VISION–BASED TIME–VARYING MOBILE ROBOT CONTROL

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Exponential stabilization to an equilibrium point for a nonlinear control system without drift is an interesting problem in control theory, which has significant applications in mobile robotics. State–feedback controllers developed for this problem require an estimate of the mobile robot's state, either from odometry or from exteroceptive sensors. The present paper considers the use of visual data, from a camera tracking a known target, in a visual–servoing scheme based on time–varying exponential stabilizers. Two such schemes are presented, one, where the system's state is explicitly reconstructed from the sensory data and another, where these data are used directly in the control loop. We study the planar case, where the camera only pans, and present some preliminary simulation results.

1 Introduction

We consider the problem of exponential stabilization to a desired equilibrium point of a mobile robot of the unicycle type using visual data to estimate its state. This can be seen as an attempt to extend the visual–servoing approach, i.e. the direct use of visual feedback in a system's control loop, which has been relatively well developed for the case of manipulator arms [5,7,6], to the case of robotic systems where nonholonomic constraints on the motion of the system are present. These constraints arise in the present case from the rolling–without–slipping of the robot's wheels on the plane supporting the system and constrain its instantaneous motion, whose component lateral to the heading direction is zero. Previous work in this area by Pissard–Gibollet and Rives [12] applied the task–function approach [5] to this problem, showing how to position a camera mounted on a mobile robot in front of a given target, without, however, explicitly controlling the final position and orientation of the mobile robot, as we are doing here.

Systems of this type can be modeled as drift–free controllable nonlinear systems with fewer controls than states, with the controls entering linearly in the state equations. Not only the linearization of these systems is uncontrollable, but also there do not exist continuous feedback control laws, involving only the state, that would asymptotically stabilize the system to an equilibrium, due to a topological obstruction pointed out by Brockett [2]. One of the approaches developed to solve the stabilization problem is the use of time–varying state feedback, i.e. control laws that depend explicitly, not only on the state, but also on time, usually in a periodic way. Samson [15] introduced them in the context of the unicycle point stabilization problem. Both the existence of smooth time–periodic feedback controls for a wide class of systems (Coron [3]) and systematic procedures for the construction of smooth asymptotic stabilizers (Pomet [13], Teel, Murray, Walsh [17]) have been developed. As noted in [15], smooth time–varying feedback controllers stabilize the system, but convergence to the desired equilibrium is only polynomial, not exponential. In fact, Lipschitz feedback can be shown to be unable to achieve exponential stabilization [9,10]. However, Coron [4] established the existence of continuous only, time–varying controls, which stabilize the system in finite time and are smooth everywhere, except at the desired equilibrium. This leads to the existence of exponential stabilizers. M'Closkey and Murray [9, 10] and Pomet and Samson [14] derived continuous non–Lipschitz periodic time–varying exponentially stabilizing controls, which make the closed–loop system homogeneous of degree zero. Morin and Samson [11] improved the design of controllers of this class, in providing ways of achieving a prespecified rate of exponential stabilization.

The implementation of a closed-loop control scheme, such as the one above, for accurate positioning and stabilization of the mobile robot at a desired configuration, depends upon our ability to estimate the state of the mobile robot at every time instant. In [10], a passive linkage was used to estimate the state of the mobile robot. In a more realistic setting, this state estimation may rely on odometry or, as in the present instance, on the use of vision sensors. To accomplish this, we suppose that the mobile robot is equipped with a vision sensor, which is able to move independently from the robot (e.g. is mounted on a pan–and–tilt platform). It is thus able to track a target of interest while the robot moves. We show how the unicycle's state can be reconstructed from the visual data using this setup. Alternatively, we propose a scheme where the visual data enter directly in the control loop, without the intermediate step of state reconstruction. The exponential time–varying state–feedback controller above, is then transformed into an image–based visual–servoing scheme for stabilizing the nonholonomic robot to a desired configuration.

In section 2 of this paper, we derive the system kinematics, velocity kinematics and vision model. In section 3 we consider the estimation of the unicycle's state from visual data. In section 4 we present the control for both the unicycle and for the camera. In section 5 we present simulation results.

2 System Model

We consider a mobile robot of the unicycle type carrying a camera in the configuration shown in fig. 1.

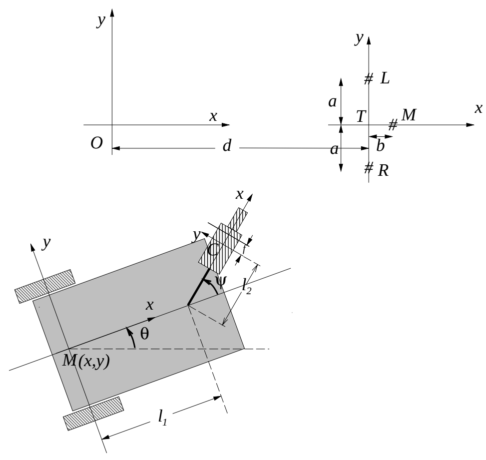


Figure 1: Unicycle with Camera

2.1 Robot Kinematics

Consider an inertial coordinate system $\{O\}$ centered at a point O of the plane, a moving coordinate system $\{M\}$ attached to the middle M of the robot's wheel axis and another moving one $\{C\}$ attached to the optical center C of the camera. Let (x, y) be the position of the point M and θ be the orientation of the mobile robot with respect to the coordinate system $\{O\}$, let ψ be the angle that the arm that carries the camera makes with the body of the mobile robot and let l_1 be the distance of the point M from the joint of the arm and l_2 be the length of this arm (fig. 1).

Consider also a fixed target containing three easily identifiable feature points arranged in the configuration of fig. 1 and let $\{T\}$ be the related coordinate frame, which we suppose to be translated along the x–axis of $\{O\}$ by a distance d. The coordinates of the three feature points with respect to $\{T\}$ are $(x_p^{\{T\}}, y_p^{\{T\}}),\ p \in \{l, m, r\}$. The distances a and b (fig. 1) are assumed to be known. Let x_{CT}, y_{CT} and θ_{CT} represent the position and orientation of $\{T\}$ with respect to $\{C\}$. The coordinates of the feature points with respect to $\{C\}$ are

$$x_p^{\{C\}} = x_{CT} + x_p^{\{T\}} \cos\theta_{CT} - y_p^{\{T\}} \sin\theta_{CT},\ y_p^{\{C\}} = y_{CT} + x_p^{\{T\}} \sin\theta_{CT} + y_p^{\{T\}} \cos\theta_{CT}. \tag{1}$$

From the kinematic chain of fig. 1, we have

$$x \quad = \quad -x_{CT}\cos\theta_{CT} - y_{CT}\sin\theta_{CT} - l_1\cos(\theta_{CT} + \psi) - l_2\cos\theta_{CT} + d,$$

$$\begin{array}{rcl} y & = & x_{CT}\sin\theta_{CT} - y_{CT}\cos\theta_{CT} + l_1\sin(\theta_{CT}+\psi) + l_2\sin\theta_{CT}, \\ \theta & = & -(\theta_{CT}+\psi). \end{array} \quad (2)$$

By differentiating the chain kinematics and, since we consider stationary targets, we get

$$\begin{pmatrix} \Xi_1^{CT} \\ \Xi_2^{CT} \\ \omega_{CT} \\ \omega_\psi \end{pmatrix} = \begin{pmatrix} -\cos(\theta+\psi) & -\sin(\theta+\psi) & l_1\sin\psi & 0 \\ \sin(\theta+\psi) & -\cos(\theta+\psi) & -(l_1\cos\psi + l_2) & -l_2 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}, \quad (3)$$

where $\omega \stackrel{\text{def}}{=} \dot{\theta}$ and $\omega_\psi \stackrel{\text{def}}{=} \dot{\psi}$. We call the above matrix $B_1(\theta,\psi)$.

The motion of the system is subject to a nonholonomic constraint due to the assumption that the wheels of the mobile robot roll without slipping on the supporting plane. This constrains the instantaneous motion of the system, imposing the requirement that the lateral component of the mobile robot's body translational velocity is zero, i.e. $-\dot{x}\sin\theta + \dot{y}\cos\theta = 0$. From this, we get the usual unicycle kinematic model

$$\dot{x} = v\cos\theta\ ,\ \ \dot{y} = v\sin\theta\ ,\ \ \dot{\theta} = \omega\ , \quad (4)$$

where $v \stackrel{\text{def}}{=} \dot{x}\cos\theta + \dot{y}\sin\theta$ is the heading velocity.

The following transformation of the states and of the inputs

$$\begin{array}{rclrcl} x_1 & = & x\cos\theta + y\sin\theta\ , & & & \\ x_2 & = & -(x\sin\theta - y\cos\theta) + \theta(x\cos\theta + y\sin\theta), & x_3 & = & \theta\ , \\ u_1 & = & v - (x\sin\theta - y\cos\theta)\,\omega\ , & u_2 & = & \omega\ , \end{array} \quad (5)$$

brings equations 4 in the so-called *chained form*[14,11]:

$$\dot{x}_1 = u_1\ ,\ \ \dot{x}_2 = x_3 u_1\ ,\ \ \dot{x}_3 = u_2\ . \quad (6)$$

2.2 Vision Model

We consider the usual pinhole camera model for our vision sensor, with perspective projection of the target's feature points (viewed as points on the plane $I\!R^2$) on a 1–dimensional image plane (analogous to a linear CCD array). This defines the projection function Y of a point of $I\!R^2$, which has coordinates (x,y) with respect to the camera coordinate frame $\{C\}$, as

$$Y : I\!R_+ \times I\!R \longrightarrow I\!R : (x,y) \longmapsto Y(x,y) = f\frac{y}{x}. \quad (7)$$

where f is the focal length of the camera. In our setup, the coordinate x corresponds to "depth".

Let $Y = Y(x, y)$ and let the projections of the target feature points on the image plane be Y_l, Y_m, Y_r, given by 7 and 1. Differentiating 7, we get the well–known equations of the optical flow [8] for the 1–dimensional case:

$$\begin{pmatrix} \dot{Y}_l \\ \dot{Y}_m \\ \dot{Y}_r \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} -\frac{1}{x_l^{\{C\}}} Y_l & \frac{1}{x_l^{\{C\}}} f & \frac{1}{f}(f^2 + Y_l^2) & 0 \\ -\frac{1}{x_m^{\{C\}}} Y_m & \frac{1}{x_m^{\{C\}}} f & \frac{1}{f}(f^2 + Y_m^2) & 0 \\ -\frac{1}{x_r^{\{C\}}} Y_r & \frac{1}{x_r^{\{C\}}} f & \frac{1}{f}(f^2 + Y_r^2) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Xi_1^{CT} \\ \Xi_2^{CT} \\ \omega_{CT} \\ \omega_\psi \end{pmatrix}. \quad (8)$$

We call the above matrix $B_2\big(Y_l, Y_m, Y_r, x_l^{\{C\}}, x_m^{\{C\}}, x_r^{\{C\}}\big)$. It corresponds to the interaction matrix of [5,12].

3 State Estimation

The problem that we consider is to stabilize the unicycle to an arbitrary configuration, which, without loss of generality, will be chosen to be zero, i.e. $(x_\star, y_\star, \theta_\star, \psi_\star) = (0, 0, 0, 0)$. The corresponding desired sensory data configuration $(Y_{l\star}, Y_{m\star}, Y_{r\star}, \psi_\star)$ can be easily specified.

Our goal then becomes to use the sensory data from the camera and from the proprioceptive sensors (e.g. encoder measuring angle ψ) to estimate the state (x, y, θ, ψ) of the system and to implement a sensor–based closed loop controller that will allow us to reach the desired configuration. Two schemes are presented below, one where the state is explicitly reconstructed and one where it is approximated directly from the sensory data.

3.1 State Reconstruction

A procedure for the estimation of the relative planar configuration of the target and the camera using three target feature points has been considered, similar to the one presented in Sugihara [16]. The setup is shown in fig. 2.

From the projections Y_l, Y_m, Y_r of the target features L, M, R on the image plane, it is possible to estimate the angles α_{LM}, α_{MR} and α_{LR}, with which the segments LM, MR, LR are seen from the optical center C. We assume that we know the target geometry, in particular the lengths of these segments. It is well known from Euclidean geometry that the locus of all points that see a given segment with a given angle, is a circle; the segment is a chord of this circle and the given angle is its angle at the circumference. The circles corresponding to the segments and the angles above can then be computed and their intersection

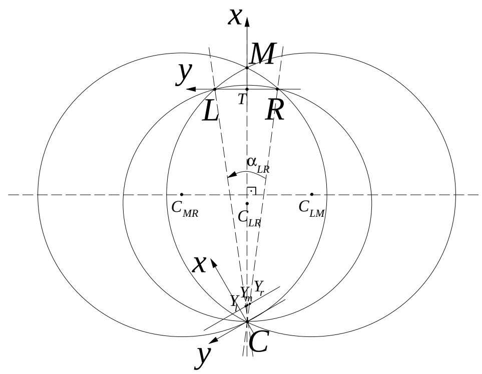


Figure 2: Unicycle State Reconstruction

is the optical center C. From this, x_{CT}, y_{CT} and θ_{CT} can be computed. Then, we can use equations 2 to estimate the state (x, y, θ).

3.2 State Approximation

We are interested in positioning the mobile robot to a desired configuration, while starting relatively close to it. We would like to do so without needing to reconstruct explicitly its state, as was done in section 3.1.

Consider the following approximation of the state of the system near the desired state $(x_\star, y_\star, \theta_\star, \psi_\star)$:

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} x_\star \\ y_\star \\ \theta_\star \\ \psi_\star \end{pmatrix} + \left(B_{2\star}\ B_{1\star}\right)^{-1} \begin{pmatrix} Y_l - Y_{l\star} \\ Y_m - Y_{m\star} \\ Y_r - Y_{r\star} \\ \psi - \psi_\star \end{pmatrix} , \tag{9}$$

where $B_{1\star}, B_{2\star}$ are the matrices B_1, B_2 defined in equations 3 and 8 respectively, evaluated at the desired configuration. Differentiating 9 we get

$$\left(\dot{\tilde{x}},\ \dot{\tilde{y}},\ \dot{\tilde{\theta}},\ \dot{\tilde{\psi}}\right)^\top = \left(B_{2\star}\ B_{1\star}\right)^{-1}\left(B_2\ B_1\right)\left(\dot{x},\ \dot{y},\ \dot{\theta},\ \dot{\psi}\right)^\top . \tag{10}$$

Thus, when the matrix $\left(B_{2\star}\ B_{1\star}\right)^{-1}\left(B_2\ B_1\right)$ is close to the identity, which is clearly the case near the desired configuration, the state approximation $(\tilde{x}, \tilde{y}, \tilde{\theta}, \tilde{\psi})$ agrees up to first order with the state of the system (x, y, θ, ψ). Its use in the control loop allows direct introduction of the sensory data in it;

moreover, the matrix $\left(B_{2\star}\ B_{1\star}\right)^{-1}$ can be computed off–line, alleviating the computational needs of this state estimation scheme.

4 Control

Assuming that the state estimation procedures described in section 3 provide us with the current state (x, y, θ, ψ) of the system at every time instant, we describe below the control used to drive the system to the desired state. An exponentially stabilizing control is considered for the unicycle, while a control that keeps the target foveated is considered for the camera.

4.1 Unicycle Exponential Stabilization

The visually estimated state (x, y, θ) can be used to compute the corresponding chained–form coordinates (x_1, x_2, x_3) by equation 5. The unicycle control that we implement is given, in terms of these (x_1, x_2, x_3), by:

$$v = u_1 + (x_1 x_3 - x_2)\, u_2 \ , \quad \omega = u_2 \ , \tag{11}$$

where u_1 and u_2 are the exponentially stabilizing time–varying state–feedback controls, developed by Morin and Samson [11] for the 3–dimensional 2–input chained–form system of equation 6, and which are:

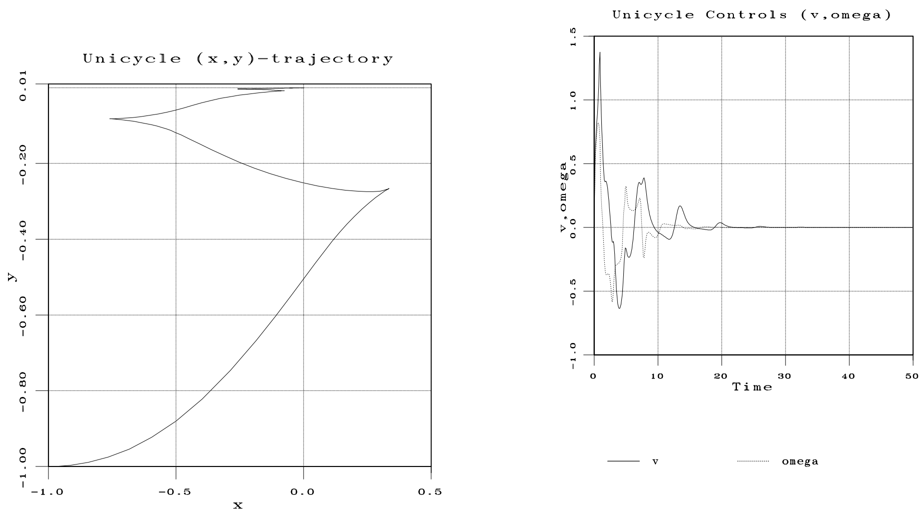
$$\begin{array}{rcl} u_1(x_1, x_2, x_3, t) &=& k_1 \left[\rho(x_1, x_2, x_3) - \alpha x_1 \sin t\right] \sin t \ , \\ u_2(x_1, x_2, x_3, t) &=& -k_3 \left[x_3 + k_2 \frac{x_2}{\rho(x_1, x_2, x_3)} \sin t\right] \ , \end{array} \tag{12}$$

where $\rho(x_1, x_2, x_3) \stackrel{\text{def}}{=} (x_1^4 + x_2^2 + x_3^4)^{\frac{1}{4}}$, $0 < \alpha < 1$ and $k_1, k_2, k_3 \geq 0$ are gains with $k_3 >> k_1, k_2$.

4.2 Camera Control

The camera is mounted on a platform of the pan–and–tilt type which allows us to reorient it at will by controlling the angle ψ that the optical axis makes with the body of the mobile robot (fig. 1). Supposing that the initial camera orientation is such that the target is in its field–of–view (FOV), we control its orientation ψ so that the target is tracked as the mobile robot moves, i.e. so that it remains almost centered in the image plane (c.f. Aloimonos and Tsakiris [1]). If the target is not in the FOV initially, an exploratory target acquisition phase may be necessary. The camera control ω_ψ used in order to achieve tracking is:

$$\omega_\psi \quad = \quad k_4 \frac{1}{n} \sum_{i=1}^{n} \frac{Y_i}{f} , \tag{13}$$



(a) (b)

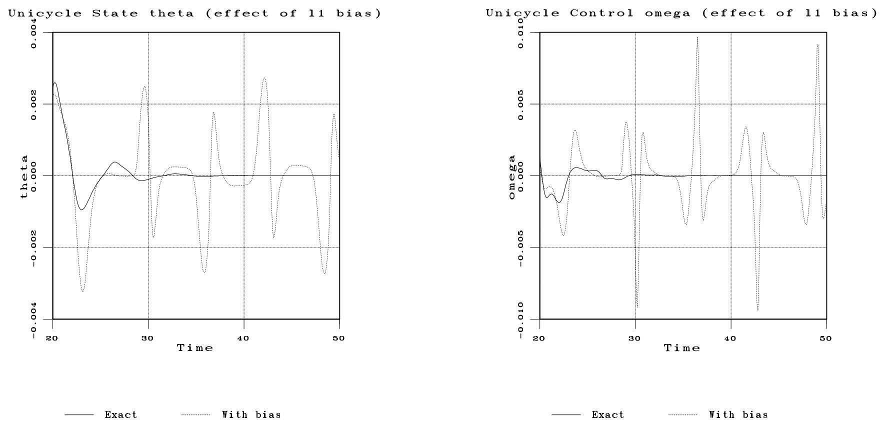
Figure 3: (a) Unicycle (x, y)–trajectory and (b) Unicycle Controls (v, ω)

where $Y_1, \ldots, Y_n$ are the projections of the n target feature points on the image plane of the camera, f is the focal length of the camera and $k_4 > 0$ is the corresponding gain.

5 Simulation Results

In the simulations presented below, we consider stabilization to zero, starting from $(x, y, \theta, \psi) = (-1, -1, 0, 0)$. We use the state approximation procedure of section 3.2. The mobile robot's (x, y)–trajectory is shown in fig. 3.a and its controls (v, ω) are shown in fig. 3.b, demonstrating the fast convergence of the system to zero.

However, uncertainty in the model parameters may prevent the system from converging, introducing oscillations in the vicinity of the desired state. This is shown in fig. 4.a: when the model parameters are known exactly, the state θ converges exponentially to zero (solid line); when a 5% bias is introduced in the parameter l_1, then θ oscillates (dotted line). The corresponding controls ω are shown in fig. 4.b. For clarity, in fig. 4, only the plots corresponding to the later part of the system's motion are shown.



(a) (b)

Figure 4: Uncertainty in model parameter l_1 : Evolution of (a) θ and (b) ω

6 Conclusions

The vision–based exponential stabilization schemes presented in this paper constitute an initial attempt to extend the visual–servoing approach to the case of robotic systems with nonholonomic constraints. Future work will consider improving the robustness of these schemes with respect to data noise and model uncertainties, accounting for the multifrequency characteristics of the system, extending our setup to the 3–dimensional case by considering a camera–carrying platform with more degrees–of–freedom and evaluating these schemes experimentally.

Acknowledgments

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