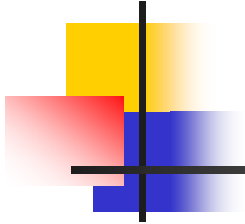




Hidden Messages in Heavy Tails: DCT-Domain Watermark Detection Using Alpha-Stable Models

- Alexia Briassouli, Panos Tsakalides, Athanasios Stouraitis
- Master's Thesis in *Signal and Image Processing Systems: Theory, Implementations, Applications, University of Patras, Greece, 2000*



- Importance of watermarking
- Watermark generation and embedding
- “Blind” watermark detection
- Statistical modeling of DCT coefficients
- Comparison of nearly optimal detectors:
generalized Gaussian, Cauchy
- Experimental Results
- Conclusions

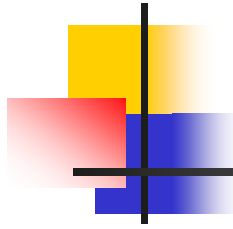


Digital information: easily reproduced and distributed without loss of fidelity

- *Watermarking:*
Embedding of a digital signal specifying legitimate owner/receiver of data *directly in the data*
- Part of a general system, not a complete solution

PROPERTIES

- Secret key known only to legal owner
- Imperceptibility, Robustness
- *Kerckhoff's Law* : The system is secure even if an attacker knows the principles and methods of watermark embedment but not the secret key

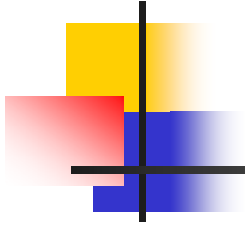


■ **WATERMARK DETECTION/EXTRACTION**

- Availability of original data
- Detection = Binary hypothesis test for watermark existence
- Extraction = extraction of message as well (*fingerprinting*)
- Accurate statistical model \Rightarrow efficient watermark detection

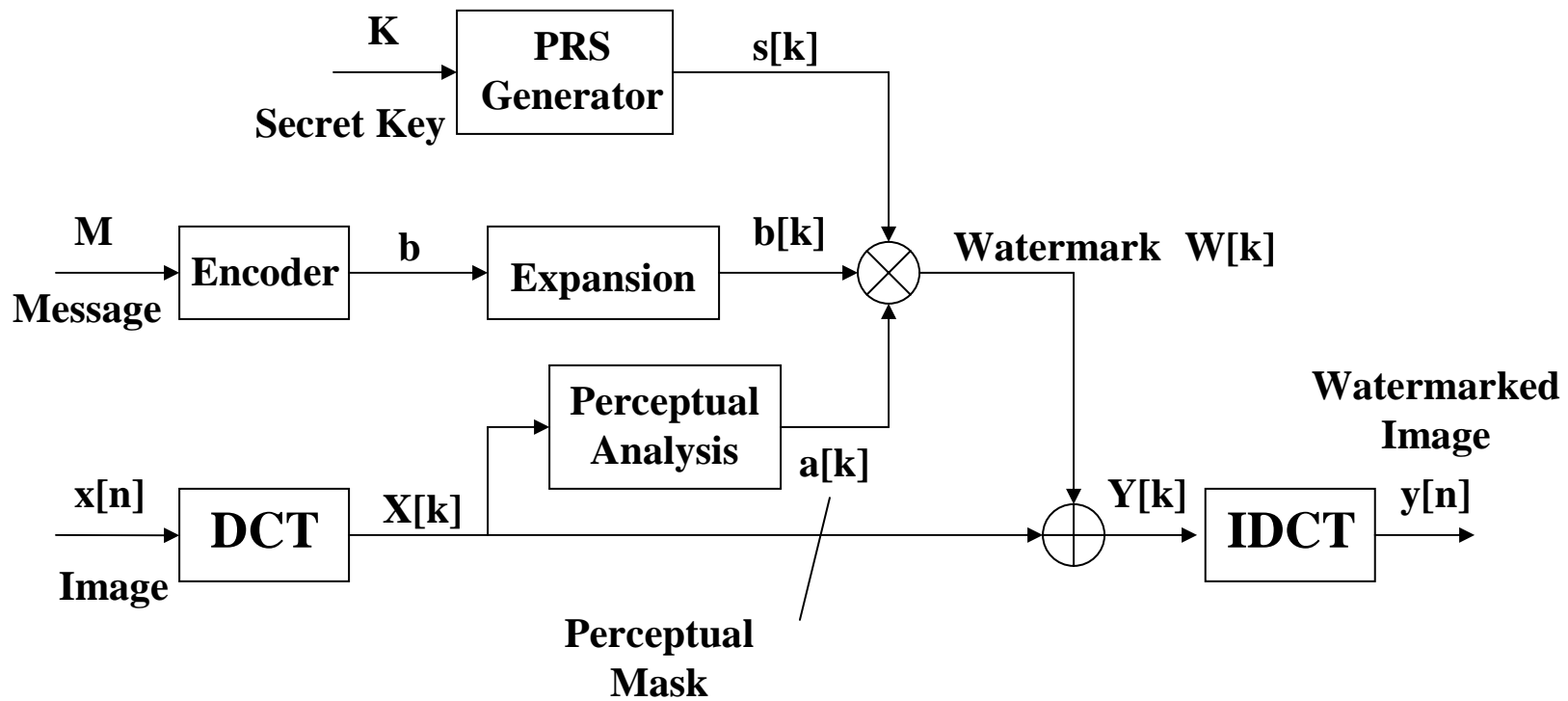
■ **SPREAD SPECTRUM WATERMARKING**

- DCT image values $x[\mathbf{k}]$ at pixels $\mathbf{k} = (i,j)$: Noise
- Anti-jamming properties of Spread Spectrum make it robust to some attacks
- Message M encoded to $N - D$ vector \mathbf{b} that is "spread" over the image (expansion process)



- Secret key K = random generator seed for the pseudorandom sequence $s[\mathbf{k}]$
- Watermark strength determined by perceptual mask for DCT data $a[\mathbf{k}]$ (*Ahumada et. al., Watson*)
- Watermark detection: one bit ($b=1, N=1$) is *repeated* over pixels – increases *robustness*
- Mask $a[\mathbf{k}]$ multiplied pixelwise by:
 - pseudorandom sequence $s[\mathbf{k}]$
 - bits $b[\mathbf{k}]$ ($b=1$ for watermark detection)to give watermark $W[\mathbf{k}] = a[\mathbf{k}]s[\mathbf{k}]b[\mathbf{k}]$

WATERMARK EMBEDDING





MODELING OF DCT COEFFICIENTS

- LAPLACIAN: tails decay exponentially with x

$$f_X(x) = \frac{b}{2} \exp(-b|x - \alpha|) \quad \begin{cases} \text{mean}(x) = a \\ \text{var}(x) = 2/b^2 \end{cases}$$

- GENERALIZED GAUSSIAN:

$$f_X(x) = A \exp(-\beta|x|^c) \quad \beta = \frac{1}{\sigma} \left(\frac{\Gamma(3/c)}{\Gamma(1/c)} \right)^{1/2}, A = \frac{\beta c}{2\Gamma(1/c)}$$

- $c = 1$ Laplacian, $c = 2$ Gauss
- c can be estimated theoretically for each DCT coefficient
- In practice $c = 0.5$ is satisfactory (*Hernandez et. al.*)
- **Cannot adequately model samples in the tails with high magnitudes**



ALPHA- STABLE MODELS :

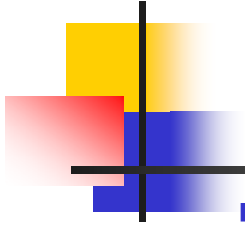
- Often used to describe heavy-tailed data
- Defined in closed form only by their characteristic function

$$\varphi_X(\omega) = E[e^{j\omega X}]$$

$$\varphi(\omega) = \exp\left(-j\delta\omega - \gamma|\omega|^\alpha [1 + j\beta \text{sign}(\omega)\phi(\omega, \alpha)]\right)$$

$$\phi(t, \alpha) = \begin{cases} \tan \frac{\alpha\pi}{2} & \alpha \neq 1 \\ \frac{2}{\pi} \log |t| & \alpha = 1 \end{cases}$$

- Parameters are estimated from the data (Max Likelihood Estimates)



- Parameters:

- *location* δ ($-\infty < \delta < \infty$) :
mean for $1 < \alpha \leq 2$, median for $0 < \alpha \leq 1$
- *scale* γ ($\gamma > 0$) : equivalent to variance
- *skewness* β ($-1 \leq \beta \leq 1$) : $\beta = 0$ for symmetric pdf
- *characteristic exponent* α ($0 < \alpha \leq 2$) : determines distribution shape: *small $\alpha \Rightarrow$ heavy tails*

Tail probabilities
$$P(X > x) = c_{\alpha} x^{-\alpha}$$

- Closed form expression of pdf only for:

$\alpha = 2 \Rightarrow$ GAUSSIAN

$\alpha = 1 \Rightarrow$ CAUCHY



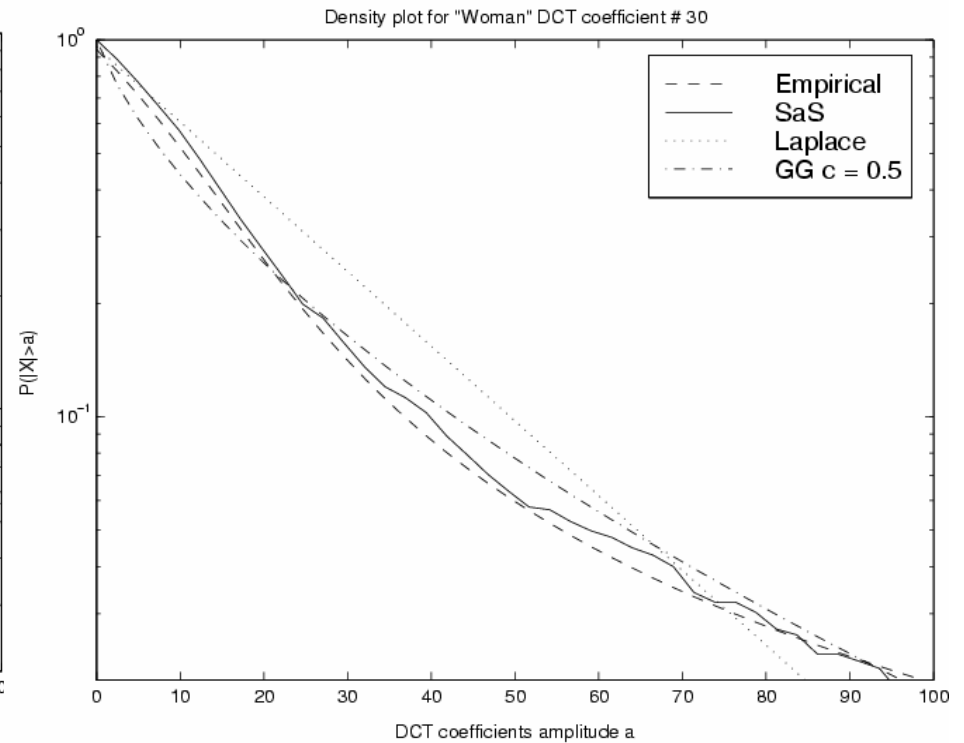
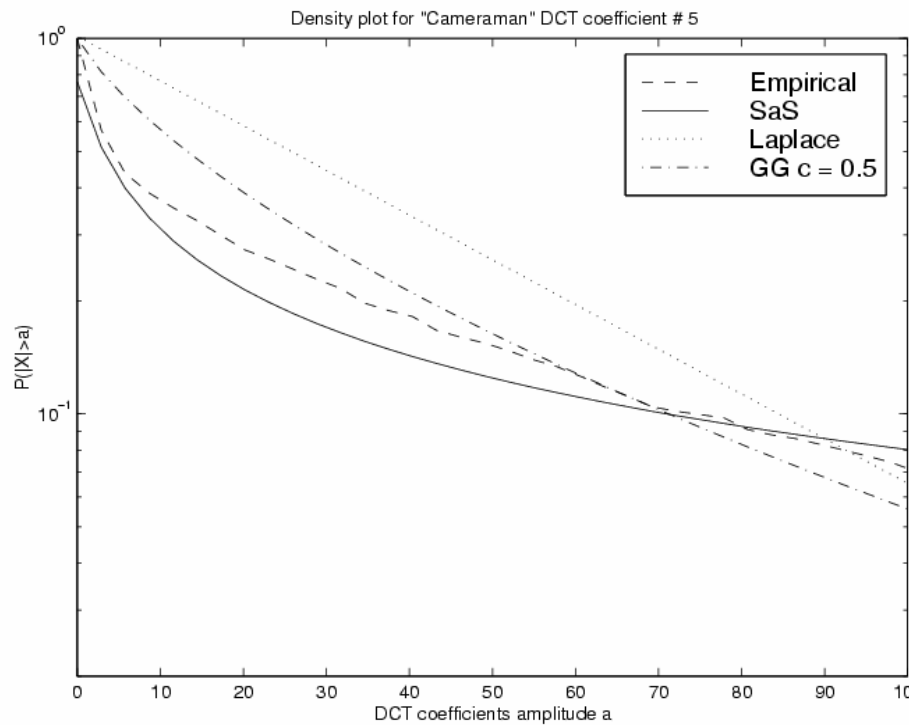
EXPERIMENTAL MODELING OF DCT COEFFICIENTS

- Model using the Amplitude Probability Density function (APD)

$$P(|X| > a)$$

- Consider Symmetric Alpha Stable (SaS, $\beta=0$) model
- Theoretical APD :
Uses ML parameter estimates from the data
- Empirical APD :
Block DCT : distribution of each coefficient over all blocks.
256x256 images : 1024x1 vector

- Cameraman DCT #5: SaS gave closest fit to empirical APD
- Woman DCT #30: SaS and gen. Gaussian give very good fit to empirical APD





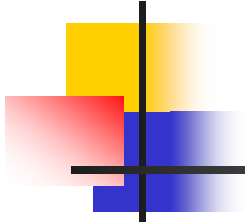
WATERMARK DETECTION

- Binary hypothesis test :

$$H_1 : Y[k] = X[k] + W[k]$$

$$H_0 : Y[k] = X[k]$$

- Log-likelihood ratio test : $l(Y) = \ln \left(\frac{f(Y | H_1)}{f(Y | H_0)} \right) > \eta$
- Watermark = signal, image = noise
- Low, mid frequency DCT coefficients
- Original and watermarked images have *similar statistical properties*



- Neyman – Pearson Testing
Receiver Operating Characteristics :

$$P_{fa} = Q\left(\frac{t - m_0}{\sigma_1}\right), P_{det} = Q\left(\frac{t - m_1}{\sigma_1}\right) \quad Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt$$

- Mean and variance of $l(Y)$: $m_0 = -m_1, \sigma_0 = \sigma_1$
- Threshold : $t = m_0 + \sigma_1 Q^{-1}(P_{fa})$

- Generalized Gaussian log – likelihood ratio:

$$l(Y) = \sum_k \beta[k]^{c[k]} \left(|Y[k]|^{c[k]} - |Y[k] - a[k]s[k]|^{c[k]} \right)$$

- Experimental verification of $l(Y)$ mean, variance

GENERALIZED GAUSSIAN (c = 0.5)				
IMAGE	m_0	m_1	σ_0^2	σ_1^2
Lena (th.)	-3.66	3.66	16.55	16.55
Lena (exp.)	-3.66	3.65	17.40	17.40
Woman (th.)	-4.71	4.71	12.41	12.41
Wom. (exp.)	-4.59	4.82	11.81	11.81



Experimental verification of $I(Y)$ mean, variance

CAUCHY				
IMAGE	m_0	m_1	σ_0^2	σ_1^2
Lena (th.)	-2.96	2.90	3.31	3.31
Lena (exp.)	-2.86	2.95	3.16	3.16
Wom. (th.)	-10.05	9.75	58.07	58.07
Wom. (exp.)	10.36	9.71	58.08	58.08

- The mean and variance of the log – likelihood ratio determine the Signal to Noise Ratio SNR:

$$SNR = m_1^2 / \sigma_1^2$$

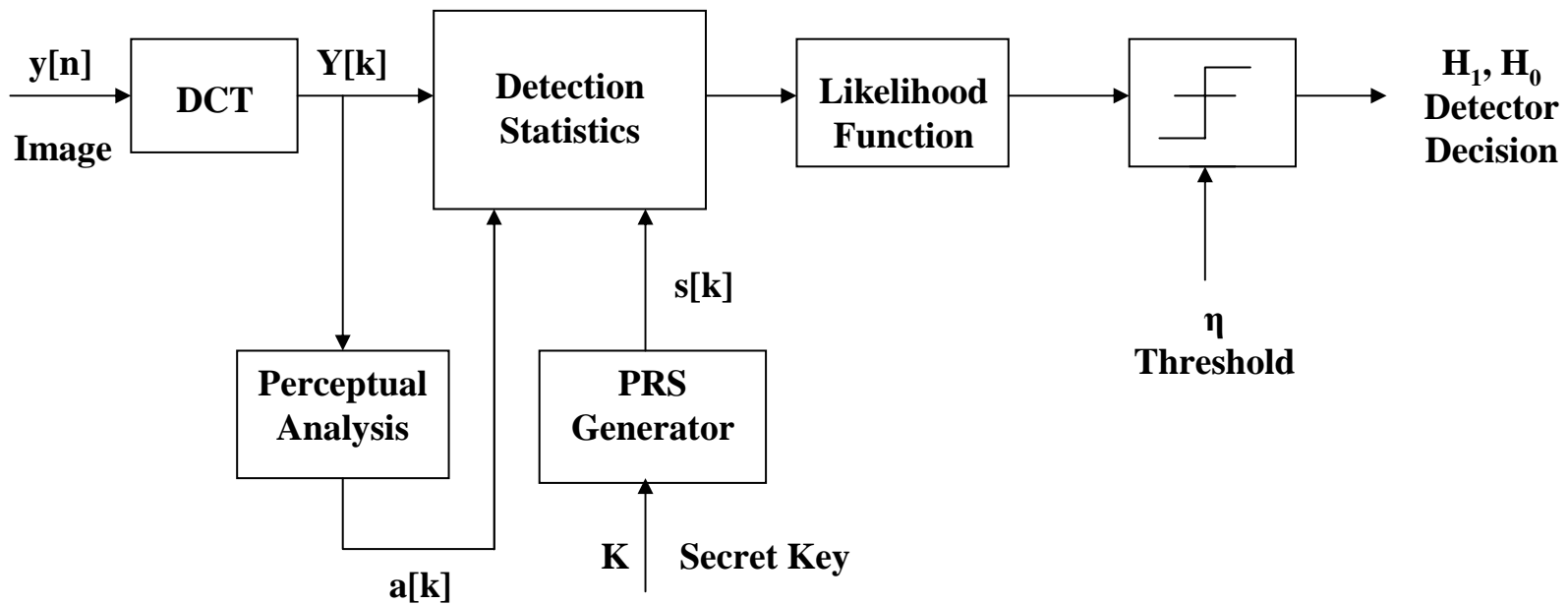
- Detection performance is determined by the ROC curves that depend only on the SNR:

$$P_{\text{det}} = Q\left(Q^{-1}(P_{fa}) - 2\sqrt{SNR}\right)$$

- High SNR gives better detection performance :

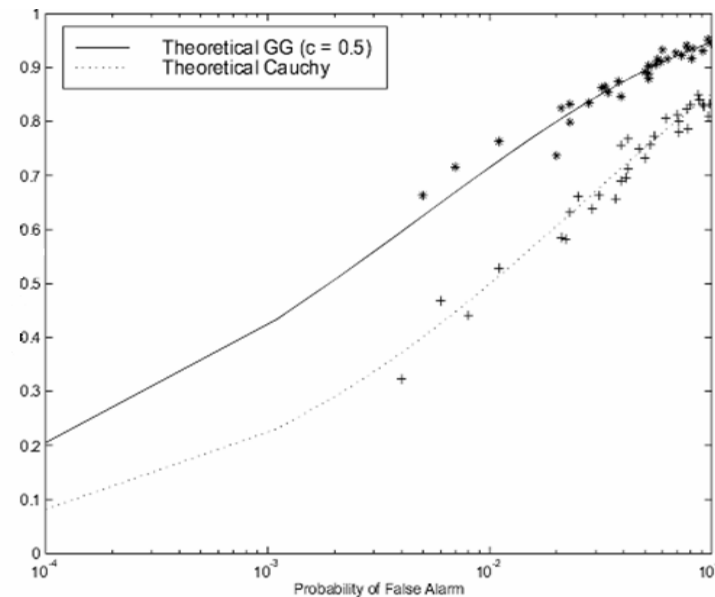
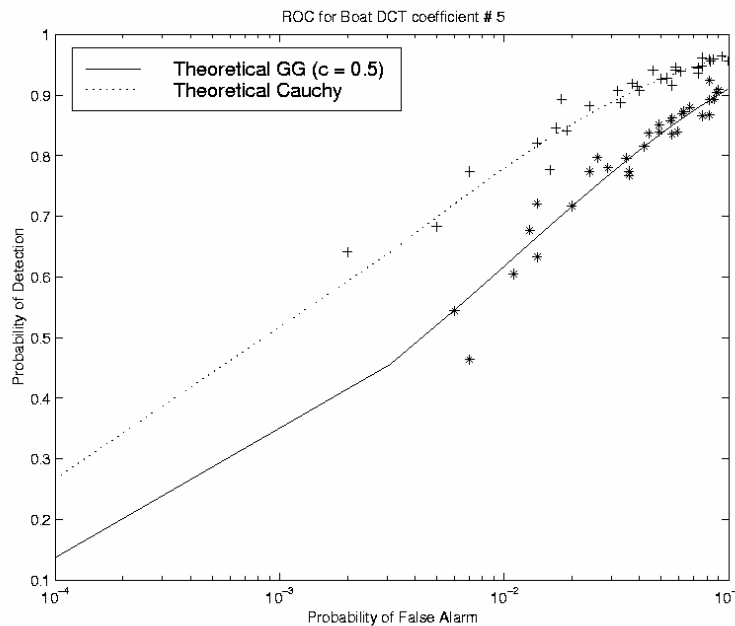
IMAGE coefficient	SNR (dB)	
	Cauchy	G.G. (c=0.5)
Boat (#10)	1.31	5.00
Cam. (#5)	5.60	3.75
Lena (#5)	4.21	2.54
Woman (#30)	3.96	2.41

WATERMARK DETECTION

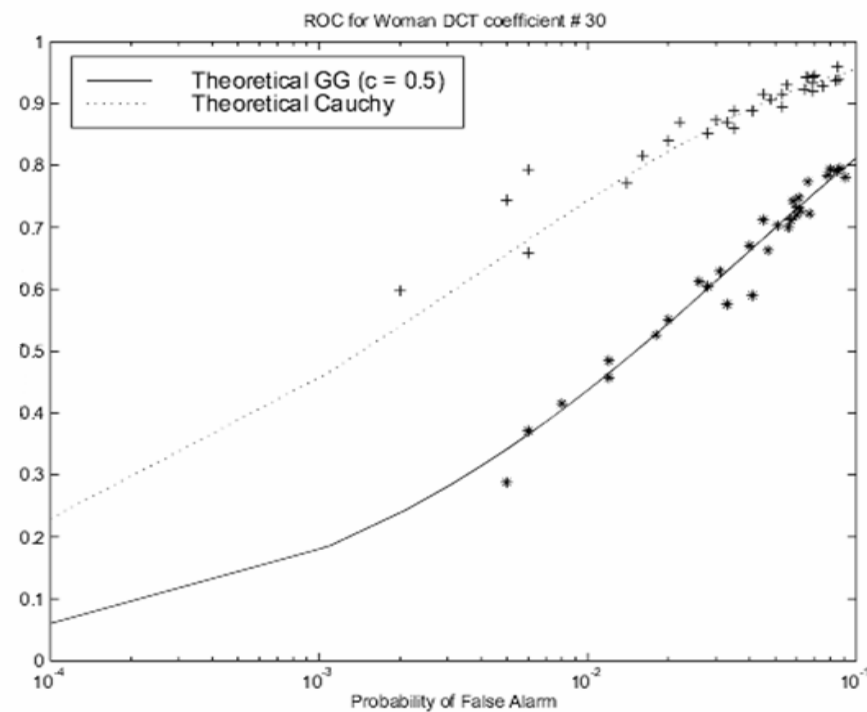
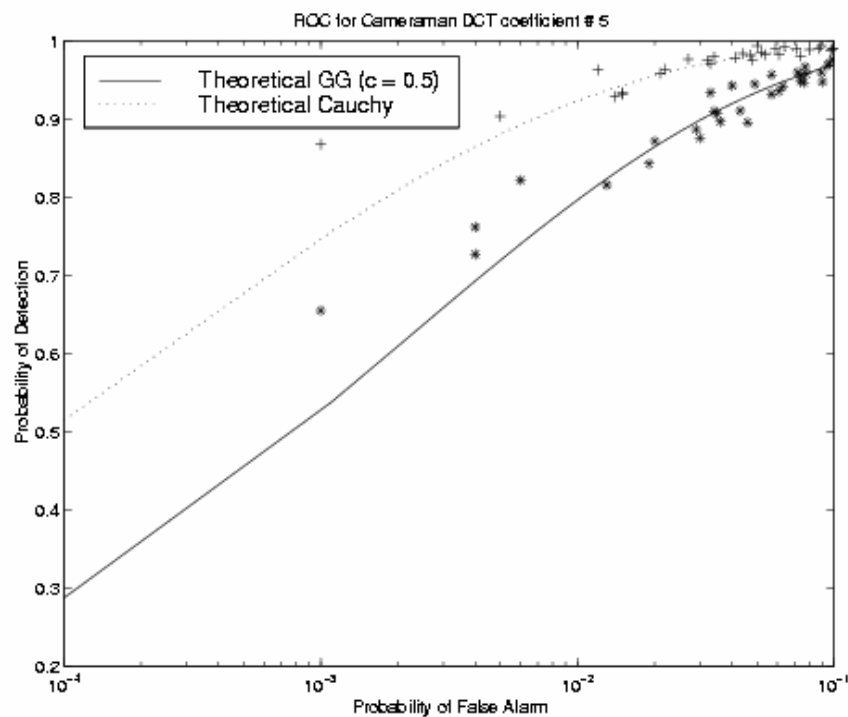


EXPERIMENTAL RESULTS

- Experimental results (Monte Carlo) verify theoretical ones.
- Boat DCT #5, #10: Cauchy detector is expected to be:
 - Better for #5 because of better modeling results
 - Worse for #10 – not so heavy tails, closer to Laplacian distribution



- Cameraman DCT #5, woman DCT #30:
Cauchy gave more accurate modeling and a higher SNR





CONCLUSIONS

- Blind watermark detector
- Improved statistical model for the data – alpha stable model
- Cauchy detector is in closed form
- *Cauchy detectors are in general very robust: their performance remains nearly optimal even for data that deviates from the Cauchy distribution*