Chapter 1

Introduction

Statistical array processing based on the linear theory of random processes with finite second-order moments has been the focus of considerable academic research. Critical problems such as high-resolution direction finding, null- and beam-steering, and detection of the number of sources illuminating an array of sensors have been studied under the assumption of a Gaussian or second-order model. Many different classes of methods, compromising optimality for the sake of computational efficiency, have been proposed under the aforementioned statistical framework [34].

Looking toward real world applications, we are interested in developing array processing methods for a larger class of random processes which include the Gaussian processes as special elements. The availability of such methods would make it possible to operate in environments which, while sharing many characteristics also differ from Gaussian environments in significant ways.

The class of stable distributions, a natural generalization of the Gaussian distribution, has some important characteristics which make it very attractive for modeling impulsive signals. Stable processes satisfy the stability property which states that linear combinations of jointly stable variables are indeed stable. They arise as limiting processes of sums of independent, identically-distributed random variables via the generalized central limit theorem. That is, if the observed randomness is the result of many cumulative effects, and these effects follow a heavy-tailed distribution, then a stable model may be appropriate. Stable processes are described by their characteristic exponent $\alpha$, taking values $0 < \alpha \leq 2$. Gaussian processes are stable processes with $\alpha = 2$. Stable distributions have heavier tails than the normal distribution, possess finite $p$th order moments only for $p < \alpha$, and are appropriate for modeling signals with outliers. Hence, non-Gaussian stable deviates have infinite variance and in some cases infinite first moment. Unlike the Gaussian distribution
which is symmetric about its mean, stable distributions may be asymmetric, i.e., they admit skewness. Therefore, in certain applications where a heavy-tailed and asymmetric model is called for, the stable model may be a viable alternative to the Gaussian distribution. The difficulty in developing signal processing methods based on stable processes is due to the fact that the linear space of a stable process is not a Hilbert space, as in the case of Gaussian processes, but either a Banach \((1 < \alpha < 2)\) or a metric space \((0 < \alpha \leq 1)\), both of which are more unyielding in their structure.

This dissertation addresses the solution of the signal parameter estimation problem through the use of sensor array data retrieved in the presence of impulsive interference. One of the most interesting problems in this area is the estimation of the direction-of-arrival (DOA) of narrow-band source signals having the same known center frequency. A related problem is the detection of the number of sources impinging on a sensor array. In the past, these problems have been studied extensively under the assumption of Gaussian distributed signals and/or noise, and a variety of methods for their solution have been proposed. As a result of the Gaussianity assumption, most methods are based on the second- or higher-order statistics of the signals.\(^{[51]}\) The following section gives a brief review of these methods.

### 1.1 Literature Review

Despite the different characteristics of several existing multisensor data systems, there exists a common methodology of processing data from multiple sensors in order to make inferences about a physical event\(^{[26]}\). A block diagram of a typical multisensor system is shown in Figure 1.1. The sensors detect incoming energy and observe a combination of intentional energy from a target as well as environmental energy such as noise, multipath signals, and interference. An additional source of incoming energy may include jamming signals, which are intentionally generated by an enemy to deceive the observer by decreasing the signal-to-noise-ratio (SNR) of the observed environment. The ensuing signal conditioning process does not alter the information content of the signal but it performs translations (e.g. frequency shifts, heterodyning, A/D or D/A changes) that facilitate subsequent processing. A data alignment function transforms the raw sensor observations into a standard set of units. An association process groups the observations into meaningful groups with every group representing observations of a single physical entity.
Our work focuses on development of methods pertaining to the estimation part of the system shown in Figure 1.1. The estimation process combines the observations to obtain an estimate of a state vector $\mathbf{X}(t)$ which best fits the observed data. Basic inferences about the observed entity’s azimuth, elevation, range, or velocity are made at this level of processing. The estimation processor is essentially an algorithm which optimizes an appropriate chosen cost function with respect to the observation vectors. The cost function is formed by assuming certain statistics about the data based on the physics of the problem and by using an optimization criterion. There exist several optimization criteria such as Least-Squares (LS), Weighted LS, Maximum Likelihood, and Constrained (Bayesian) which, until recently, were based on a second-order moments statistical model.

Maximum Likelihood (ML) was one of the first methods to be applied in the area of array processing [34]. When applying the ML technique to the source localization problem, two different assumptions for the signal waveforms result into two different methods. According to the Stochastic ML (SML), the signals are modeled as Gaussian random processes. This is often motivated by the Central Limit Theorem and results in mathematically convenient expressions. On the other hand, in the Deterministic ML (DML) the signals are considered as unknown, deterministic quantities that need to be estimated.
in conjunction with the direction of arrival. This is a natural model for digital communication applications where the signals are far from being normal random variables, and where estimation of the signal is of equal interest.

The importance of the ML technique comes from the mathematical property that, under certain regularity conditions, the ML estimator is known to be asymptotically efficient, i.e., it achieves the Cramér-Rao bound (CRB) for the estimation error variance. In this sense, ML has the best possible asymptotic properties.

Many researchers have studied the ML technique as a means of approaching the source localization problem in the presence of Gaussian additive noise. Stoica and Nehorai examined the ML performance and its asymptotic properties, derived expressions for the CRB and established some of its properties [71]. Additionally, they investigated the relationship between the ML and large sample approximation methods such as MUSIC. With high computational cost constituting the main shortcoming of the ML technique, Ziskind and Wax introduced a computationally attractive method for calculating the ML estimator [97]. Their method was based on an iterative technique named “Alternating Projection” (AP) that transformed the multivariate optimization problem into a sequence of simpler one-dimensional optimization problems.

However, due to the high computational load of the multivariate nonlinear maximization problem involved in the ML estimator, sub-optimal methods have also been developed [13, 30, 41, 57, 61, 66, 80, 81, 84, 85]. The better known ones are cited here: Minimum Variance Distortionless method of Capon [13] and the so-called eigenvector-based methods including the MUSIC [66], Minimum Norm [41, 57], and the ESPRIT method [61]. The performance of the aforementioned methods is inferior to that of the ML method, especially for low SNR values or when the number of observation snapshots is small.

The MUSIC method, a generalization of Pisarenko’s harmonic retrieval method, has received the most attention and triggered the development of a large number of algorithms referred to as eigenvector or subspace techniques. Besides offering a new geometric interpretation of the array processing problem, MUSIC uses concepts from complex vector spaces and well-known tools from linear algebra, such as the singular value decomposition (SVD), in order to achieve high resolution while keeping the computational complexity relatively low compared to that of the ML methods.

Source localization can be considered as an important initial step in adaptive array processing. Once the desired signal and interference directions are determined, adaptive
statistically optimum beamforming maximizes the signal-to-noise ratio of the array output for optimal signal detection. The multiple sidelobe canceller (MSC) is perhaps the earliest statistically optimum beamformer introduced by Applebaum et al. [1]. Widrow and associates studied the adaptive antenna problem by minimizing the mean square error between the beamformer output and a reference signal [92]. Reed et al. concentrated on adaptive radar applications. They developed an adaptive processor which maximizes the probability of detection for a fixed false-alarm rate [4]. They also introduced a direct method of adaptive weight computation, based on the sample covariance matrix of the noise field, which provides rapid convergence [5].

Recently, special interest has been shown in relaxing some of the assumptions concerning the statistical nature of the noise in the bearing estimation problem [19, 20, 39]. One such method is the bispectrum beamformer introduced by Forster and Nikias [19]. It was demonstrated that, for the case of spatially correlated Gaussian additive noise with unknown cross-spectral matrix (CSM), the bispectrum beamformer may provide asymptotically better bearing estimates than the stochastic ML method with known CSM.

Often in actual applications the Gaussian noise assumption proves inadequate, as systems designed under this assumption exhibit a significant performance degradation. There exist physical processes generating interferences which contain noise components that are impulsive in nature. These processes can be natural, as well as man-made, and include radar clutter, underwater acoustic signals, lightning in the atmosphere, and transients in power lines and car ignitions. In modeling this type of signals, the stable distribution law provides a very attractive theoretical tool. It was proven that under broad conditions, a general class of impulsive noise follows the stable law [68]. As a result, considerable research interest has been shown in designing robust signal processing algorithms for detection, direction finding, and equalization, that can do well not only in the presence of Gaussian noise, but also, more importantly, in the presence of impulsive noise environments [75, 78, 79].

1.2 Dissertation Organization and Contribution

This dissertation is devoted to the detection and localization problem of multiple sources in the presence of impulsive additive interference which can be modeled as a complex isotropic \( \alpha \)-stable process. The dissertation is organized as follows: In Chapter 2, we formulate the direction-of-arrival estimation problem. In Chapter 3, we introduce the statistical model,
based on the class of bivariate symmetric \( \alpha \)-stable (S\( \alpha \)S) distributions. This model is well-suited for describing signal and/or noise processes that are impulsive in nature and contains the Gaussian process as a special case. In Chapter 4, we develop the deterministic ML estimator and derive the Cramér-Rao bound for the Cauchy case. We also discuss the application of the ML method for the general case of S\( \alpha \)S processes and we present Monte-Carlo simulation results. In Chapter 5, we develop large-sample, subspace-based bearing estimation techniques based on the eigendecomposition of the array covariance matrix. In Chapter 6, we extend our methods to handle the case of wide-band source signals. Finally, in Chapter 7, we propose future research directions.

### 1.3 Abbreviations

The following abbreviations are used in this dissertation:

- **CRB**: Cramér-Rao bound
- **DML**: deterministic maximum likelihood
- **DOA**: direction-of-arrival
- **FLOM**: fractional lower-order moment
- **GSNR**: generalized signal-to-noise ratio
- **IM**: incoherent MUSIC
- **IRM**: incoherent ROC-MUSIC
- **ML**: maximum likelihood
- **MLC**: maximum likelihood Cauchy
- **MLG**: maximum likelihood Gaussian
- **MSE**: mean-square error
- **MUSIC**: multiple signal classification
- **ROC-MUSIC**: robust covariance-based MUSIC
- **PSNR**: pseudo signal-to-noise ratio
- **S\( \alpha \)S**: symmetric \( \alpha \)-stable
- **SML**: stochastic maximum likelihood
- **SNR**: signal-to-noise ratio
- **SSC**: steered spectral covariance
- **STMV**: steered minimum variance