated using the true (i.e. not the estimated) calibration and fundamental matrices. These values represent lower bounds for the error in angle measurements from images, since in this case the \( A \) and \( F \) matrices are known with perfect accuracy and the only source of error is the noise corrupting the retinal projections. As can be verified from table 1, the introduction of the estimated matrices in the measurement process increases the error only slightly. This implies that both the calibration and the fundamental matrices have been obtained with satisfactory accuracy.

6.2 Experiments with Real Images

Three experiments performed with real images are reported in this section. In all of these experiments, point matches and the associated fundamental matrices have been computed with the aid of [2]. The line segments employed have been extracted automatically with the aid of a line detector. Finally, throughout all experiments, the skew angle \( \theta \) has been assumed known and equal to \( \frac{\pi}{2} \); matrix \( K \) in Eq.(26) is thus parameterized using four unknowns.

The first experiment is performed using five 512 \( \times \) 768 images of the church of the village of Valbonne. Images 0 and 2 of this sequence are shown in Figs. 1(a) and (b). Self-calibration has been performed using the ten fundamental matrices defined by all possible image pairs. The estimated intrinsic calibration matrix is shown in the top part of table 2. Figures 1(c) and (d) illustrate the line segments used along with the recovered calibration to compute the angles reported in the lower part of table 2. More specifically, the left column of this table indicates the pairs of image line segments defining the angles, the second column from the left corresponds to the ground truth angle values, the third column from the left supplies the cosine of the angles as computed from the im-
### Table 1. Synthetic experiments results.
The left column shows the standard deviation of the noise added to image points, the middle column shows the estimated A matrices and the right column contains the error statistics for the measured angles. The true values for the A matrix are given in the top row of the table.

<table>
<thead>
<tr>
<th>Noise std dev</th>
<th>Estimated A matrix</th>
<th>Angle relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>840.7 / 0 310.1</td>
<td>m 8E-4 (1.04E-5)</td>
</tr>
<tr>
<td></td>
<td>770.6 / 270.6</td>
<td>sd</td>
</tr>
<tr>
<td>0.1</td>
<td>839.0 / 0 314.9</td>
<td>m 0.03 (0.02)</td>
</tr>
<tr>
<td></td>
<td>754.8 / 294.1</td>
<td>sd 0.07 (0.07)</td>
</tr>
<tr>
<td>0.5</td>
<td>841.2 / 0 366.9</td>
<td>m 0.10 (0.08)</td>
</tr>
<tr>
<td></td>
<td>813.2 / 253.5</td>
<td>sd 0.13 (0.12)</td>
</tr>
<tr>
<td>1.0</td>
<td>864.3 / 0 318.1</td>
<td>m 0.14 (0.09)</td>
</tr>
<tr>
<td></td>
<td>784.8 / 385.5</td>
<td>sd 0.12 (0.11)</td>
</tr>
<tr>
<td>1.5</td>
<td>890.6 / 0 210.7</td>
<td>m 0.15 (0.13)</td>
</tr>
<tr>
<td></td>
<td>688.6 / 353.5</td>
<td>sd 0.22 (0.23)</td>
</tr>
</tbody>
</table>

### Table 2. Valbonne church sequence: Estimated intrinsic calibration matrix (top), ground truth and estimated angle values (bottom).

<table>
<thead>
<tr>
<th>Angle segments</th>
<th>Actual angle (deg)</th>
<th>Estimated cosine</th>
<th>Estimated angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 1</td>
<td>90</td>
<td>0.00020432</td>
<td>89.882933</td>
</tr>
<tr>
<td>2 - 3</td>
<td>90</td>
<td>0.02075855</td>
<td>88.810540</td>
</tr>
<tr>
<td>4 - 5</td>
<td>90</td>
<td>0.0293123</td>
<td>88.320287</td>
</tr>
<tr>
<td>6 - 7</td>
<td>90</td>
<td>0.0287784</td>
<td>88.350891</td>
</tr>
<tr>
<td>8 - 9</td>
<td>0</td>
<td>0.9999115</td>
<td>2.411304</td>
</tr>
<tr>
<td>7 - 10</td>
<td>0</td>
<td>0.999947</td>
<td>0.588870</td>
</tr>
<tr>
<td>11 - 12</td>
<td>0</td>
<td>0.9995942</td>
<td>5.163766</td>
</tr>
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<td>13 - 14</td>
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### Figure 1. The Valbonne church sequence: (a)-(b) frames 0 and 2, (c)-(d) the line segments from frames 0 and 2 that are used for measuring 3D angles.

### 7 Conclusions
Camera calibration is a prerequisite to a variety of vision tasks. In this paper, a novel method for self-calibration has been proposed. The method employs a simplification of the Kruppa equations which relies solely on the SVD of the fundamental matrix and avoids recovering noise-sensitive quantities such as the epipoles. The simplified Kruppa equations form the basis for a non-linear minimization scheme that yields the intrinsic calibration parameters. Experimental results using synthetic data as well as three image sequences have been reported. As can be verified from the Euclidean measurements performed with the aid of the recovered calibration matrices, the estimates of the latter are of acceptable accuracy. A detailed performance comparison of the proposed method with existing techniques can be found in [11].

### Acknowledgements
The authors thank Théo Papadopoulo for providing part of the code for the estimation of the uncertainty associated with the SVD of the fundamental matrix. He also provided valuable advise related to practical issues.

### References
[3] Gabriella Csurka, Cyril Zeller, Zhengyou Zhang, and Olivier Faugeras. Characterizing the uncertainty of the fundamen-
Table 3. INRIA building sequence: estimated intrinsic calibration matrix (top), ground truth and estimated angle values (bottom).

<table>
<thead>
<tr>
<th>Angle segments</th>
<th>Actual angle (deg)</th>
<th>Estimated cosine</th>
<th>Estimated angle (deg)</th>
</tr>
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<td>0</td>
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<tr>
<td>0 - 2</td>
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<td>0.999943</td>
<td>0.609908</td>
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<tr>
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Table 4. Arcades Square sequence: estimated intrinsic calibration matrix (top), ground truth and estimated angle values (bottom).

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<th>Estimated angle (deg)</th>
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