INDOOR POSITIONING IN WIRELESS LANS USING COMPRESSIVE SENSING SIGNAL-STRENGTH FINGERPRINTS

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ABSTRACT

Accurate indoor localization is a significant task for many ubiquitous and pervasive computing applications, with numerous solutions based on IEEE802.11, bluetooth, ultrasound and infrared technologies being proposed. The inherent sparsity present in the problem of location estimation motivates in a natural fashion the use of the recently introduced theory of compressive sensing (CS), which states that a signal having a sparse representation in an appropriate basis can be reconstructed with high accuracy from a small number of random linear projections. In the present work, we exploit the framework of CS to perform accurate indoor localization based on signal-strength measurements, while reducing significantly the amount of information transmitted from a wireless device with limited power, storage, and processing capabilities to a central server. Equally importantly, the inherent property of CS acting as a weak encryption process is demonstrated by showing that the proposed approach presents an increased robustness to potential intrusions of an unauthorized entity. The experimental evaluation reveals that the proposed CS-based localization technique is superior in terms of an increased localization accuracy in conjunction with a low computational complexity when compared with previous statistical fingerprint-based methods.

1. INTRODUCTION

Location estimation systems have a great potential in several distinct areas, such as in navigation, transportation, medical community, security, and entertainment. With the wide deployment of mobile wireless systems and networks, location-based services are made possible on mobile devices. The existing communication technology is employed in such environments, with characteristic examples being the IEEE802.11 [1], as well as infrared [2], ultrasonic [3], bluetooth [4] or touch sensors in order to estimate the position of a mobile user. There are also systems that combine optical, acoustic, and signal-strength measurements, along with motion attributes for location estimation [5].

Due to the wide deployment of wireless local area networks (WLAN), specifically referred to the IEEE802.11 infrastructure, many indoor positioning systems make use of WLANs for estimating the position of a user. Received signal-strength (RSS) values is a typical metric used in WLAN positioning systems, as it can be obtained directly from access points (APs) by any device that uses a network adapter. The IEEE802.11 infrastructure does not require any specific hardware or installation costs. However, due to the nature of the indoor environment, transient phenomena, such as shadowing and multipath fading, lead to radio channel obstructions and variations of the RSS. This makes the design of accurate positioning systems a difficult task and location estimation a challenging area of research.

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The RSS-based location estimation systems can be classified in two categories, namely, the fingerprint- or map-based and prediction-based architectures. The fingerprint-based techniques consist of two distinct phases. First, during a *training phase* a wireless device that listens to a channel receives the beacons sent by APs periodically and records their RSS values at known positions of the physical space [1, 6]. In a subsequent *runtime phase* the system also records the RSS values from the received beacons but at random unknown positions. In both phases the wireless client scans all the available channels. The cell with a training signature that has the smallest distance from the runtime signature is reported as the estimated position. On the other hand, the prediction-based techniques employ RSS and radio-propagation models to find the distance of a wireless user (peer) from an AP (or landmark), (*e.g.*, CLS) [7].

However, due to the unpredictable nature of the RSS measurements most of the fingerprint-based systems have increased computational cost. On the other hand, the inherent sparsity of the physical space motivated in a natural fashion the use of the recently introduced theory of *compressive sensing* (CS) [8, 9] in the problem of target localization [10]. CS states that signals which are sparse or compressible in a suitable transform basis can be recovered from a highly reduced number of incoherent random projections, in contrast to the traditional methods dominated by the well-established Nyquist-Shannon sampling theory.

In a recent work [11], a CS-based localization method was introduced. In particular, the location estimation algorithm is carried out on the mobile device by using the average RSS values in order to construct the transform basis. Our proposed work differs from the previous one in several aspects, from the way we acquire the compressed set of measurements to the way we perform the location estimation. For instance, in contrast to [11] where the estimation is performed by the wireless device with the potentially limited resources, in our system the computational burden is put on the server, where increased storage and processing resources are available. Besides, in the proposed localization technique the CS approach is applied directly on the raw RSS measurements and not on their average as in [11], and thus exploiting their time-varying behavior. Equally importantly, the inherent property of CS acting as a weak encryption module is exploited to guarantee with high probability that the communication between the device and the server is secured against potential intrusions of an unauthorized entity.

The paper is organized as follows: Section 2, overviews related fingerprint-based positioning systems for mobile computing. In Section 3, the proposed CS-based WLAN localization method is analyzed in detail, while in Section 4 the encryption capability of CS is introduced. In Section 5, the performance of the proposed method is compared with recent fingerprint-based algorithms. Finally, Section 6 summarizes our main results and gives directions for future work.

2. OVERVIEW OF RECENT FINGERPRINT-BASED LOCATION ESTIMATION METHODS

In the following, we introduce in brief some recent fingerprint-based localization techniques, which were shown to be efficient in several indoor environments, and with which we compare the performance of the proposed CS-based architecture.

A region-based algorithm was introduced in [12], which improved the location estimation accuracy against previous methods by developing a statistical method based on a multivariate Gaussian (MvG) model to fit the statistics of the received RSS measurements. More specifically, the physical space is discretized first in a grid consisting of cells with known coordinates. Then, a statistical signature is extracted for each cell in the training phase by modeling the RSS values received from a set of APs using a multivariate Gaussian distribution. In the runtime phase, a similar statistical signature is generated at the unknown position, which is then compared with the training signatures by means of a statistical similarity measure, namely, the Kullback-Leibler divergence (KLD). The estimated location $[x_R^*, y_R^*]$ is given by the coordinates of the i^*-th cell that minimizes the KLD, that is,

$$i^* = \arg \min_{i=1,\ldots,C} D(p_R || p_{i,T}) \ , \tag{1}$$

where C is the number of cells, while p_R and $p_{i,T}$ correspond to the multivariate Gaussians in the runtime and training phase (for the i-th cell), respectively. The accuracy of the algorithm is improved through an iterative scheme applied in multiple spatial scales (*regions*), where the fingerprint of each region is generated by employing all the RSS measurements from all APs collected at positions within that region. Then, a refinement step is employed by comparing the signature of the unknown cell with the signatures of the cells in the best-matched region. This process reduces the likelihood of selecting a single false region/cell over the correct one. The closest region is found by minimizing the following KLD,

$$s^* = \arg \min_{s=1,\ldots,S} D(p_R || G_{s,T}) \ , \tag{2}$$

where S is the number of regions and $G_{s,T}$ denotes the total multivariate Gaussian whose parameters are estimated over all cells of the s-th region during the training phase.

Another common approach in location estimation problems is the use of the k-Nearest Neighbor algorithm (kNN) [13], where an RSS map is constructed by averaging separately the signal-strength values received from each AP. Let $\mu_R = [\mu_1, \ldots, \mu_P]$ be the signature vector of the unknown runtime cell c_R, where μ_i is the average RSS received from the i-th AP ($i = 1, \ldots, P$). Similarly, let $\nu_{T,c} = [\nu_1^c, \ldots, \nu_P^c]$ be the signature vector of the cell c extracted during the training phase. Then, the algorithm computes the Euclidean distance between the runtime and all the training cells, $d(c_R, c) = \|\mu_R - \nu_{T,c}\|_2$ ($c = 1, \ldots, C$), and reports the k closest neighbors by sorting the distances in increasing order. The final estimated position is given by computing the centroid of these k closest neighbors.

In a recent work [14], the problem of location estimation was treated in a framework that also takes advantage of the spatial sparsity. In particular, the location estimation is formulated as a constrained ℓ_1-norm minimization problem based on a suitably learned dictionary. A signature is associated to each AP by averaging the RSS measurements which would be received by the AP from each potential cell of the discrete spatial domain. Then, the system builds the dictionary by concatenating the signatures from all APs. A similar signature is generated at the unknown runtime cell, which is then projected on the dictionary to form the vector of measurements. However, the lack of a random measurement matrix required when working in the framework of CS may decrease the system's performance under unpredictable environmental conditions, while also the communication of the projected measurements from the wireless device to the APs, where the localization takes place, could pose undesired security issues.

3. CS-WLAN ARCHITECTURE

Let $\Psi \in \mathbb{R}^{N \times N}$ be a matrix whose columns correspond to a transform basis. In terms of signal approximation it has been demonstrated [8, 9] that if a signal $\mathbf{x} \in \mathbb{R}^N$ is K-sparse in basis Ψ (meaning that the signal is exactly or approximately represented by K elements of this basis), then it can be reconstructed from $M = rK \ll N$ non-adaptive linear projections onto a second measurement basis, which is incoherent with the sparsity basis, and where $r > 1$ is a small overmeasuring constant. The measurement model in the original space-domain is written as

$$\mathbf{g} = \Phi \mathbf{x} \ , \tag{3}$$

or via its equivalent transform-domain representation,

$$\mathbf{g} = \Phi \Psi \mathbf{w} \ , \tag{4}$$

where $\mathbf{g} \in \mathbb{R}^M$ is the measurement vector, $\Phi \in \mathbb{R}^{M \times N}$ denotes the measurement matrix, and $\mathbf{w}$ denotes the sparse vector of transform coefficients. Examples of measurement matrices, which are incoherent with any fixed transform basis with high probability (universality property [9]), are the random matrices with independent and identically distributed (i.i.d.) Gaussian or Bernoulli entries.

By employing the M compressive measurements and given the K-sparsity property in the transform basis, the original signal can be recovered by taking a number of different approaches. The majority of these approaches solve constrained optimization problems, while a number of recently introduced CS methods proceed in a Bayesian framework. In the first case, commonly used approaches are based on convex relaxation [8, 16], and greedy strategies (*e.g.*, Orthogonal Matching Pursuit (OMP) [17, 18]). The CS methods of the second class proceed by formulating a posterior probability distribution for the unknown sparse signal and then by seeking for its maximum. Characteristic examples of Bayesian CS (BCS) methods are the standard BCS [19] and a recently introduced BCS approach based on Gaussian Scale Mixtures (GSM) [20], which yielded a superior performance compared with the standard BCS by employing a GSM as a sparsity-enforcing prior. In the subsequent analysis, methods from both classes are tested for the experimental evaluation of the proposed localization algorithm.

As it was mentioned before, a common characteristic of all RSS-based fingerprint methods is their implementation in two distinct phases, namely, a *training phase* (off-line), where the central server is mainly involved, and a *runtime phase* (on-line), which concerns the wireless device to be localized. When working in a CS framework, several requirements should be specified individually for the two phases, and this is the issue described in the following two subsections. Besides, for convenience the following notations are used in the subsequent derivations: i) y_T denotes any quantity y that is related with the training phase, ii) y_R denotes that y is associated with the runtime phase.

In a localization scenario, the area of interest is first divided into a set of C non-overlapping cells of predetermined size $x_s \times y_s$, and then it is covered with P access points (AP). The inherent sparsity in the problem of location estimation comes from the fact that the device to be localized can be placed in exactly one of these cells. Let $\mathbf{w} = [0 \ 0 \ \cdots \ 0 \ 1 \ 0 \ \cdots \ 0]^T \in \mathbb{R}^C$ be an indicator vector with its j-th component being equal to "1" if the device is located in the j-th cell. Thus, in the framework of CS, the problem of localization is reduced to a problem of recovering the sparse vector $\mathbf{w}$.

3.1 Training phase

During the training phase, a set of RSS samples is collected at each cell from each AP. Let $\psi_{T,j}^i \in \mathbb{R}^{n_{j,i}}$ denote the vector of training RSS measurements received at cell j from the AP i. In general $n_{j,i} \neq n_{j',i'}$ for $j \neq j'$, $i \neq i'$. Then, these vectors are collected from all cells by a central server, which forms a single matrix $\Psi_T^i \in \mathbb{R}^{N_i \times C}$ for the i-th AP by concatenating the corresponding C vectors. In general, the length of these vectors varies from cell to cell, thus in our implementation we set $N_i = \min_j \{n_{j,i}\}$, $i = 1, \ldots, P$, $j = 1, \ldots, C$.

The reconstruction performance of a CS method is highly affected by the choice of an appropriate sparsifying transformation represented by the matrix Ψ. In several signal processing applications common choices for the Ψ are the Discrete Cosine Transform (DCT) and the Discrete Wavelet Transform (DWT). However, in our case the degree of sparsity can be improved by selecting Ψ_T^i as a transform matrix. This is motivated by the intuition that the vector of RSS measurements at a given cell j received from AP i will be closer to the corresponding vectors of its neighboring cells, and thus it could be expressed as a linear combination of a small subset of the columns of Ψ_T^i.

Moreover, a measurement matrix $\Phi_T^i \in \mathbb{R}^{M_i \times N_i}$ must be associated with each transform matrix Ψ_T^i, where M_i is the number of CS measurements ($M_i \ll N_i$). In the proposed algorithm a standard Gaussian matrix with its columns normalized to unit Euclidean norm is employed to acquire the measurements, while in general $M_i \neq M_{i'}$, $i \neq i'$. The overall measurement model associated with the i-th AP is given by

$$\mathbf{g}_i = \Phi_T^i \Psi_T^i \mathbf{w} \ . \tag{5}$$

3.2 Runtime phase

A similar process to the one described in the previous section is followed during the runtime phase. More specifically, at the current unknown cell c the device collects a number of RSS measurements from all the APs. Let $\psi_{R,c}^i \in \mathbb{R}^{n_{c,i}}$ be the vector of RSS measurements received from the i-th AP. Notice that, since the acquisition time interval during the runtime phase is smaller than that in the training phase, a reduced amount of RSS readings is acquired, that is, $n_{c,i} < n'_{c,i}$, where $n'_{c,i}$ denotes the length of the corresponding RSS vector generated at the same cell during the training phase. The CS measurement model associated with the cell c and AP i is written as

$$\mathbf{g}_{c,i} = \Phi_R^i \psi_{R,c}^i \ , \tag{6}$$

where $\Phi_R^i \in \mathbb{R}^{M_{c,i} \times n_{c,i}}$ denotes the corresponding measurement matrix during the runtime phase.

For simplicity let us consider the case of a single AP. In the ideal case, if c^* is the true location and ψ_{R,c^*}^i is among the columns of Ψ_T^i, then the inversion of (5), given the operator $\Phi_T^i \Psi_T^i$ and the measurements $\mathbf{g}_{c^*,i}$, would yield a one-sparse vector $\mathbf{w}$ with the "1" placed exactly at its c^*-th component. However, due to the varying environmental conditions between the training and the runtime phase, as well as the differences in the dimension of the received RSS measurement vectors mentioned above, the best we could expect is the vector ψ_{R,c^*}^i to be close to ψ_{T,c^*}^i.

In order to overcome the dimensionality constraints, while maintaining the robustness of the reconstruction procedure, we select Φ_R^i to be a subset of Φ_T^i with an appropriate number of rows such as to maintain equal measurement ratios, $\frac{M_i}{N_i} = \frac{M_{c,i}}{n_{c,i}}$. For this purpose, the wireless device requests from the server the corresponding measurement sub-matrix by transmitting the dimension of the received RSS vector. Then, the measurement vector $\mathbf{g}_{c,i}$ is formed for each AP i according to (6) and transmitted to the server, where the reconstruction takes place. We emphasize at this point the significant conservation of the processing and bandwidth resources of the wireless device, by computing only low-dimensional matrix-vector products to form $\mathbf{g}_{c,i}$ ($i = 1, \ldots, P$) and then transmitting a highly reduced amount of data ($M_{c,i} \ll n_{c,i}$). Then, the CS reconstruction is performed at the server for each AP independently and the final location estimate is the centroid of the estimated cells. The reason is that, due to the network configuration, the RSS values are received independently.

The amount of transmitted data is further reduced in the proposed implementation by selecting to process the RSS readings of only the top P' strongest APs, that is, the APs with the highest mean RSS value of the corresponding vectors $\psi_{R,c}^i$. An additional advantage of this process is that in many cases we discard the potentially confusing information from APs from which either there was not any reception at all, or there was a link failure with the device during the runtime phase. The experimental evaluation presented in the next section reveals an increased estimation accuracy of the proposed CS localization algorithm. Finally, the overall system architecture is shown in Fig. 1.

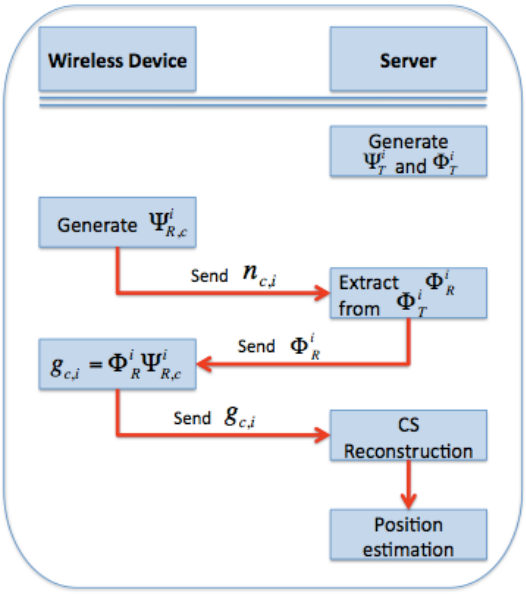

Figure 1: Flow diagram of the proposed CS-WLAN localization scheme.

4. CS AND SECURITY

Due to their acquisition process, CS measurements can be viewed as "weakly encrypted" for an attacker without knowledge of the measurement matrices Φ_T^i. CS-based encryption provides both signal compression and encryption guarantees, without the additional computational cost of a separate encryption protocol and thus it could be useful in location estimation, where the implementation of an additional software layer for cryptography could be costly. The encryption property of a CS approach relies on the fact that the matrix Φ_T^i is unknown to an unauthorized entity, since Φ_T^i can be generated using a (time-varying) cryptographic key that only the device and the server share. An attack could be considered as the attempt to estimate the key by trying to find the special structure of the Φ_T^i matrix [21].

In our proposed approach no cryptographic key is required, since it is based only on the matrices Φ_T^i ($i = 1, \ldots, P$). More specifically, the server extracts the sub-matrix Φ_R^i from Φ_T^i and then permutes its lines forming a new $\Phi_{R,p}^i$, which is then sent to the wireless device, where the associated measurement vector $\mathbf{g}_{c,i} = \Phi_{R,p}^i \psi_{R,c}^i$ is computed. A potential attacker has two options, either to try capturing $\Phi_{R,p}^i$ by intercepting the server$\rightarrow$device direction, or by acquiring $\mathbf{g}_{c,i}$ by intercepting the opposite direction. In the first case, modern network cryptographic protocols could guarantee that the decryption of $\Phi_{R,p}^i$ is almost infeasible in practice due to the combinatorial nature of the inverse problem. In the second case, as it will be illustrated in the next section, even the exact knowledge of $\mathbf{g}_{c,i}$ is insufficient, resulting in a significantly increased estimation error, when the attacker does not achieve the exact estimate of $\Phi_{R,p}^i$.

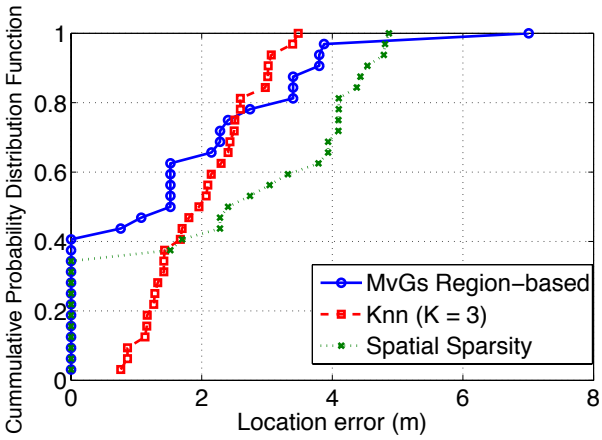

Figure 2: Performance comparison of previous fingerprint-based localization methods for the INRIA dataset.

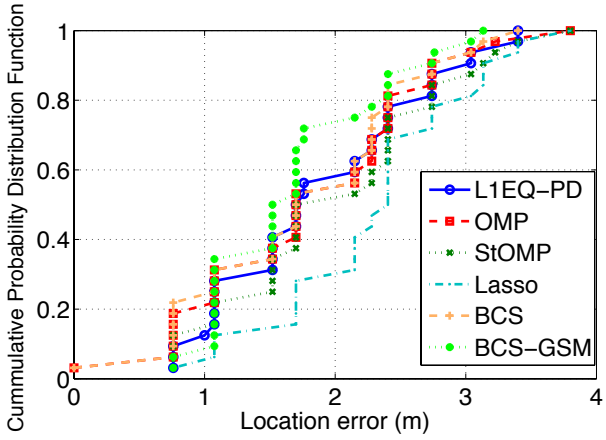

Figure 3: Performance evaluation of the CS-WLAN localization method for several reconstruction algorithms.

5. EXPERIMENTAL RESULTS

In this section, the performance of the proposed CS-WLAN localization method is evaluated and compared with previous fingerprint-based algorithms.

The dataset used in the present evaluation was acquired in the building 21 of INRIA, at Rocquencourt campus (Paris)[1]. The wireless coverage is achieved by employing an infrastructure consisting of five IEEE802.11 APs. The area used in the formation of the RSS map is discretized in cells of equal dimensions 0.76 m $\times$ 0.76 m. The RSS map consists of measurements from different cells and for an average number of five APs per cell. The time intervals during the acquisition in the training and runtime phase were set to 90 sec and 30 sec, respectively.

The estimation accuracy of the methods tested hereafter is evaluated in terms of the localization error, which is defined as the Euclidean distance between the centers of the estimated cell and the true cell where the mobile user is located at runtime. Runtime measurements in 32 distinct cells are employed in the subsequent evaluation.

Fig. 2 presents the localization error of the three methods introduced briefly in Section 2 (region-based MvG, kNN ($k = 3$), and spatial sparsity-based). The median error is equal to 1.99 m and 2.34 m for the kNN, and the spatial sparsity-based methods, respectively, while the MvG approach results in a median error of 1.56 m. Fig. 3 shows the estimation error for the proposed CS-based method, averaged over 100 Monte-Carlo runs, where in each run a distinct measurement matrix is generated. The reconstruction performance is compared between several widely-used norm-based techniques and Bayesian CS algorithms. More specifically, the following methods are employed[2]: 1) ℓ_1-norm minimization using the primal-dual interior point method (L1EQ-PD), 2) Orthogonal Matching Pursuit (OMP), 3) Stagewise Orthogonal Matching Pursuit (StOMP), 4) LASSO, 5) BCS, and 6) BCS-GSM [20]. As it can be seen, the BCS and BCS-GSM methods outperform the others with a median error of 1.89 m and 1.78 m, respectively (ℓ_1 2 m, OMP 2.03 m, StOMP 2.01 m, and Lasso 2.30 m). In this experiment only 8% of the total runtime RSS measurements vector is employed.

[1]The proposed method has been also evaluated by generating simulated RSS sequences using an appropriate propagation model in a more complicated indoor environment. Due to space limitations, the results and the simulated database can be found in [22].

[2]For the implementation of methods 1)-5) the MATLAB codes can be found in: `http://sparselab.stanford.edu/`, `http://www.acm.caltech.edu/l1magic`, `http://people.ee.duke.edu/~lcarin/BCS.html`

The effect of the number of CS measurements in the estimation accuracy is further examined for the top two candidates, namely, the BCS and BCS-GSM. Fig. 4 shows the corresponding localization error as a function of the percentage of the number of RSS measurements ($M = rN$ with $r \in \{5\%, 10\%, 15\%, 20\%\}$). As we expected, the localization accuracy increases by increasing the number of CS measurements, and for 15% of the RSS values the proposed approach outperforms the MvG method, which was the best among the previous fingerprint-based techniques.

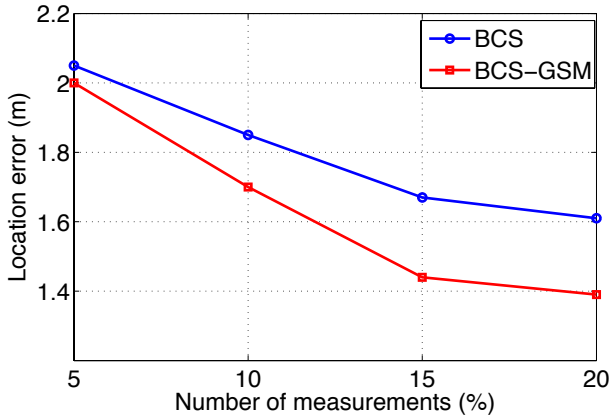

Figure 4: Localization accuracy of CS-WLAN using BCS and BCS-GSM, as a function of the number of CS measurements.

Fig. 5 compares the location error of the MvG and the BCS-GSM (with 25% of RSS measurements) methods, as a function of the input SNR. Each RSS vector is corrupted by additive white Gaussian noise with the SNR varying from 10 to 40 dB. As it can be seen, the proposed CS-based approach presents a clear superiority against MvG, especially for lower SNR values.

Finally, Fig. 6 illustrates the encryption capabilities of the proposed CS localization method for the BCS and BCS-GSM algorithms. In particular, the average localization error (over 100 Monte-Carlo runs) is shown as a function of the percentage of permuted lines ($\{0\% : 20\% : 100\%\}$) of the true matrices Φ_R^i, where the reconstruction is performed by considering exact knowledge of the measurement vectors $\mathbf{g}_{c,i}$. The results agree with our intuition that as the complexity of the permutation increases, the estimation accuracy decreases without an exact estimate of the true measurement matrix.

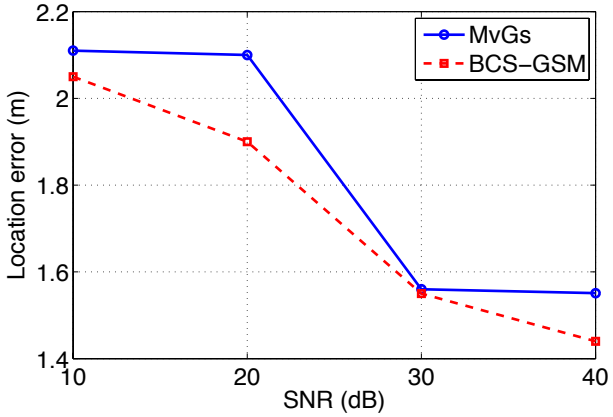

Figure 5: Localization accuracy of the MvG and BCS-GSM methods for varying input SNR.

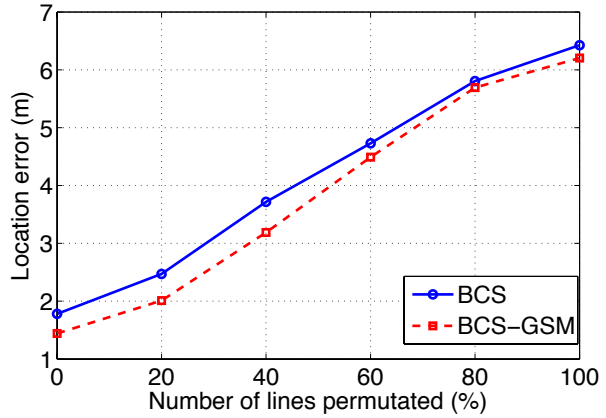

Figure 6: Evaluation of CS-WLAN encryption property using BCS and BCS-GSM, for a varying number of permuted lines of Φ_R^i.

6. CONCLUSION

This paper introduced an indoor localization method based on CS RSS fingerprints. The experimental results revealed that both BCS and BCS-GSM approaches achieve a higher estimation accuracy when compared with other CS recovery schemes, as well as with previous fingerprint-based methods. The enhanced encryption capabilities of the proposed CS-WLAN architecture, without the additional computational cost of a separate encryption protocol, were also evaluated. In the present work, the unknown location was estimated by performing separate reconstruction for each AP. A straightforward extension will be the use of the joint sparsity structure of the indicator vector $\mathbf{w}$ among the APs for the simultaneous location estimate. Moreover, a more thorough study should be carried out for the robustness of the inherent encryption property in terms of the several network parameters.

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