Color Image Enhancement Through 3-D Histogram Equalization

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Abstract

The well known method of histogram equalization for grey-level image enhancement is extended to color images in this paper. A method of direct 3-D histogram equalization is proposed which results in a uniform histogram of the RGB values. Due to the correlation between the bands, the principle on which grey-level (1-D) histogram equalization is based is not valid in the case of color images (3-D). This problem is alleviated by employing a histogram specification method where a uniform histogram is specified.

1 Introduction

The histogram of a grey-level image presents the relative frequency of occurrence of the various levels in the image. Histogram equalization has been proposed as an efficient technique for the enhancement of grey-scale images [6]. This technique modifies an image so that the histogram of the resulting image is uniform (flat). Variations of this technique known as histogram modification and histogram specification, which result in a histogram having a desired shape, have also been proposed [2, 3]. The extension of histogram equalization to color images is not trivial due to the multidimensional nature of the histogram in color images. For this reason, various methods have been proposed for color histogram equalization which spread the histogram either along the principal component axes of the original histogram [4] or spread repeatedly the three two dimensional histograms [3]. Other enhancement methods have been recently proposed which operate mainly on the brightness component of the original image [5, 1].

In this paper we present a new method for direct 3-D histogram equalization. This method is actually a histogram specification method. The specified histogram is a uniform 3-D histogram and, consequently, histogram equalization is achieved.

In the next sections the theoretical background of the proposed method is first presented and then issues concerning the computational implementation are discussed. Some experimental results from the application of this method are presented and the paper concludes with a brief discussion.

2 Histogram Equalization

In histogram equalization, the goal is to obtain a uniform histogram for the output image. The theoretical foundation, underlying histogram equalization, can be found in probability theory. If we consider an absolute continuous random variable (grey level) $X, a \leq X \leq b$, with cumulative probability density function $F_X(x) = Prob(X \leq x)$, then the random variable $Y = F_X(X)$ will be uniformly distributed over $(0, 1)$. In the discrete case, the assumption of continuity of variable X is not satisfied and therefore Y will be uniformly distributed only approximately. However, despite the approximate uniform distribution of Y, histogram equalization effectively spreads the grey-level values resulting in a powerful technique for image enhancement.

2.1 3-D Histogram Equalization

For the 3-D color space we can proceed in an analogous manner. Let us consider three random continuous variables R, G, and B (representing the three color components) with a joint probability density function $f_{R,G,B}(r, g, b)$ and joint probability distribution function $F_{R,G,B}(r, g, b) = Prob(R \leq r, G \leq g, B \leq b)$. As in the 1-D case we define three new variables R', G', and B' as

$$\begin{aligned} R' &= F_R(R), \\ G' &= F_G(G), and \\ B' &= F_B(B) \end{aligned} \tag{1}$$

The joint probability distribution function of R', G', and B' is given as

$$\begin{aligned} &F_{R',G',B'}(r', g', b') \\ &= Prob(R' \leq r', G' \leq g', B' \leq b') \\ &= Prob(F_R(R) \leq r', F_G(G) \leq g', F_B(B) \leq b') \\ &= Prob(R \leq F_R^{-1}(r'), G \leq F_G^{-1}(g'), B \leq F_B^{-1}(b')) \\ &= F_{R,G,B}(F_R^{-1}(r'), F_G^{-1}(g'), F_B^{-1}(b')) \end{aligned} \tag{2}$$

Assuming now independence of the R, G, and B components we can further decompose the last equation as a product of the probability distribution functions of the three color components:

$$\begin{aligned} &F_{R',G',B'}(r', g', b') \\ &= F_R(F_R^{-1}(r'))F_G(F_G^{-1}(g'))F_B(F_B^{-1}(b')) \\ &= r'g'b' \end{aligned} \tag{3}$$

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From (3) we have

$$f_{R',G',B'} = \frac{\vartheta}{\vartheta r'}\frac{\vartheta}{\vartheta g'}\frac{\vartheta}{\vartheta b'}F_{R',G',B'}(r',g',b') = 1 \quad (4)$$

From the above result it is concluded that the uniform distribution of the histogram in the R',G',B' space is only guaranteed in the case of independent R, G, and B components. However, it is known that these components are generally correlated and, consequently, this assumption is not valid especially in cases of highly correlated data (e.g. satellite images). Methods to overcome this difficulty have been proposed which spread the histogram along the principal component axes of the original histogram [4] or spread repeatedly the three two dimensional histograms [3].

A different approach is taken in this paper which results in a direct 3-D histogram equalization. Since the aim is a uniform 3-D histogram we approach this problem through a histogram specification method. Namely, a 3-D uniform histogram is specified as the output histogram and, therefore, histogram equalization is achieved.

In the 1-D case, the method of histogram specification works as follows. Let X and Y be the input and output variables that take the values x_{i_x} and y_{i_y}, $i_x, i_y = 0,\ldots,L-1$ (L is the number of discrete grey levels) with probabilities $p_X(x_{i_x})$ and $p_Y(y_{i_y})$, respectively. Define two new variables

$$C_{I_x} = \sum_{i_x=0}^{I_x} p_X(x_{i_x}),\ I_x = 0,\ldots,L-1 \quad (5)$$

$$\overline{C}_{I_y} = \sum_{i_y=0}^{I_y} p_Y(y_{i_y}),\ I_y = 0,\ldots,L-1 \quad (6)$$

Let the value of $\overline{C}_{I_y}$ for which $\overline{C}_{I_y} - C_{I_x} \geq 0$, for the smallest value of I_y. Then y_{I_y} is the output corresponding to the input value x_{I_x}.

For the 3-D case, the following method is proposed for uniform histogram specification. Let X and Y be the input and output (vector) variables which assume as values triplets $(x_{r_x}, x_{g_x}, x_{b_x})$ and $(y_{r_y}, y_{g_y}, y_{b_y})$, $r_x, g_x, b_x, r_y, g_y, b_y, = 0,\ldots,L-1$, with probabilities $p_X(x_{r_x}, x_{g_x}, x_{b_x})$ and $p_Y(y_{r_y}, y_{g_y}, y_{b_y})$, respectively. The probabilities p_X are computed from the original color image histogram. The probabilities p_Y are all set to $\frac{1}{L^3}$ since there are L^3 histogram entries and we want all to have the same probability (uniform distribution). $C_{R_xG_xB_x}$ and $\overline{C}_{R_yG_yB_y}$ (the 3-D equivalents of C_{I_x} and $\overline{C}_{I_y}$ defined in eqns.(5) and (6) above) are computed from the probabilities p_X and p_Y as follows:

$$C_{R_xG_xB_x} = \sum_{r_x=0}^{R_x}\sum_{g_x=0}^{G_x}\sum_{b_x=0}^{B_x} p_X(x_{r_x}, x_{g_x}, x_{b_x}) \quad (7)$$

$$\begin{aligned}\overline{C}_{R_yG_yB_y} &= \sum_{r_y=0}^{R_y}\sum_{g_y=0}^{G_y}\sum_{b_y=0}^{B_y} p_Y(y_{r_y}, y_{g_y}, y_{b_y}) \\ &= \sum_{r_y=0}^{R_y}\sum_{g_y=0}^{G_y}\sum_{b_y=0}^{B_y} \frac{1}{L^3} \\ &= \frac{(R_y+1)(G_y+1)(B_y+1)}{L^3} \end{aligned} \quad (8)$$

Equation (8) shows that $\overline{C}_{R_yG_yB_y}$ is simply computed as a product instead of a triple summation. Following that, the smallest R_y, G_y, B_y for which the inequality

$$\overline{C}_{R_yG_yB_y} - C_{R_xG_xB_x} \geq 0 \quad (9)$$

is true are found. The output produced, for input (R_x, G_x, B_x), is (R_y, G_y, B_y).

Summarizing, the following three steps constitute the above described method for 3-D histogram specification:

1. Compute the original histogram
2. Compute $C_{R_xG_xB_x}$ and $\overline{C}_{R_yG_yB_y}$ using eqns.(7) and (8), respectively.
3. For each input value (R_x, G_x, B_x), find the smallest R_y, G_y, B_y such that eqn.(9) is satisfied. The set of values (R_y, G_y, B_y) is the output produced.

3 Computational Considerations

Computationally, step 1 is implemented in just one pass through the image data. Step 2 can be implemented recursively, reducing drastically the execution time and memory requirements. Dropping out for simplicity the case where either of R_x, G_x, or B_x is zero, then $C_{R_xG_xB_x}$ is computed as

$$\begin{aligned}C_{R_xG_xB_x} &= C_{R_x-1,G_x-1,B_x-1} \\ &+ C_{R_x-1,G_xB_x} + C_{R_xG_x-1,B_x} + C_{R_xG_xB_x-1} \\ &- C_{R_x-1,G_x-1,B_x} - C_{R_x-1,G_xB_x-1} - C_{R_xG_x-1,B_x-1} \\ &+ p_X(R_x, G_x, B_x)\end{aligned} \quad (10)$$

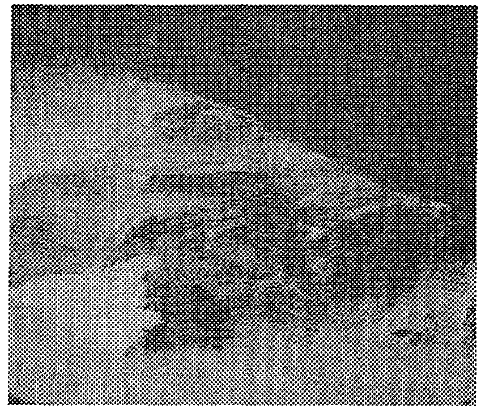
Step 3 presents an ambiguity since many solutions for the R_y, G_y, B_y exist that satisfy eqn.(9). This ambiguity is remedied as follows. The computed value of $C_{R_xG_xB_x}$ at (R_x, G_x, B_x) is initially compared to the product $P = \frac{1}{L^3}(R_x+1)(G_x+1)(B_x+1)$ [the value of $\overline{C}_{R_yG_yB_y}$ at (R_x, G_x, B_x)] since, for a uniform histogram, the value of this product should also be the value of $C_{R_x G_x B_x}$. In case of equality the input value is not changed. If $C_{R_x G_x B_x}$ is greater (less) than P, then the indexes R_x, G_x, and B_x are repeatedly increased (decreased), one at a time, until eqn.(9) is satisfied. The final values constitute the output produced. The merit of this is twofold: (a) histogram stretching is achieved simultaneously in all three directions, and (b) the computational requirements are reduced (only a few values are checked). The overall computational complexity is manifested by step 2 which computes $C_{R_x G_x B_x}$ (cumulative histogram) for a total of $L \cdot L \cdot L = L^3$ histogram entries resulting in a $O(L^3)$ complexity. Since L is usually 256 (8 bits for each color), the $O(L^3)$ complexity does not allow for real time implementation in small computer installations, unless special purpose computer architectures are used.

4 Experimental Results

The proposed method of color image histogram equalization has been implemented and tested on a set of color images. It has also been compared experimentally to the histogram equalization method that operates independently in each color. A quantitative measure of the performance of these methods is difficult to derive since the only criterion for the performance of such a method (and of all the image enhancement methods) is the visual appearance of the produced images. This implies that only qualitative comparisons can be performed. Our method has been verified to produce clear images with the colors being closer to the original ones than in the case of independent histogram equalization. This is as expected since the histogram is stretched along the diagonal axis (line of greys) and the colors are not affected. The later method, when compared to ours, has the tendency to overstretch the histogram resulting in unnatural images. An important feature of the 3-D histogram equalization method is the fact that it does not "destroy" an image portion with a(n almost) uniform histogram. This is not necessarily the case for other methods that operate on separate components. For a visual observation, the results obtained for two images are shown here. Fig.1b shows the results of 3-D histogram equalization applied to the "everest" image of Fig.1a. In Fig.1c separate histogram equalization on each of the three components has been applied. As can be verified, better visual results are obtained with the direct 3-D histogram equalization. Fig.2 presents the same results for a "girl" image. the effect of overstretching the histogram by independent equalization of the three bands is illustrated in this figure. However, our method achieves to enhance the image but without producing any color artifacts.

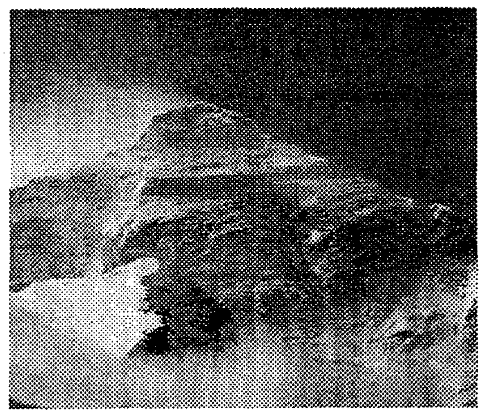
5 Discussion

A method of direct 3-D histogram equalization has been presented in this paper. This method is actually a histogram specification method, where a 3-D uniform histogram is specified. In this way, the problem of correlation between bands is alleviated and uniform stretching of values is achieved.



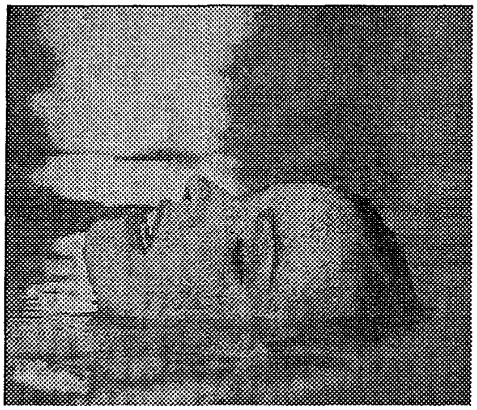
(a)

(b)

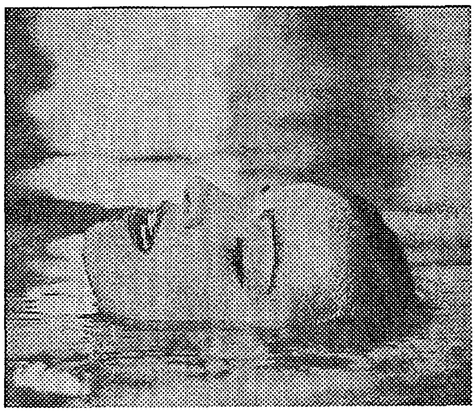


(c)

Figure 1: (a) Original Image (b) 3-D Histogram Equalization (c) Independent Histogram Equalization



(a)



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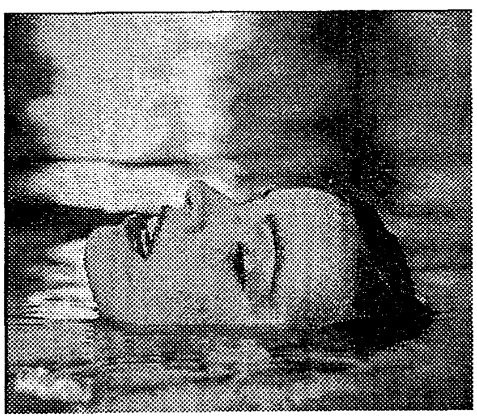

(c)

Figure 2: (a) Original Image (b) 3-D Histogram Equalization (c) Independent Histogram Equalization

This method has been experimentally verified as producing very good results when applied to hihgly correlated images. However, a drawback can be specified in the computational complexity associated with this method. The $O(L^3)$ complexity does not allow real time implementation in small computing facilities which restricts its application for real time data imagery. However, this method may be employed for enhancement of highly correlated data and in cases where real time performance is not the key issue, rather the quality of the produced output.

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