Vector Directional Filters—A New Class of Multichannel Image Processing Filters

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Abstract—Vector directional filters (VDF) for multichannel image processing are introduced and studied in this paper. These filters separate the processing of vector-valued signals into directional processing and magnitude processing. This provides a link between single-channel image processing, where only magnitude processing is essentially performed, and multichannel image processing where both the direction and the magnitude of the image vectors play an important role in the resulting (processed) image.

VDF find applications in satellite image data processing, color image processing, and multispectral biomedical image processing. In this paper, results are presented for the case of color images, as an important example of multichannel image processing. It is shown that VDF can achieve very good filtering results for various noise source models.

I. INTRODUCTION

Although conventional approaches to multichannel image processing are based on processing the image channels separately, they fail to utilize the inherent correlation that is usually present in multichannel images. Consequently, vector processing of multichannel images is desirable [1]. Recently, this has been adopted by many researchers [2]–[4]. An important class of vector image processing operators are the vector median filters (VMF) that have been introduced as extension of scalar median filters [5]. VMF can be derived 1) as maximum likelihood estimates when the underlying probability densities are double-exponential or 2) using vector order statistics [6]. In the latter case, the vector median of a population is defined as the minimal vector according to the aggregate ordering technique [6]. Based on vector order statistics, extensions or modifications of VMF have also been proposed [2], [7].

The operation of the above-mentioned filters can be described according to some distance criterion that is applied to the set of vectors inside the processing window. However, the features that uniquely characterize a vector, namely direction and magnitude, are not considered by such an operation and this may produce erroneous results in certain cases. Such an example is shown in Fig. 1, where VMF is applied to the set of vectors \( f_1, \ldots, f_5 \). The output produced is vector \( f_4 \), although vector \( f_2 \) would be a better candidate to output.

This paper approaches the aforementioned problem by explicitly considering the vector features and separating the processing of vector-valued signals into two steps: directional processing and magnitude processing. A new class of filters is introduced, called vector directional filters (VDF). VDF perform the first step, namely directional processing. They operate on the direction of the image vectors aiming at eliminating vectors with atypical directions in the vector space. This is achieved by employing a novel vector ordering technique in which the angle between the image vectors serves as the ordering criterion. The term “directional processing” used here denotes the processing performed according to the vectors’ direction in the vector space. This term has been adopted by other authors to denote processing in certain directions in the image plane [8]. Here it is used in the context of vector spaces and hence it should not bring any confusion. Similarly, the term “magnitude processing” denotes the processing of image data where only the vector magnitudes are taken into account.

The application of VDF results in the removal of vectors with atypical directions and a set containing vectors with approximately the same direction in the vector space is produced as the output set. Since the vectors in this set are approximately collinear, a magnitude processing operation (second step) can be applied to produce a single output vector at each image pixel. This step can be performed by any classical gray-level image processing filter.\(^1\)

\(^1\)This is obvious since gray-level image processing filters operate on the magnitude at each pixel location.
This property of VDF (separation of processing) establishes a link between multichannel image processing and single-channel image processing. After directional processing a multichannel signal can locally be seen as a single-channel signal, since this processing results in vectors being approximately collinear. Consequently, the bulk of techniques developed for gray-level image processing can be employed in the step of magnitude processing. This is a major advantage of this approach, since it facilitates the use of many efficient image processing operators (order statistics, α-trimmed mean, morphological) to multichannel images.

II. VECTOR DIRECTIONAL FILTERS

Basic Vector Directional Filter

Let \( f(x) : Z^d \rightarrow Z^m \), represent a multichannel signal and let \( W \in Z^d \) be a window of finite size \( n \) (filter length). \( f \) represents the signal dimensions and \( m \) represents the number of signal channels. The pixels in \( W \) will be denoted as \( x, i = 1, 2, \ldots, n \) and \( f(x, i) \) will be denoted as \( f_i, f \), are \( m \)-dimensional (m-D) vectors in the vector space defined by the \( m \) signal channels. A window size \( n \) is implied in all subsequent operations, if not stated otherwise.

The definition of the basic vector directional filter (BVDF) follows. This is a special case of VDF. However, it facilitates their introduction and mathematical treatment.

**Definition 1:** The output of the BVDF, for input \( \{ f_i, i = 1, 2, \ldots, n \} \), is \( f_{BD} = BVDF[ f_1, f_2, \ldots, f_n ] \) such that

\[
 f_{BD} \in \{ f_i, i = 1, 2, \ldots, n \}
\]

and

\[
 \sum_{i=1}^{n} A(f_{BD}, f_i) \leq \sum_{i=1}^{n} A(f_j, f_i), \forall j = 1, 2, \ldots, n
\]

where \( A(f_i, f_j) \) denotes the angle between the vectors \( f_i, f_j \), and \( 0 \leq A(f_i, f_j) \leq \pi \).

BVDF outputs the vector from the input set that minimizes the sum of the angles with the other vectors. In other words, it chooses the vector most centrally located, without considering magnitudes. The operation of BVDF can be parallelized with the operation of VMF which outputs the vector that minimizes the sum of the distances to the other vectors.

The application of BVDF to a vector-valued 1-D signal is illustrated in Fig. 2. The BVDF output is shown in Fig. 2(b) for the input shown in Fig. 2(a). It should be explicitly noted that for the special case of BVDF processing, a single output vector is produced at each pixel and the next step, i.e. magnitude processing, is not applicable. In other words, BVDF filtering considers only directional information in producing the output vector signal.

For 2-D vectors, the angle of \( f_{BD} \) is the least error estimate of the angle location.

**Proof:** With respect to Fig. 1, we observe that the angles of the vectors with the \( x \)-axis form a sequence of numbers with median the angle of \( f_{BD} \) (\( f_{BD} \)). It is well known that the median is the least absolute error estimate of the location of a set of numbers, which, since the angles are positive, proves the above statement. Q.E.D.

In 3-D the above statement holds approximately for small angles between the vectors.

**Proof:** Without loss of generality we consider a set of vectors \( \{ f_i, i = 1, 2, \ldots, n \} \) of equal length. Then, the least error estimate of the location of the vectors \( \{ f_i \} \) (constrained to the set \( f_i \)) is the vector median \( f_{VM} \), since

\[
 \sum_{i} \| f_i - f_{VM} \| \leq \sum_{i} \| f_i - f_j \|, \forall j = 1, 2, \ldots, n
\]

i.e. the minimum of \( \sum_i \| f_i - f_j \| \) is obtained for \( f_i = f_{VM} \). Since the vectors are of equal length, say \( l(>0) \), \( \| f_i - f_{VM} \| \) can be
expressed as

\[ \| f_i - f_{VM} \| = 2f \sin \frac{\phi_i}{2} \]  

(4)

where, \( \phi_i = A(f_i, f_{VM}) \). Since \( \sum_i \| f_i - f_{VM} \| \) is a minimum, \( \sum_i \sin \frac{\phi_i}{2} \) is a minimum. By assumption, the angles \( \phi_i \) are small and the \( \sin \frac{\phi_i}{2} \) can be substituted with the angles \( \frac{\phi_i}{2} \) (expressed in radians). Q.E.D.

From the above proof it is evident that the outputs of VMF and BVDF are the same for equal-length vectors. However, this is not the case for vectors of unequal length. In the latter case, the BVDF output is still the vector that would have resulted if the vectors were of equal length. The VMF, however, would generally produce different results. This is demonstrated in Fig. 1 where the BVDF output is vector \( f_2 \) whereas the VMF output is vector \( f_4 \).

**Generalized Vector Directional Filters**

BVDF may perform well when the vector magnitudes are of no importance and the direction of the vectors is the major issue. However, this may not be the case in some multichannel signal processing applications and the magnitudes of the vectors should also be considered. This can be accomplished by generalizing BVDF so that the output at each pixel is not a vector \( f_i \in W \) but instead a set of vectors \( f_i \in W, i = 1, 2, \ldots, n \).

**Definition 2:** The output of the generalized vector directional filter (GVDF), for input \( \{ f_i, i = 1, 2, \ldots, n \} \), is the set \( S_{GD} = GVDF[f_1, f_2, \ldots, f_n] \), where

\[ S_{GD} = \{ f^{(1)}, f^{(2)}, \ldots, f^{(k)} \}, f^{(i)} \in \{ f_j, j = 1, 2, \ldots, n \} \]

\( \forall i = 1, 2, \ldots, k \)  

(5)

Let \( \alpha_i \) correspond to \( f_i \) and be defined as

\[ \alpha_i = \sum_{j=1}^{n} A(f_i, f_j), i = 1, 2, \ldots, n. \]  

(6)

An ordering of the \( \alpha_i \)s

\[ \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_k \leq \cdots \leq \alpha_n \]  

(7)

implies the same ordering to the corresponding \( f_i \)s

\[ f^{(1)} \leq f^{(2)} \leq \cdots \leq f^{(k)} \leq \cdots \leq f^{(n)} \]  

(8)

The first \( k \) terms of the ordered sequence \( f^{(i)} \) constitute the output of the GVDF.

We note here that the first term in (8), \( f^{(1)} \), is the BVDF output. GVDF generalizes BVDF in the sense that its output is a superset of the (single) BVDF output. GVDF outputs the set of vectors whose angle from all the other vectors is small as opposed to the BVDF which outputs the vector whose angle from all the other vectors is a minimum. In other words, the output set of the GVDF consists of vectors centrally located in the population with approximately the same direction in the vector space. The application of the GVDF to the signal of Fig. 2(a) is illustrated in Fig. 2(c) where a set of vectors is output at each sample point.

After the application of GVDF (directional processing) the output produced is a set of \( k \) vectors with approximately the same direction and the magnitude processing step should be applied to produce a single output vector at each pixel. The whole operation is illustrated in Fig. 3, where the GVDF is cascaded with the \( F \) module (filter).

Since \( F \) processes the vectors using only magnitude information, it can be any gray-scale image processing filter [9], [10]. This is illustrated in Figs. 2(c), (d). The GVDF output (Fig. 2(c)) has been passed through a gray-scale median filter (\( F \)) to produce the final output shown in Fig. 2(d).

Some properties that constitute VDF appropriate for image processing have been studied elsewhere [11]. We mention here the preservation of step edges, invariance under scaling and rotation, and existence of and convergence to root signals. The behavior of VDF in the case of vector edges can be illustrated by the following 1-D example, where the window \( W \) of size five is centered over vector \( f_4 \) and \( f_2, \ldots, f_6 \) are inside \( W \).

\[
\begin{array}{cccccc}
\cdots & f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & \cdots \\
\end{array}
\]

\[
\begin{array}{cccccc}
\cdots & \backslash & \backslash & \backslash & \backslash & \backslash & \cdots \\
W
\end{array}
\]

\(^2\) A proof of this property has been derived for the BVDF of length three only. However, experimentation with larger filter lengths has demonstrated this property.
vector in the color space. Such a color vector is shown in Fig. 4. The three axes that define the color cube represent the three primaries (R,G,B). A particular color may be described as a point in the color cube, as shown in Fig. 4. The point marked with a "x" denotes the intersection point of the color vector with the Maxwell triangle (the triangle drawn between the three primaries, R,G,B). It is well known that the intersection point of a color vector with the Maxwell triangle gives an indication of the chromaticity of the color (hue and saturation) in terms of the distances of the point from the vertices of the triangle [12]. It is evident that this point depends only on the direction of a color vector and not on its magnitude. Therefore, the operation of VDF can be described in terms of the color chromaticity. Since BVDF results in the least chromaticity error. In the case of GVDF, a set of vectors is rendered. It should be clear at this point that for the case of color image processing, VDF operate actually as chromaticity filters. The significance of this stems from the fact that color is generally perceived first in terms of the hue component [15, p. 31] and then in other terms such as brightness, richness, purity, and saturation. On the other hand, luminance filtering can be efficiently performed subsequently using classical grey-level image processing filters.

### Table I

<table>
<thead>
<tr>
<th>Noise Model</th>
<th>VMF</th>
<th>BVDF</th>
<th>GVDF³</th>
<th>GVDF²</th>
<th>GVDF¹</th>
<th>GMSE (x10⁻²) for the &quot;Lena&quot; Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>3x3</td>
<td>3x3</td>
<td>3x3</td>
<td>5x5</td>
<td>3x3</td>
<td>5x5</td>
</tr>
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<td>1.60</td>
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<td>3.41</td>
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<td>1.08*</td>
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<td>3.52</td>
<td>2.67</td>
<td>1.53</td>
<td>1.13*</td>
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<tr>
<td>impulsive</td>
<td>0.60</td>
<td>0.66</td>
<td>0.61</td>
<td>0.89</td>
<td>0.36*</td>
<td>0.56</td>
</tr>
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</table>

### Table II

<table>
<thead>
<tr>
<th>Noise Model</th>
<th>VMF</th>
<th>BVDF</th>
<th>GVDF³</th>
<th>GVDF²</th>
<th>GVDF¹</th>
<th>GMSE (x10⁻²) for the &quot;Lake&quot; Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>1.54</td>
<td>1.20</td>
<td>3.86</td>
<td>4.20</td>
<td>1.52</td>
<td>1.16*</td>
</tr>
<tr>
<td>Gaussian/impulsive</td>
<td>1.64</td>
<td>1.29*</td>
<td>4.04</td>
<td>4.53</td>
<td>1.64</td>
<td>1.33</td>
</tr>
<tr>
<td>double-exponential</td>
<td>0.66</td>
<td>1.14</td>
<td>1.35</td>
<td>2.37</td>
<td>0.63</td>
<td>1.38</td>
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</table>

### Table III

<table>
<thead>
<tr>
<th>Noise Model</th>
<th>VMF</th>
<th>BVDF</th>
<th>GVDF³</th>
<th>GVDF²</th>
<th>GVDF¹</th>
<th>MCRE for the &quot;Lena&quot; Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>7.48</td>
<td>5.71</td>
<td>5.96</td>
<td>4.45</td>
<td>6.05</td>
<td>4.20*</td>
</tr>
<tr>
<td>Gaussian/impulsive</td>
<td>7.64</td>
<td>5.60</td>
<td>6.30</td>
<td>5.04</td>
<td>6.22</td>
<td>4.37*</td>
</tr>
<tr>
<td>double-exponential</td>
<td>2.98</td>
<td>2.90</td>
<td>2.81</td>
<td>2.73</td>
<td>2.82</td>
<td>2.66*</td>
</tr>
<tr>
<td>impulsive</td>
<td>2.14</td>
<td>2.48</td>
<td>1.97</td>
<td>2.46</td>
<td>2.02</td>
<td>2.39</td>
</tr>
</tbody>
</table>

Ordering of \( f_3, \ldots, f_1 \) will result in \( f_1 = f_2 = f_3 < f_4 \). It is evident that the vectors in the high ranks (\( f_3, f_2 \)) will be eliminated after VDF processing and will not affect the vector edge.

### III. MOTIVATION AND INTUITIVE ISSUES

The introduction of VDF for multichannel image processing has been motivated by a number of factors mostly related to color imaging. In this section we attempt to introduce the reader to these concepts. As already explained, VDF attempt to process vector images by exploiting the vector features, direction and magnitude. They separate the processing of multichannel images into directional processing and magnitude processing. This allows the bridging of scalar image processing and vector image processing and also enables the use of all the successful scalar image processing techniques in multichannel images.

As a special (and important) case of multichannel images, we may consider the case of color images. Color image processing can be regarded from the perspective of the color cube, as shown in Fig. 4. The three axes that define the color cube represent the three primaries (R,G,B). A particular color may be described as a vector in the color space. Such a color vector is shown in Fig. 4. The point marked with a "x" denotes the intersection point of the color vector with the Maxwell triangle (the triangle drawn between the three primaries, R,G,B). It is well known that the intersection point of a color vector with the Maxwell triangle gives an indication of the chromaticity of the color.
TABLE IV
MCRE FOR THE "LAKE" IMAGE

<table>
<thead>
<tr>
<th>noise model</th>
<th>VMF</th>
<th>BVDF</th>
<th>GVDF(^1)</th>
<th>GVDF(^2)</th>
<th>GVDF(^3)</th>
<th>GVDF(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>3 × 3</td>
<td>5 × 5</td>
<td>3 × 3</td>
<td>5 × 5</td>
<td>3 × 3</td>
<td>5 × 5</td>
</tr>
<tr>
<td>Gaussian/impulsive</td>
<td>10.84</td>
<td>8.89</td>
<td>8.99</td>
<td>8.37</td>
<td>8.95</td>
<td>7.09*</td>
</tr>
<tr>
<td>double-exponential</td>
<td>5.12</td>
<td>5.57</td>
<td>4.92</td>
<td>5.42</td>
<td>4.96</td>
<td>5.32</td>
</tr>
<tr>
<td>impulsive</td>
<td>4.77</td>
<td>5.44</td>
<td>4.94</td>
<td>6.35</td>
<td>4.74</td>
<td>5.43</td>
</tr>
</tbody>
</table>

GVDF\(^1\): GVDF followed by \(\alpha\)-trimmed mean
GVDF\(^2\): GVDF followed by morphological open-close
GVDF\(^3\): GVDF followed by multistage max/median

Fig. 7. (a) “Lena” corrupted with Gaussian noise (\(\sigma = 30\)). (b) “Lena” corrupted with 4% impulsive noise. (c), (d) 3 × 3 VMF of (a) and (b), respectively.

In the case of GVDF, three filters have been used for the step of magnitude processing: the \(\alpha\)-trimmed mean [9], the morphological open-close [16], and the multistage max/median [8]. Two quantitative measures have been employed. The first is the normalized mean squared error (NMSE) which is a standard measure also used by other authors in evaluation experiments [2], [8]:

\[
NMSE = \frac{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} ||f(i,j) - \hat{f}(i,j)||^2}{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} ||f(i,j)||^2}
\]

(9)
where \( N_1 \) and \( N_2 \) are the image dimensions, and \( f(i,j) \) and \( \hat{f}(i,j) \) denote the original and the estimated image vector at pixel \((i,j)\), respectively. The second measure is related to the color chromaticity and is referred to as the mean chromaticity error (MCRE). We have introduced this measure since VDF operate as chromaticity filters and, consequently, their performance in terms of chromaticity error should be evaluated. MCRE is defined as

\[
MCRE = \frac{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} C[f(i,j), \hat{f}(i,j)]}{N_1N_2}
\]

where \( N_1, N_2, f(i,j) \) and \( \hat{f}(i,j) \) are as in (9) and \( C[f(i,j), \hat{f}(i,j)] \) is the chromaticity error between vectors \( f(i,j) \) and \( \hat{f}(i,j) \). It is defined as the distance \( PP \) between the two points \( P \) and \( \hat{P} \), which are the intersection points of \( f(i,j) \) and \( \hat{f}(i,j) \) with the Maxwell triangle, respectively. This is shown graphically in Fig. 5.

We have applied VDF to a number of color images ranging from detailed indoor and outdoor scenes to human faces. We present here the results for two images, “Lena” and “Lake,” shown in Fig. 6. The test images have been contaminated using the following noise distributions: (a) Gaussian \((\sigma = 30)\), (b) Gaussian \((\sigma = 30)\) contaminated with 2% of impulses, (c) double-exponential \((\sigma = 40)\), and (d) impulsive (4%). The impulsive noise has been simulated in two steps. In the first step each image channel is corrupted independently with 4% impulsive noise. In the second step, a correlation factor \( \rho = 0.5 \) is used to further determine the corruption of pixel \((i,j)\) in channel \( C \), if the same pixel \((i,j)\) is corrupted in any of the two other channels. The second step simulates the channel correlation in multichannel images. It is noted that, although vector order statistics based filters are not optimal for Gaussian noise, this type of noise has been used in our experiments since, for a suitable selection of the magnitude processing filter \( e.g., \alpha\)-trimmed mean), VDF are expected to perform efficiently in this case, too.

Tables I and II show the NMSE results for the two test images and for filter windows \( 3 \times 3 \) and \( 5 \times 5 \). A “*” in a table entry indicates the best filter performance in the corresponding row (noise distribution). From these results it can be concluded that VDF perform at least as good and in most cases better than VMF. It can be observed that the GVDF followed by an \( \alpha \)-trimmed mean filter has very good performance in short tailed noise. Similarly, with a
multistage max/median filter as the magnitude processing filter, very good performance in long tailed noise is obtained. Tables III and IV show the MCRE results for the same images. As can be observed, VDF result always in a better chrominance estimate than VMF and this justifies their employment in color image processing.

In addition to the quantitative evaluation presented above, a qualitative evaluation seems worthwhile since the topic is image processing and the visual assessment of the processed images is, ultimately, the best subjective measure of the efficacy of any method. Therefore, we present sample processing results in Fig. 7. Fig. 7(a) shows the "Lena" image corrupted with Gaussian noise ($\sigma = 30$) and Fig. 7(b) shows the same image corrupted with 4% impulsive noise. Figs. 7(c) and(d) show the VMF results for Figs. 7(a) and (b), respectively, and Figs. 7(e), (f), (g), and (h) refer to the VDF processing results. Figs. 7(e) and (f) show the GVDF/a-trimmed mean results and Figs. 7(g) and (h) show the GVDF/multistage max/median results. The superiority of the GVDF/a-trimmed mean in the case of Gaussian noise and the GVDF/multistage max/median in the case of impulsive noise is illustrated in this example. In effect, the properties of the gray-scale processing filters that make them appropriate for monochrome image processing have also been retained in the case of color images. In other words, the o-trimmed mean is still efficient in low pass filtering the Gaussian noise, whereas the multistage max/median possesses the detail preserving property. This demonstrates the advantage of combining the directional filters with efficient gray-scale filters. The former filter out vectors with "atypical" direction producing thus a set of vectors with almost the same direction; the latter can, subsequently, perform on this output set as if it has been produced by a single-channel source.

It should be noted that the correlated noise has no effect on VDF (and VMF) since they operate directly on the image vectors and not on the individual channels of a multichannel image. This can also be verified from the results obtained.

REFERENCES