ELSEVIER

Signal Processing 80 (2000) 381–396

SIGNAL PROCESSING

www.elsevier.nl/locate/sigpro

Robust spatial filtering of coherent sources for wireless communications

Marilli Rupi[a], Panagiotis Tsakalides[b,*], Enrico Del Re[a], Chrisostomos L. Nikias[b]

[a] *Dip di Ingegneria Elettronica, Università di Firenze, via S. marta 3, 50139 Firenze, Italy*
[b] *Department of Electrical Engineering, University of Southern California, Los Angeles, 90089-2564 CA, USA*

Received 7 September 1998; received in revised form 13 September 1999

Abstract

This paper addresses the inter-signal coherence problem in mobile communications applications where multipath parameters, such as path angle of arrival, must be estimated with an array of sensors operating in impulsive interference. We reduce the measured coherence by using the spatial smoothing approach to the structure of the measurement matrix while at the same time we mitigate the effects of the heavy-tailed background noise by employing a signed-power nonlinearity to the array data. The combination of these two processing modules gives rise to a family of robust smoothed subspace array processing methods based on fractional lower-order statistics (FLOS), which are able to identify all incident angles regardless of their correlation structure. The improved performance of the proposed techniques is demonstrated via Monte Carlo simulations © 2000 Elsevier Science B.V. All rights reserved.

Zusammenfassung

Dieser Artikel behandelt das Signalkohärenzproblem in der mobilen Kommunikation, wenn Mehrwegeparameter wie z.B. der Einfallswinkel eines Pfades mit Hilfe einer Sensorgruppe geschätzt werden muß, die in impulshafter Störung arbeitet. Durch räumliche Gläuung reduzieren wir die gemessene Kohärenz auf die Struktur der Datenmatrix. Dabei wird das stark ausgedehnte Hintergrundgeräusch durch eine Vorzeichen-Potenz-Nichtlinearität abgeschwächt, die auf die Daten der Sensorgruppe angewandt wird. Die Kombination dieser zwei Signalverarbeitungsmodule ist die Ausgangsbasis für eine Familie von robusten glättenden Unterraum-Methoden zur Sensorgruppensignalverarbeitung, die auf fractional lower order statistics (FLOS) basieren, die sämtliche Einfallswinkel unabhängig von ihrer Korrelationsstruktur identifizieren können. Das verbesserte Leistungsverhalten der vorgestellten Verfahren wird anhand von Monte Carlo Simulationen demonstriert. © 2000 Elsevier Science B.V. All rights reserved.

Résumé

Cet article porte sur le problème de la cohérence inter-signaux dans les applications de communications mobiles dans lesquelles de paramètres de propagations multiples, tels que l'angle d'arrivée, doivent être estimés avec un réseau de capteurs opérant en interférence impulsive. Nous réduisons la cohérence mesurée en utilisant l'approche de lissage spatial à la structure de la matrice de mesure tandis que dans le même temos nous mitigeons les effets du bruit de fond à densité de probabilité étendue en employant une non-linéarité de type puissance signée sur les données du réseau. La

* Correspondence address: VLSI Design Laboratory, Department of ECE, University of Patras, 261 10 Rio, Greece. Tel.: + 30-61-997-283; fax: + 30-61-994-798.

E-mail address: tsakalid@ee.upatras.gr (P. Tsakalides)

PII: S0165-1684(99)00138-3

combinaison de ces deux modules de traitement permet de créer une famille de méthodes de traitement de réseau en sous-espace lissées robustes basées sur les statistiques d'ordre inférieur fractionnaires (FLOS), qui sont capables d'identifier tous les angles d'incidence quelle que soit leur structure de corrélation. Les performances supérieures des techniques proposées sont mises en lumière par des simulations de type Monte Carlo.

Keywords: Symmetric alpha-stable (SαS) distributions; Eigen-based methods; Multipath environment; Coherent sources; Spatial smoothing; Fractional lower-order statistics

1. Introduction

The rapidly increasing demands for personal mobile communications have created an avid interest in new array processing algorithms and antenna architectures. The appropriate choice for a particular antenna array model depends on its capability to discriminate among different incoming signals, which can be generated from various digitally modulated sources. Specifically, in wireless communications the transmitted signal arrives to the receiver via multiple paths due to physical phenomena such as diffraction and reflection, which are caused by obstacles present in the line of view between the transmitter and the receiver. Multipath can affect the incoming signal and destroy completely the information sequence [13]. Besides, coherent interference can arise when smart jammers deliberately redirect scaled and delayed replicas of the same signal to the receiver. The antenna array should be able to process the incoming signal and improve its quality by applying techniques based on interference cancellation and spatial diversity [4].

In recent years, considerable effort has been directed towards the development of high-resolution techniques for estimating the direction of arrival of multiple signals using antenna arrays. The class of eigen-based methods has been proven to be an effective means to achieve high-resolution source localization even when the signals are partially correlated [20]. However, when some of the signals are perfectly correlated, eigen-based or subspace techniques designed to locate uncorrelated sources cannot be used directly since the incoherence assumption is violated.

To address the coherent signal localization problem, a spatial smoothing scheme was initially suggested by Johnson et al. [6], and subsequently studied by Shan et al. [15]. The proposed methods used sub-aperture sampling in uniform linear arrays that essentially decorrelates the coherent signals. To compensate for the effective array aperture reduction induced by the smoothing techniques, Johnson et al. introduced the modified spatial smoothing method [6]. Other advanced techniques used forward and complex conjugate backward sub-arrays of the original array [12].

The majority of the spatial smoothing techniques assume that the array operates in an additive white noise background. By and large, the Gaussian distribution is the favorite noise model commonly employed in radio communications mainly because it often leads to closed-form solutions and to linear processors. However, multiuser interference, atmospheric noise (thunderstorms), car ignitions, and other types of naturally occurring or man-made noise sources result in an aggregate noise component that may exhibit high amplitudes for small duration time intervals [10,16]. Recent experimental measurements have demonstrated that the ambient channel noise is decidedly non-Gaussian due mostly to impulsive phenomena (see [19] and references therein). Indeed, it has been shown that electromagnetic noise in urban mobile-radio channels is heavy tailed in nature and can be better modeled by using distributions with algebraic tails rather than the Gaussian or other exponentially tailed distributions [8,9,11].

Detection and estimation algorithms designed under the Gaussian assumption exhibit various degrees of performance degradation, depending on the non-Gaussian nature of the noise. This is due to the lack of robustness of linear and quadratic types of signal processors to many types of non-Gaussian environments [5]. On the other hand, non-Gaussian noise may actually be beneficial to a system's performance if appropriately modeled and treated [19]. For this reason, there is a need to use more general and realistic non-Gaussian models and design robust signal processing techniques that take into account the heavy-tail nature of the data.

Recently, a statistical model of heavy-tailed interference, based on the theory of alpha-stable random processes, has been proposed for signal processing applications [11]. The family of alpha-stable distributions arises under very general assumptions and describes a broad class of impulsive interference. It is a parsimonious statistical–physical model defined (in its most general form) by only four parameters that can be efficiently estimated directly from the data. Furthermore, the alpha-stable model is the only one whose members obey the stability property and the Generalized Central Limit Theorem. For these and other reasons, explained in greater detail in Section 2.1, probabilists, statisticians, economists, signal processing engineers, and other scientists scattered through a variety of disciplines have embraced alpha-stable processes as the model of choice for heavy-tailed data [1].

In this paper, we address the problem of direction of arrival (DOA) estimation with an array of sensors operating in heavy-tailed noise, under the assumption of fully coherent incident signals. We reduce the measured coherence by using the spatial smoothing approach while at the same time we mitigate the effects of the heavy-tailed background noise environment by employing a signed-power nonlinearity to the array data.

The paper is organized as follows: a brief review of the alpha-stable family is undertaken in Section 2.1. In Section 2.2, we describe the adopted signal model. In Section 3, we combine spatial smoothing with fractional lower-order statistics to achieve high-resolution DOA estimation of narrow-band coherent sources in the presence of impulsive noise. In Section 5, we demonstrate the improved performance of the proposed method via simulation examples. Finally, in Section 6, we summarize the results and present avenues of future research.

2. Problem statement

2.1. Alpha stable distributions

In this section, we give a brief description of alpha-stable distributions, which are well suited for describing signals that are impulsive in nature. A review of the state of the art on stable processes from a statistical point of view is provided by a volume of papers edited by Cambanis et al. [3], and a textbook by Samorodnitsky and Taqqu [14]. An extensive review of stable processes from a signal processing point of view can be found in a monograph by Shao and Nikias [11].

The alpha-stable family is most conveniently described by its characteristic function as follows:

$$\varphi(t) = e^{\jmath at - \gamma|t|^{\alpha}[1 + \jmath\beta\,\mathrm{sign}(t)\omega(t,\alpha)]}, \tag{1}$$

where $\omega(t,\alpha)$ is $\tan(\alpha\pi/2)$ if $\alpha \neq 1$ and $(2/\pi)\log|t|$ if $\alpha = 1$, and $\mathrm{sign}(t)$ is $|t|$ if $t \neq 0$ and 0 if $t = 0$. As seen in (1), alpha-stable distributions are defined by four parameters: (i) the characteristic exponent $0 < \alpha \leqslant 2$ that determines the heaviness of the tails of the distribution, (ii) the skewness parameter $-1 \leqslant \beta \leqslant 1$ (the distribution is symmetric when $\beta = 0$), (iii) the dispersion $\gamma > 0$ that determines the spread of the density, (iv) and the location parameter $-\infty \leqslant a \leqslant \infty$, (which corresponds to the mean when $1 < \alpha \leqslant 2$ and the median when $0 < \alpha < 1$) [11].

In the following, we will consider only symmetric alpha-stable (SαS) distributions for which the skewness parameter is equal to zero. Noise modeled with a SαS density takes negative and positive values with equal probability. Fig. 1 shows plots of SαS probability density functions for several values of α. One can see that the SαS tails decay at a lower rate than the Gaussian density tails. We should note that the Gaussian distribution is an important member of the SαS family (when $\alpha = 2$). While the Gaussian density has exponential tails, the stable densities have algebraic tails. The smaller the characteristic exponent α is, the heavier the tails of the SαS density. This fact implies that random variables following SαS distributions with small characteristic exponents are *highly impulsive*. It is this heavy-tail characteristic that makes the SαS densities appropriate for

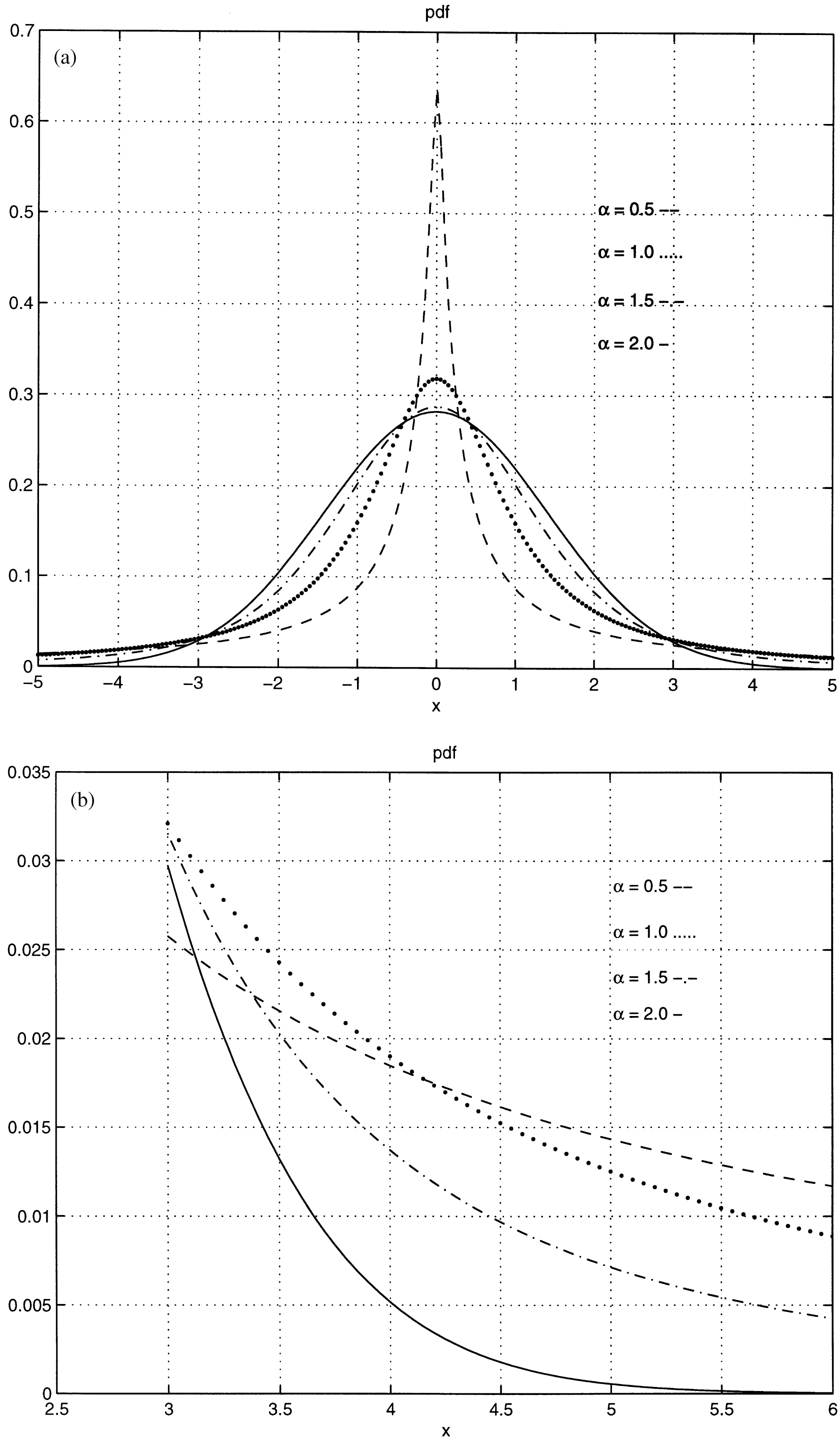


Fig. 1. (a) SαS probability density functions. (b) Tails of SαS probability density functions.

modeling noise and interference which is impulsive in nature.

The alpha-stable family shares many properties with the Gaussian distribution that further justify its role in data modeling. Namely, alpha-stable densities satisfy the stability property, which states that linear combinations of jointly stable variables are indeed stable. Moreover, they arise as limiting distributions of sums of independent, identically distributed random variables via the generalized central limit theorem.

SαS random variables possess finite pth-order moments only for $p < \alpha$, so it is clear that for all non-Gaussian SαS variables finite second- or higher-order statistics do not exist. Hence, for signal processing algorithmic development, it is necessary to define new tools based on fractional lower-order statistics (FLOS) [11]. For example, consider two random variables x, y of zero location parameter and finite pth-order moment for some possibly fractional p, $0 < p \leqslant 2$. Then, the pth-order correlation of x with y is defined as [2]

$$\langle x, y\rangle_p = E[xy^{\langle p-1\rangle}], \tag{2}$$

where $y^{\langle p-1\rangle} = |y|^{p-2}y^*$. Clearly, when $p = 2$, the above definition reduces to the usual correlation function between x and y. Fractional lower-order statistics of the form (2) have been used in the design of signal processing algorithms that are robust to the existence of heavy-tailed noise in the data [11,17].

2.2. Signal model

Fig. 2 shows a block diagram of the implementation of our proposed processor that successively restores the information sequence with techniques of spatial smoothing and impulsive interference suppression. We consider a uniformly spaced linear antenna array consisting of N elements. The distance d between two sensors is half the operating wavelength of the antenna. The array receives the contribution from multiple signal paths due to the surrounding environment, commonly present in a mobile radio communication scenario.

The transmitted waveform is a general memoryless quadrature amplitude signal with complex

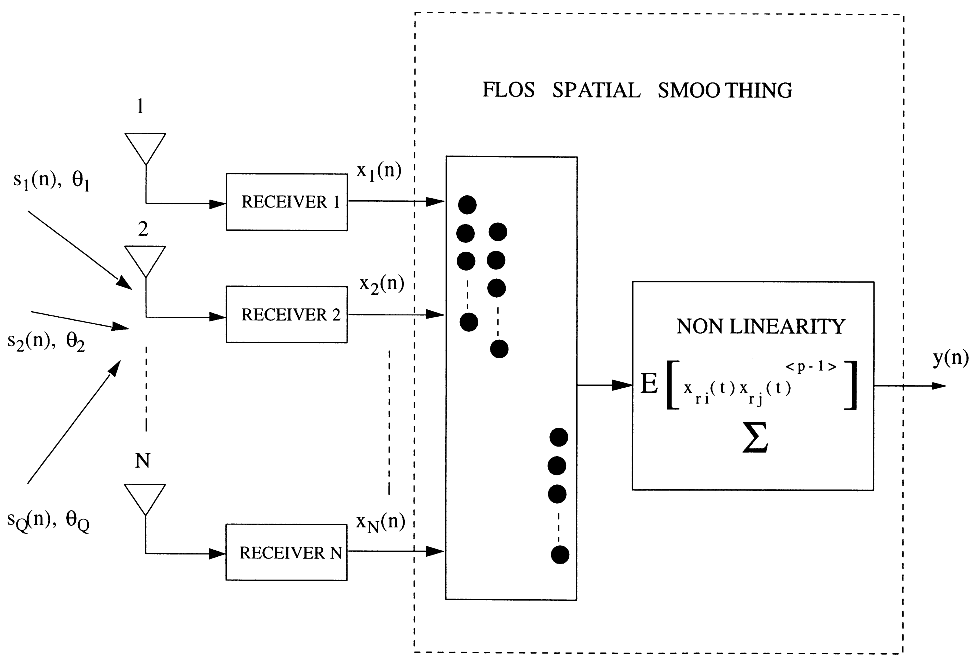


Fig. 2. Proposed receiver structure based on spatial smoothing and fractional lower-order statistics.

baseband representation

$$s(t)=\sum_k I_k u(t-kT), \tag{3}$$

where $\{I_k\}$ represents the set of symbols, modeled as independent identically distributed i.i.d. random variables, $u(t)$ is the data pulse, and T is the symbol duration. We assume the existence of Q fully correlated signals $s_k(t)$, where each signal is an attenuated version of the transmitted sequence $s(t)$ by a constant real variable, a_k. Moreover, since we consider a mobile radio scenario, a fixed phase-shift, f_k is also taken into account. This shift is due to the relative motion between the transmitter and the receiver. Under the narrowband operating assumption, the propagation delay across the array is much smaller than the reciprocal of the signal bandwidth, and it follows that, by using a complex envelop representation, the incident signal to the nth array element, $n=1,\dots,N$, can be written as [13,4]

$$x_n(t)=\sum_{k=1}^{Q} s_k(t)\mathrm{e}^{-\mathrm{j}2\pi(d/\lambda)(n-1)\sin\theta_k}$$
$$=\sum_{k=1}^{Q} a_k s(t)\mathrm{e}^{\mathrm{j}2\pi f_k}\mathrm{e}^{-\mathrm{j}2\pi(d/\lambda)(n-1)\sin\theta_k}. \tag{4}$$

Using vector notation, the array output can be expressed as [6]

$$\boldsymbol{x}(t)=\boldsymbol{V}(\boldsymbol{\vartheta})\boldsymbol{s}(t)+\boldsymbol{n}_\alpha(t), \tag{5}$$

where $\boldsymbol{x}(t)$ is the $N\times 1$ array output vector and $\boldsymbol{s}(t)=[s_1(t),\dots,s_Q(t)]^{\mathrm{T}}$ is the $Q\times 1$ signal vector received by the array, whose elements are the attenuated and phase shifted versions of the band limited information signal $s(t)$.

In the following sections, we address the problem of direction-of-arrival (DOA) estimation in the presence of heavy-tailed noise and fully coherent incoming sources. To introduce the reader to the use of fractional lower-order statistics for robust DOA estimation, we first assume an all SαS model both for the uncorrelated information signal and the noise. Then, we consider a processor that detects fully correlated communication signals in an arbitrary heavy-tailed noise background.

3. Subspace methods based on FLOS

We start our analysis by first considering the case of uncorrelated incoming signals. To address the problem of parameter estimation in the presence of heavy-tailed noise environments, the concept of the covariation matrix was introduced in [17] to characterize the correlation properties of the signal/noise field. In addition, application of eigen-decomposition methods to the covariation matrix resulted to robust DOA estimates of independent signal sources in SαS noise.

The covariation matrix, $\boldsymbol{\Gamma}_X$, of the observed vector process $\boldsymbol{x}(t)$ is defined as the matrix whose elements are the covariations $[x_i(t)\quad x_j(t)]_\alpha$ of the components of $\boldsymbol{x}(t)$, given by

$$[x_i(t)\quad x_j(t)]_\alpha=\int_S\left(\sum_{k=1}^{Q} v_i(\vartheta_k)s_k+n_{\alpha_i}\right)\times\left(\sum_{h=1}^{Q} v_j(\vartheta_h)s_h+n_{\alpha_j}\right)^{\langle\alpha-1\rangle}\mu(\mathrm{d}\boldsymbol{S}), \tag{6}$$

where $\mu(\cdot)$ is the spectral measure of the process $x_i(t)$, and $v^{\langle\alpha-1\rangle}=|v|^{(\alpha-2)}v^*$ denotes the so-called signed-power nonlinearity of the complex element v. Assuming that the incoming signals are SαS uncorrelated with each other and with the SαS noise vector, it is possible to obtain the following expression for the covariation matrix of the observation vector [17]

$$\boldsymbol{\Gamma}_X\triangleq[\boldsymbol{x}(t)\quad\boldsymbol{x}(t)]_\alpha=\boldsymbol{V}(\boldsymbol{\vartheta})\boldsymbol{\Gamma}_S\boldsymbol{V}^{\langle\alpha-1\rangle}(\boldsymbol{\vartheta})+\gamma_{n_\alpha}\boldsymbol{I}, \tag{7}$$

where $\boldsymbol{\Gamma}_S$ is the covariation matrix of the incident signals, γ_{n_α} is the noise covariation, and $\boldsymbol{I}$ is the identity matrix. Because of the Vandermonde structure of the steering matrix $\boldsymbol{V}(\boldsymbol{\vartheta})$, the (i,j)th element of $\boldsymbol{V}^{\langle\alpha-1\rangle}(\boldsymbol{\vartheta})$ is the complex conjugate of the (j,i)th element of matrix $\boldsymbol{V}(\boldsymbol{\vartheta})$: $[\boldsymbol{V}^{\langle\alpha-1\rangle}(\boldsymbol{\vartheta})]_{i,j}=|[\boldsymbol{V}(\boldsymbol{\vartheta})]_{j,i}|^{\alpha-2}[\boldsymbol{V}(\boldsymbol{\vartheta})]^*_{j,i}=[\boldsymbol{V}(\boldsymbol{\vartheta})]^*_{j,i}$. As a result the covariation matrix can be written as

$$\boldsymbol{\Gamma}_X=\boldsymbol{V}(\boldsymbol{\vartheta})\boldsymbol{\Gamma}_S\boldsymbol{V}^{\mathrm{H}}(\boldsymbol{\vartheta})+\gamma_{n_\alpha}\boldsymbol{I}. \tag{8}$$

Observing (8), it is possible to conclude that standard subspace techniques can be applied to the covariation of the observation vector to extract the bearing information. One such algorithm, named ROC-MUSIC, was introduced and studied in [17].

A point of interest, is the relation between the second-order (SO) correlation matrix and the FLOS-based covariation matrix. Note in (7) that when $\alpha = 2$, i.e., for Gaussian distributed noise, the expression for the covariation matrix becomes identical to the well-known expression for the covariance matrix, where the signal and noise covariation matrices are replaced by the signal and noise covariance matrices, respectively. Truly, by changing the parameter α in (7), we obtain a class of spatial correlation matrices that result into processors that may provide considerable flexibility for optimization purposes when operating in various degrees of heavy-tailed noise environments. Naturally, a important issue is the choice of the appropriate value of the non-linearity α, which has to be estimated from the data. The interested reader may find extensive studies on this issue in [18,7].

4. Subspace methods based on FLOS and spatial smoothing

In a realistic communication scenario, the transmitted signal $s(t)$ is not alpha stable but instead can be described by the expression given in (3). Furthermore, for the case of correlated sources, the measurement vector at the receiver may be written as

$$\boldsymbol{x}(t) = \boldsymbol{V}(\boldsymbol{\vartheta})\boldsymbol{a}s(t) + \boldsymbol{n}_\alpha(t) = \boldsymbol{V}(\boldsymbol{\vartheta})\boldsymbol{s}(t) + \boldsymbol{n}_\alpha(t), \tag{9}$$

where $\boldsymbol{a}$ is the $Q \times 1$ complex constant attenuation vector and $\boldsymbol{s}(t) = \boldsymbol{a}s(t)$.

To address signal coherence phenomena in an impulsive noise background, we introduce a *spatial smoothing* version of the FLOS-based ROC-MUSIC algorithm. The basic idea behind spatial smoothing is to divide the array of dimension $N > Q$ into uniformly overlapping sub-arrays of dimension $P > Q$, in such a way that each sub-array shares with an adjacent sub-array all but one of its sensors. Let the rth sub-array consist of the elements $(r, r+1, \dots, r+P-1)$. Then, we can write the fractional lower-order correlation function of the ith with the jth element of this sub-array as

$$\langle x_{r_i} x_{r_j} \rangle_p = E\left[\left(\sum_{k=1}^{Q} v_{r_i}(\vartheta_k)s_k + n_{\alpha_i}\right) \times \left(\sum_{h=1}^{Q} v_{r_j}(\vartheta_h)s_h + n_{\alpha_j}\right)^{\langle p-1\rangle}\right], \tag{10}$$

where $i,j = 1, \dots, P$ are the element indices and $r = 1, \dots, N-P+1$ is the sub-array index. In (10) and the following derivations, we drop the time index, t, for notational convenience. By taking the expectation operation within the first product term of (10) we have that

$$\langle x_{r_i} x_{r_j} \rangle_p = E\left[\sum_{k=1}^{Q} v_{r_i}(\vartheta_k)s_k \left(\sum_{h=1}^{Q} v_{r_j}(\vartheta_h)s_h + n_{\alpha_j}\right)^{\langle p-1\rangle}\right] + E\left[n_{\alpha_i}\left(\sum_{h=1}^{Q} v_{r_j}(\vartheta_h)s_h + n_{\alpha_j}\right)^{\langle p-1\rangle}\right]. \tag{11}$$

In the first expectation term in (11), all products with terms of the $\langle p-1\rangle$th power that contain fractional powers of only the noise component, n_{α_j}, are zero since s_k and n_{α_j} are independent and $E[s_k] = 0$. Hence, the first expectation term of (11) may be written as

$$\langle x_{r_i} x_{r_j} \rangle_p^1 = \sum_{k=1}^{Q}\sum_{h=1}^{Q} v_{r_i}(\vartheta_k) v_{r_j}(\vartheta_h)^{\langle p-1\rangle} E[s_k s_h^{\langle p-1\rangle}] + E\left[g\left(\sum_{k=1}^{Q} v_{r_i}(\vartheta_k)s_k, \sum_{h=1}^{Q} v_{r_j}(\vartheta_h)^a s_h^b, n_{\alpha_j}^c\right)\right], \tag{12}$$

where $g(\cdot)$ is a function of x, and a, b, c are three fractional values strictly less then $p-1$, with the constraint $b \neq 0$, $c \neq 1$.

A similar argument holds for the second expectation term in (11). In this case, all products with terms of the $\langle p-1\rangle$th power which contain fractional powers of the signal s_h are zero since s_h and n_{α_i} are independent and $E[n_{\alpha_i}] = 0$. Hence, the second expectation contributes terms of the form

$$\langle x_{r_i} x_{r_j} \rangle_p^2 = E[n_{\alpha_i} n_{\alpha_j}^{\langle p-1\rangle}] = \gamma_{n_\alpha} \delta_{i,j}, \tag{13}$$

since the noise components are independent and with zero location parameter. Using (12) and (13), we can express (11) as

$$\langle x_{r_i} x_{r_j} \rangle_p = \langle x_{r_i} x_{r_j} \rangle_p^1 + \langle x_{r_i} x_{r_j} \rangle_p^2$$
$$= \sum_{k,h=1}^{Q} v_{r_i}(\vartheta_k) v_{r_j}(\vartheta_h)^{\langle p-1\rangle} E[s_k s_h^{\langle p-1\rangle}]$$
$$+ \gamma_{n_\alpha} \delta_{i,j} + \Phi_{i,j}, \tag{14}$$

where

$$\Phi_{i,j} = E\left[g\left(\sum_{k=1}^{Q} v_{r_i}(\vartheta_k) s_k, \sum_{h=1}^{Q} v_{r_j}(\vartheta_h)^{\langle a\rangle} s_h^b, n_{\alpha_j}^c \right)\right], \tag{15}$$

we will call the *corruption factors*. The terms in (15) are finite quantities since all the fractional powers a, b, c, d are strictly less than $p-1$ and $n_{\alpha_i}, n_{\alpha_j}$ are SαS noise components that have finite moments of order $p < \alpha$. The effect of the corruption factor is that of a correlated noise field which will adversely affect the performance of the method. However, as we demonstrate in the simulation section, the advantage of using the FLOS formulation in the presence of heavy-tailed environment noise outweighs the negative effect of the induced corruption factors.

Eq. (14) can be written in a compact form so that the FLOS matrix of the rth sub-array is given by

$$\boldsymbol{\Gamma}_{X_P}^{(r)} \triangleq \boldsymbol{V}_P(\boldsymbol{\vartheta}) \boldsymbol{D}^{r-1} \boldsymbol{\Gamma}_S \boldsymbol{D}^{r-1^{\langle p-1\rangle}} \boldsymbol{V}_P^{\langle p-1\rangle}(\boldsymbol{\vartheta})$$
$$+ \gamma_{n_\alpha} \boldsymbol{I}_P + \boldsymbol{\Phi}_P^{(r)}, \tag{16}$$

where $\boldsymbol{V}_P(\boldsymbol{\vartheta})$ is the set of steering vectors for a sub-array of length P and $\boldsymbol{D}$ is a diagonal matrix whose lth element is equal to $e^{-j2\pi(d/\lambda)\sin(\vartheta_l)}$. The (i,j)th element of $\boldsymbol{V}_P^{\langle p-1\rangle}(\boldsymbol{\vartheta})$ is the (j,i)th element of $\boldsymbol{V}_P(\boldsymbol{\vartheta})$ to the signed power of $^{\langle p-1\rangle}$. Since the elements of $\boldsymbol{V}_P(\boldsymbol{\vartheta})$ and $\boldsymbol{D}^{r-1}$ have unit magnitude, it follows that

$$\boldsymbol{\Gamma}_{X_P}^{(r)} = \boldsymbol{V}_P(\boldsymbol{\vartheta}) \boldsymbol{D}^{r-1} \boldsymbol{\Gamma}_S \boldsymbol{D}^{r-1^{\mathrm{H}}} \boldsymbol{V}_P(\boldsymbol{\vartheta})^{\mathrm{H}} + \gamma_{n_\alpha} \boldsymbol{I}_P + \boldsymbol{\Phi}_P^{(r)}. \tag{17}$$

In practice, the FLOS matrix is evaluated as the average of the sub-array FLOS matrices:

$$\bar{\boldsymbol{\Gamma}}_X = \frac{1}{N-P+1} \sum_{r=1}^{N-P+1} \boldsymbol{\Gamma}_{X_P}^{(r)}. \tag{18}$$

Using (18) along with (17), we can write the FLOS matrix as

$$\bar{\boldsymbol{\Gamma}}_X = \boldsymbol{V}_P(\boldsymbol{\vartheta}) \bar{\boldsymbol{\Gamma}}_S \boldsymbol{V}_P(\boldsymbol{\vartheta})^{\mathrm{H}} + \bar{\boldsymbol{\Gamma}}_{n_\alpha}$$
$$+ \frac{1}{N-P+1} \sum_{r=1}^{N-P+1} \boldsymbol{\Phi}_P^{(r)}, \tag{19}$$

where

$$\bar{\boldsymbol{\Gamma}}_S \triangleq \frac{1}{N-P+1} \sum_{r=1}^{N-P+1} \boldsymbol{D}^{r-1} \boldsymbol{\Gamma}_S \boldsymbol{D}^{r-1^{\mathrm{H}}}, \tag{20}$$

$$\bar{\boldsymbol{\Gamma}}_{n_\alpha} \triangleq \frac{1}{N-P+1} \sum_{r=1}^{N-P+1} \gamma_{n_\alpha} \boldsymbol{I}_P. \tag{21}$$

As a result of the spatial smoothing operation, the matrix in (19) is nonsingular [15]. Hence, we can apply an eigen decomposition of $\bar{\boldsymbol{\Gamma}}_X$ to achieve robust DOA estimation in a fading environment with heavy-tailed noise. We call the resulting method the *ROC-MUSIC smoothing* algorithm.

5. Simulation results

In this section, we show comparative results on the resolution capability of ROC-MUSIC smoothing versus ROC-MUSIC, MUSIC and MUSIC smoothing in several scenarios of signal and noise environments. In our simulations, we used a linear array with 12 sensors spaced a half-wavelength apart. The spatial smoothing techniques employed overlapped sub-arrays each containing six sensors. A total of $M = 500$ snapshots for every experiment is available to the algorithms.

Four multipath signals impinge on the array from directions $\vartheta = [30°, -40°, 60°, -15°]$. The relative power attenuation factors $|a_k|^2$ for the four paths were chosen to be $\{-3, 0, -2, -6$ dB$\}$. The constant phase shifts of a_k were taken uniformly distributed over $(-\pi, \pi]$. The transmitted waveform is a QPSK signal, filtered with a squared root raised cosine. The received signal is filtered with the same matched squared root raised cosine filter.

The signals are received by the array in the presence of additive alpha-stable noise. Since the alpha-stable family for $\alpha < 2$ defines processes with infinite variance, we define the *fractional effective* SNR (FESNR) measure as the ratio of the fractional signal power over the fractional noise power:

$$\mathrm{FESNR} = 10\log\left(\frac{\sum_{t=1}^{M}|s(t)|^{\alpha}}{\sum_{t=1}^{M}|n_{\alpha}(t)|^{\alpha}}\right). \tag{22}$$

According to this choice of an SNR metric, the SαS noise samples are power scaled by the corresponding characteristic exponent α before contributing to the SNR calculation. Note that, for Gaussian noise ($\alpha = 2$), expression (22) coincides with the usual SNR measure.

The proposed ROC MUSIC smoothing algorithm has to estimate the FLOS matrix in (19) from the sensor measurements. We used the approach followed in [7,17], by calculating the instantaneous value of the expectation $E[\boldsymbol{x}^{\langle p_1\rangle}\cdot\boldsymbol{x}^{\langle p_2\rangle^{\mathrm{H}}}]$, where the two parameters p_1, p_2 have to follow the constraint $p_1 < \alpha/2$ and $p_2 < \alpha/2$. In real life, the statistics of the additive noise data are unknown. This gives rise to the need for fast, simple, and efficient estimators of the alpha-stable parameters (especially, the characteristic exponent, α) from real data. Several such estimators compromising optimality for the sake of computational efficiency, have been proposed in the past and they are described in [18] and references therein.

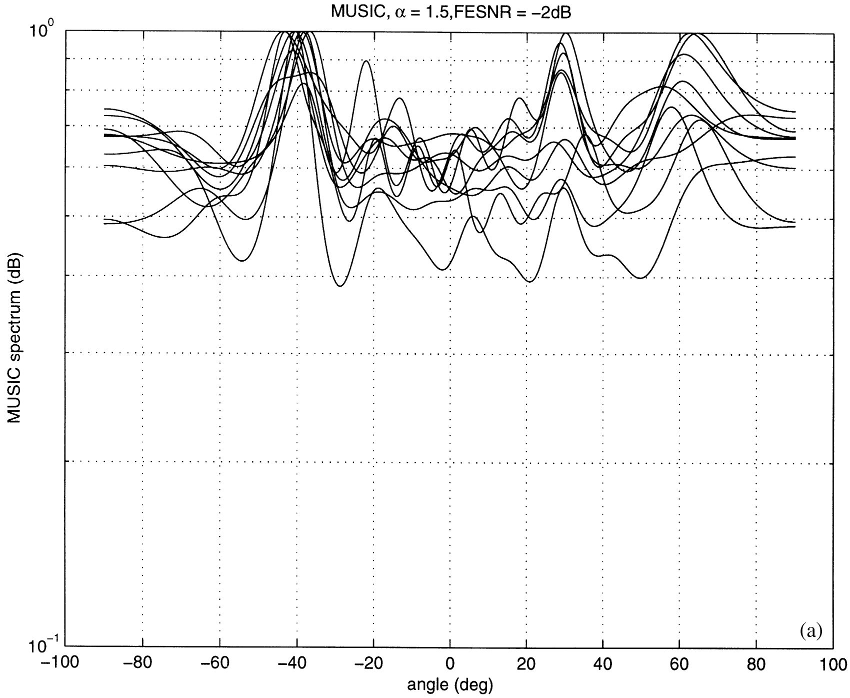
Figs. 3–5 show results on the resolution capabilities of the methods for various values of the characteristic exponent α of the noise. First, by comparing Figs. 3(a),(b) to (c),(d), we see the improvement in coherent source DOA estimation achieved with spatial smoothing. Of course, because of the presence of heavy-tailed noise, the improvement is mostly apparent for the case of the introduced FLOS-based method. Indeed, comparing Figs. 3(c) and (d) we see that, for a fairly impulsive noise environment ($\alpha = 1.5$, $\mathrm{FESNR} = -2$ dB), the SO-based MUSIC smoothing method exhibits low-resolution as it cannot clearly identify the multipath signals. On the other hand, the proposed ROC-MUSIC smoothing processor places clearly distinguished peaks to all four DOAs (cf. Fig. 3(d)).

For more closely spaced sources, which are located at $\vartheta = [15^\circ, -40^\circ, 40^\circ, -30^\circ]$ Fig. 4 demonstrates that the ROC-MUSIC Smoothing algorithm is still able to resolve the two adjacent paths from -40° and -30°, something that the MUSIC smoothing method is not able to achieve at this particular FESNR.

Even when the statistical behavior of the noise is close to Gaussian ($\alpha = 1.85$), the ROC-MUSIC smoothing method still performs better than the MUSIC Smoothing (cf. Figs. 5(a) and (b)). Finally, when operating in a Gaussian noise environment with finite second-order statistics, both methods exhibit good performance (cf. Figs. 5(c) and (d)).

The property of the introduced FLOS-based processor to operate robustly in various interference backgrounds, which was demonstrated in Figs. 3–5, was also quantified by performing Monte Carlo runs to measure the associated root mean-square error (RMSE) of the DOA estimates. For this part of the simulations, we used two signals coming from directions $[30^\circ - 40^\circ]$ and an eight-element array with a sub-array dimension equal to 4. Table 1 reports the results of both SO- and FLOS-based methods when estimating the source at 30°, as a function of the additive noise characteristic exponent and the FESNR. Clearly, the table demonstrates that the DOA estimates associated with the FLOS-based processor have significantly smaller RMSE for all non-Gaussian noise backgrounds and all FESNR values. For the case of Gaussian noise, the performance of the two methods is comparable, with the SO-based processor having slightly better performance, as expected.

In conclusion, the proposed spatial smoothing method based on the fractional lower-order statistics is shown to be able to resolve coherent sources in various types of noise environments. The FLOS-based method exhibits increasing performance improvement over the conventional second-order based techniques for heavier noise backgrounds.



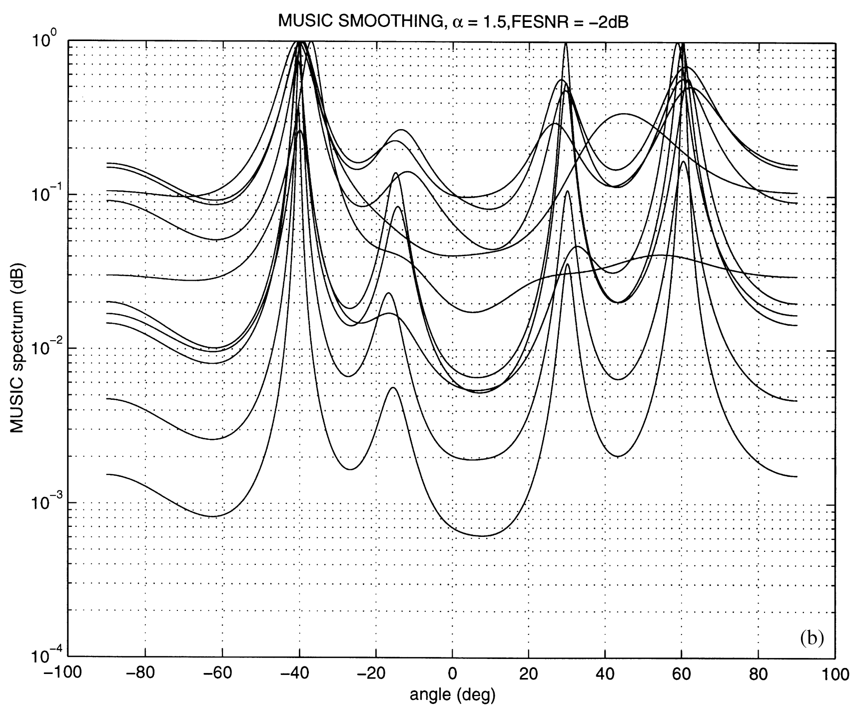
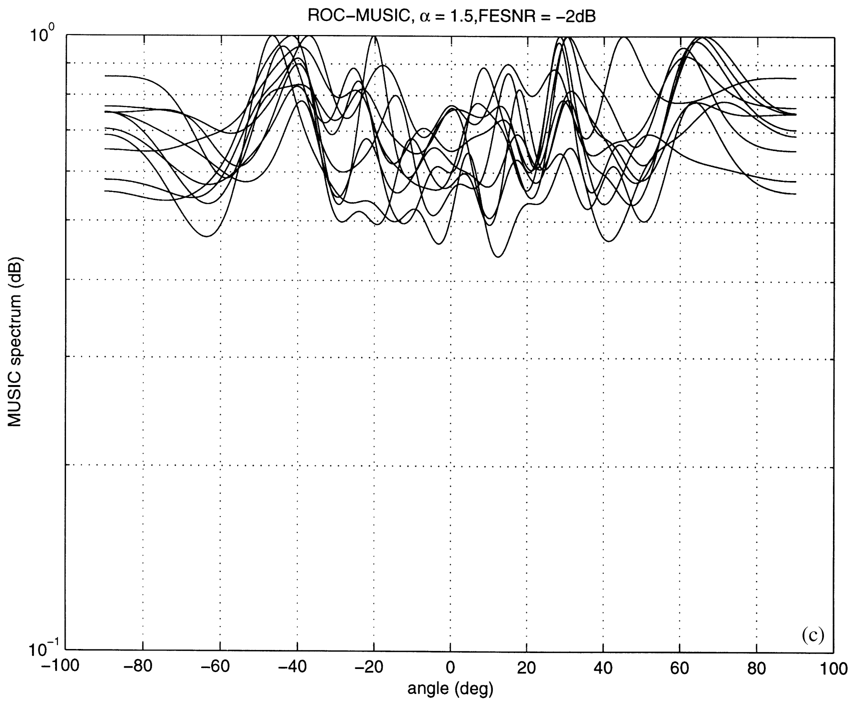


Fig. 3. (a) Spatial spectral estimates in additive alpha-stable noise with $\alpha = 1.5$ and FESNR $= -2$ dB: (a) MUSIC; (b) MUSIC smoothing; (c) ROC-MUSIC; and (d) ROC-MUSIC smoothing.



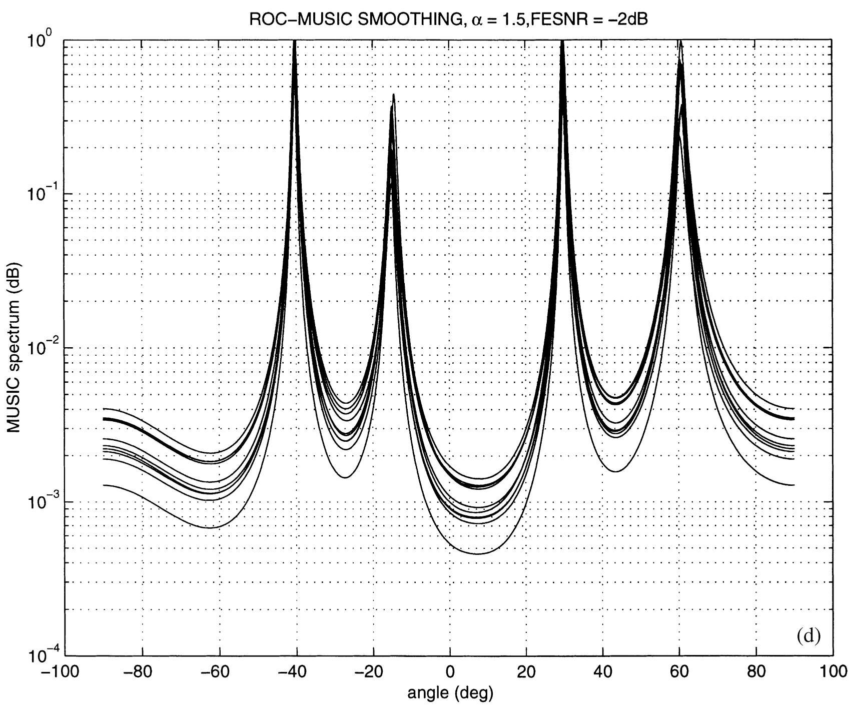


Fig. 3. (continued).

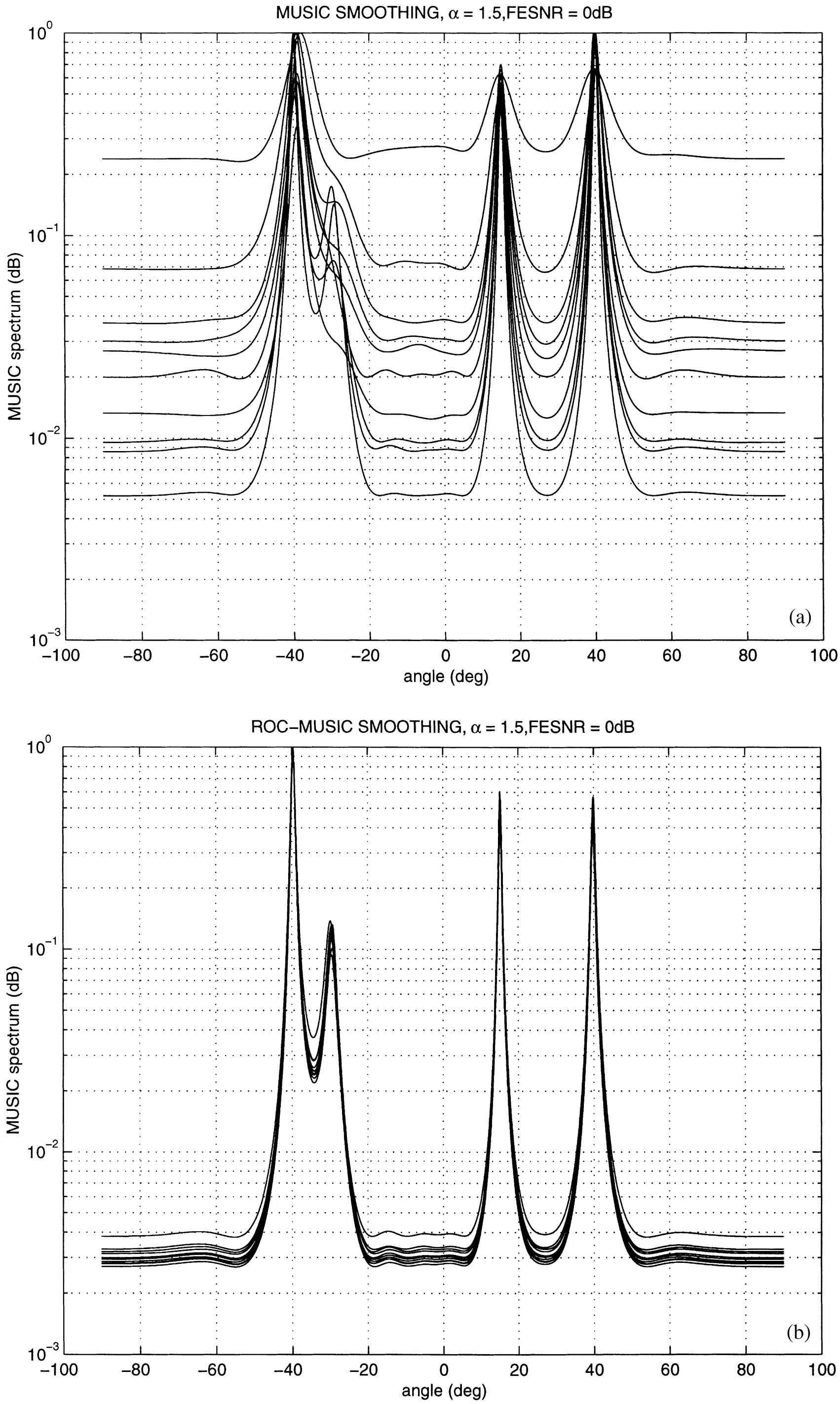
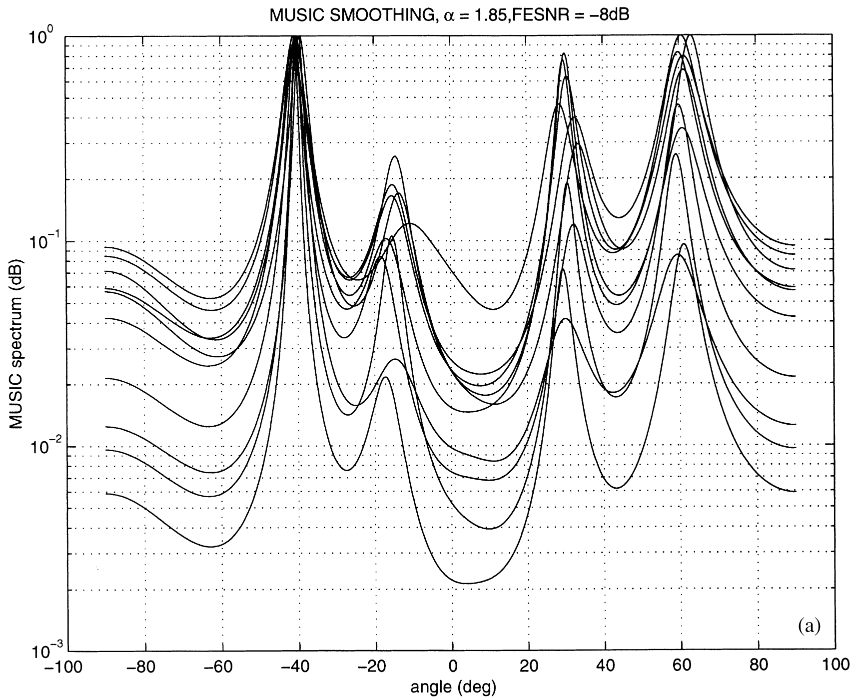


Fig. 4. Spatial spectral estimates in additive alpha-stable noise with $\alpha = 1.5$ and FESNR $= 0$ dB. (a) MUSIC smoothing; (b) ROC-MUSIC smoothing.



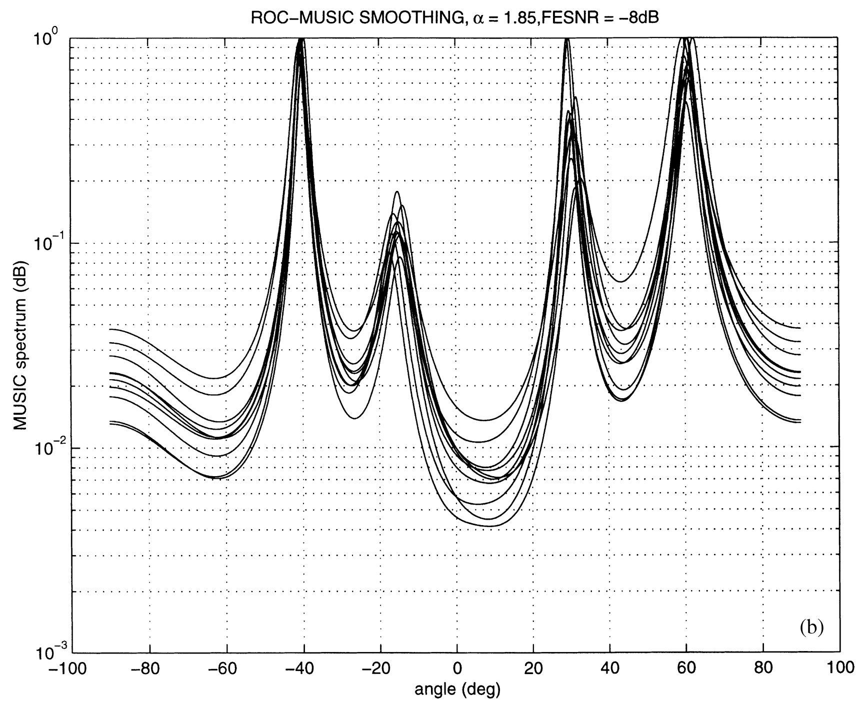
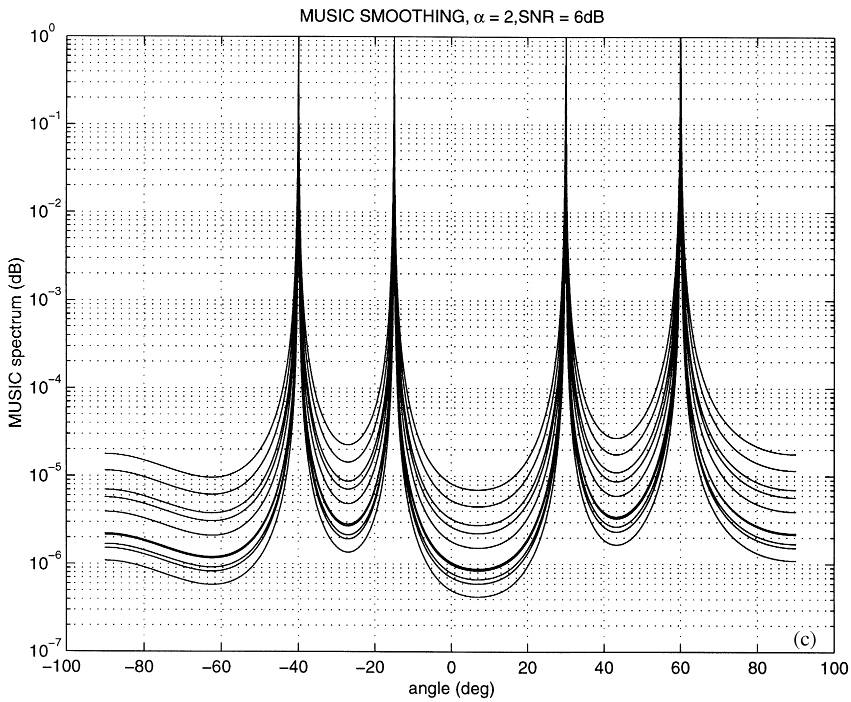


Fig. 5. (a) Music smoothing and ROC-MUSIC smoothing for $\alpha = 1.85$ and FESNR $= -8$ dB (a)–(b), and for $\alpha = 2$ and SNR $= 6$ dB (c)–(d).



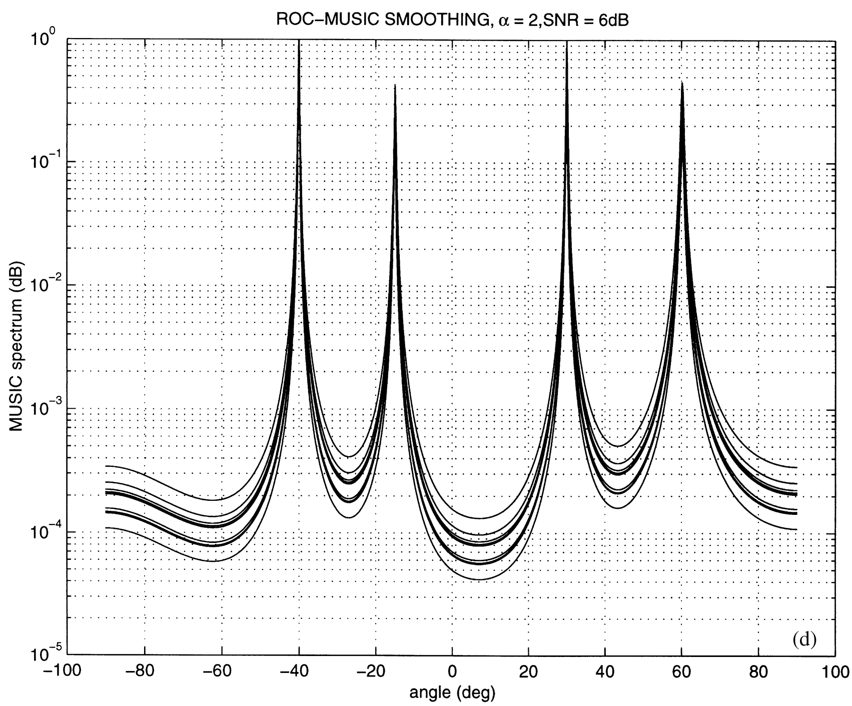


Fig. 5. (continued).

Table 1
RMSE of DOA estimation as a function of the noise characteristic exponent and the FESNR (MS: MUSIC Smoothing, R-MS: ROC-MUSIC Smoothing)

	$\alpha = 1.5$		$\alpha = 1.85$			$\alpha = 2$	
dB/	R-MS	MS	R-MS	MS	dB/	R-MS	MS
− 12	1.8	23.8	5.6	13.11	4	0.14	0.1
− 8	0.76	15.72	1.1	2.01	6	0.12	0.07
− 4	0.46	2.5	0.4	0.7	8	0.09	0.05
0	0.26	1.06	0.22	0.25	10	0.08	0.04
4	0.2	0.42	0.09	0.08	12	0.05	0.02

6. Conclusion

Conventional high-resolution eigen-decomposition techniques perform poorly in coherent receiving environments. Spatial smoothing methods proposed in the past address the signal coherence problem but fail to operate reliably in a heavy-tailed non-Gaussian noise. The method proposed in this paper is able to achieve high resolution performance when operating in both multipath and impulsive noise environments. The new algorithm is based on spatial smoothing of the FLOS matrix of an antenna array and it is shown to exhibit better resolution performance in a wide range of noise environments without considerably increasing the complexity of the system. Several limitations of the proposed method need to be addressed in the future, including unequally spaced non-linear arrays, and correlated additive noise structures.

References

[1] R. Adler, R. Feldman, M.S. Taqqu, A Practical Guide to Heavy Tails: Statistical Techniques and Applications, Birkhauser, Boston, MA, 1998.

[2] S. Cambanis, G. Miller, Linear problems in *p*th order and stable processes, SIAM J. Appl. Math. 41 (1981) 43–69.

[3] S. Cambanis, G. Samorodnitsky, M.S. Taqqu (Eds.), Stable Processes and Related Topics, Birkhauser, Boston, MA, 1991.

[4] R.T. Compton Jr., Adaptive antennas, Concepts and Performance, Prentice-Hall, Englewood Cliffs, NJ, 1988.

[5] S.A. Kassam, H.V. Poor, Robust techniques for signal processing: a survey, Proc. IEEE 73 (March 1985) 433–481.

[6] J.R. Johnson, J.E. Evans, D.F. Sun, High resolution angular spectrum estimation techniques for terrain scattering analysis and angle of arrival estimation, Proceedings of the First ASSP Workshop Spectral Estimation, 1981.

[7] X. Ma, C.L. Nikias, Joint estimation of time delay and frequency delay in impulsive noise using fractional lower-order statistics, IEEE Trans. Signal Process. 44 (November 1996) 2669–2687.

[8] D. Middleton, Statistical-physical models of urban radio noise environment – Part i: foundations, IEEE Trans. Electron Comput. 14 (1972) 38–56.

[9] U. Mitra, H.V. Poor, Detection of spread-spectrum signals in multi-user environment, in: Proceedings of IEEE ICASSP, Detroit, Michigan, May 9–12, 1995, pp. 1844–1847.

[10] C.L. Nikias, I.S. Reed, V. Prasanna, Multidisciplinary research on advanced high-speed, adaptive signal processing for radar sensor, Technical Report, University of Southern California, June 1996.

[11] C.L. Nikias, M. Shao, Signal Processing with Alpha-Stable Distributions and Applications, Wiley, New York, 1995.

[12] S.U. Pillai, B.H. Kwon, Forward/backward spatial smoothing techniques for coherent signal identification, IEEE Trans. Acoust. Speech Signal Process. 37 (January 1989) 8–14.

[13] J.G. Proakis, Digital Communications, McGraw-Hill, New York, 1995.

[14] G. Samorodnitsky, M.S. Taqqu, Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance, Chapman & Hall, New York, 1994.

[15] T.-J. Shan, M. Wax, T. Kailath, On spatial smoothing for direction-of-arrival estimation of coherent signals, IEEE Trans. Acoust. Speech, Signal Process. 33 (August 1985) 806–811.

[16] B.W. Stuck, B. Kleiner, A statistical analysis of telephone noise, Technical Report, Bell. Syst. Tech. J. June 1974.

[17] P. Tsakalides, C.L. Nikias, The robust covariation-based MUSIC (ROC-MUSIC) algorithm for bearing estimation

in impulsive noise environments, IEEE Trans. Signal Process. 44 (July 1996) 1623–1633.

[18] P. Tsakalides, R. Raspanti, C.L. Nikias, Angle/doppler estimation in heavy-tailed clutter backgrounds, IEEE Trans. Aerosp. Electron. System 35 (April 1999) 419–436.

[19] X. Wang, H.V. Poor, Robust multiuser detection in non-gaussian channels, IEEE Trans. Signal Process. 47 (February 1999) 289–305.

[20] M. Wax, T.-J. Shan, T. Kailath, Spatio-temporal spectral analysis by eigenstructure methods, IEEE Trans. Acoust. Speech, Signal Process. 32 (August 1984) 817–827.