Signal Detection in Severely Heavy-Tailed Radar Clutter *

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Abstract

Alpha-stable distributions have recently been recognized in the signal processing community as simple, yet very accurate, *two-parameter* statistical models for signals and noises that contain an impulsive component of various degrees of severity. On the basis of this finding, several signal processing problems have been addressed and solved within the framework of alpha-stable distributions and with the use of fractional, lower-order statistics. In this paper, we attempt to popularize these new signal processing tools within the radar community. In particular, we evaluate the goodness-of-fit of alpha-stable models in the radar environment and test the performance of new signal processing algorithms for signal detection and classification on real radar, sea-clutter data.

Key words - Symmetric Alpha-Stable Processes, Fractional Lower-Order Statistics, Radar Signal Processing, Radar Target Detection

1. Introduction

The design of radar systems constitutes a highly challenging problem, characterized by very weak signal levels and strong interferences from either unintentional (clutter) or intentional (jammers) sources. In this highly unfavorable, nonstationary, and unpredictable environment, complex tasks need to be performed by the radar sensors, such as detection and classification of very weak targets (embedded in clutter, noise, and intentional jamming), robust beamforming to null interference (especially in constantly changing environments with minimal signal knowledge, moving arrays, and uncertain element location and response), and automatic image analysis, segmentation, and classification.

One main prerequisite for the efficient accomplishment of the above tasks is the development of proper statistical models for the interference that is present in radar returns and attains the form of spikes, due to clutter sources such as ocean waves, and glints, due to reflections from large, flat surfaces such as buildings or vehicles. The presence of these spikes obscurs the target detection capability of the radar and significantly degrades its performance. Usually, the K-distribution is considered as the model for the amplitude statistics of sea-clutter [10, 1]:

$$f_K(c,\nu;x) = \frac{4c}{\Gamma(\nu)}(cx)^{\nu}K_{\nu-1}(2cx).$$

In the above equation, $K_\nu(x)$ is a modified Bessel function, c is a scale parameter, and ν is a shape parameter. The K-distributed model for radar clutter arises from the assumption that the radar return consists of the sum of a large number of independent returns ("speckle") that vary in intensity with time. Other models that have been proposed to statistically describe clutter include the Weibull and the log-normal. Target detection is usually done using either a bank of coherent detectors or a bank of quadratic energy detectors [24]. However, in very spiky sea-clutter, the number of false alarms can be very high and cannot be reduced by varying the detection threshold [24]. Recently, the principles of higher-order statistics (HOS) have been

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used to propose tests to detect deterministic and non-Gaussian stochastic signals in Gaussian noise [4, 8, 11]. However, the success of these methods in combatting the spiky nature of the interference has been very limited, especially for short observations [11, 20].

On many occasions [25], the empirical data indicate that the probability density functions (pdfs) of the associated noise processes maintain a similarity to the Gaussian pdf, being bell-shaped, smooth, and symmetric, but at the same time have significantly heavier tails. For example, atmospheric noise may be considered as arising from a superposition of many statistically independent sources so that central limit theorems are applicable in the evaluation of its pdf. The empirical fact [25] of algebraic (inverse power) tails in the pdf of atmospheric noise naturally leads to the assumption of a stable pdf. On the other hand, a certain class of non-Gaussian pdfs was considered in [7], which was parameterized in such a way that the Gaussian pdf was obtained at a certain limit. A similar parameterization of the class of stable random processes, in the characteristic function domain, is possible [6] in a manner that the Gaussian pdf is again included as a certain special case. The above evidence, combined with a recent, increasing interest in the application of the theory of stable random variables and processes in statistical signal processing [14, 16], clearly suggests that possible quite accurate models for large classes of impulsive noise in communication links may be the stable pdfs [15]. Very recently, it was also theoretically shown that, under general assumptions, a broad class of impulsive noise follows a stable distribution [15]. The stable model was then tested with a variety of real data and was found to match the data with excellent fidelity [9], at least equal to that of the Middleton models.

The performance of optimum and suboptimum receivers in the presence of SαS impulsive interference was examined in [21], both theoretically and via Monte-Carlo simulation, and a method was presented for the real time implementation of the optimum nonlinearities. From this study, it was found that the corresponding optimum receivers perform in the presence of SαS impulsive interference quite well, while the performance of Gaussian and other suboptimum receivers is unacceptably low. It was also shown that a receiver designed on a Cauchy assumption for the first order distribution of the impulsive interference performed only slightly below the corresponding optimum receiver, provided that a reasonable estimate of the noise dispersion was available, which for real-time signal processing purposes could be obtained via the fast algorithms in [19].

The study in [21] was, however, limited to coherent reception only, in which the amplitude and phase of the signals is assumed to be exactly known. The optimum demodulation algorithm for reception of signals with random phase in impulsive intereference and its corresponding performance was derived in [22] and tested against the traditional incoherent Gaussian receiver [13, Ch. 4]. Finally, the performance of asymptotically optimum multichannel structures for incoherent detection of amplitude-fluctuating bandpass signals in impulsive noise modeled as a bivariate, isotropic, symmetric, alpha-stable (BISαS) process was evaluated in [23]. In particular, our attention in [23] was directed to detector structures in which the different observation channels corresponded to spatially diverse receiving elements. However, our general findings hold for communication receivers of arbitrary diversity. We derived the proper test statistic, by generalizing the detector proposed by Izzo and Paura [5] to take into account the infinite variance in the noise model, and showed that exact knowledge of the noise distribution was *not* required for almost optimum performance. We also showed that receiver diversity did *not* improve the performance of the Gaussian receiver when operating in non-Gaussian impulsive noise and, therefore, a non-Gaussian detection algorithm could substitute for receiver diversity.

The present paper is devoted to an appraisal of the applicability of the alpha-stable model in the radar environment. More specifically, we evaluate the goodness-of-fit of the alpha-stable models in the radar environment and test the performance of the recently proposed new algorithms on real radar sea-clutter data. Our goal is twofold: (i) To popularize the concepts of alpha-stable distributions and fractional, lower-order statistics, as well as the basic signal processing algorithms that have been developed so far, among the radar community and (ii) To test the proposed algortihms on real radar, sea-clutter data. In particular, the paper is organized as follows: Section 2 summarizes the key definitions and properties of Fractional, Lower-Order Moments of SαS processes. In Section 3, we evaluate moment tests for random signal detection problems. We summarize the paper in Section 4, in which we also draw conclusions and suggest possible future research topics of interest to the radar community.

2. Fractional, Lower-Order Statistics

We consider a random variable ζ such that its fractional, lower-order pth moment is finite

$$\mathcal{E}\{|\zeta|^p\} < \infty, \tag{1}$$

where $0 < p < \infty$. We will call ζ a pth-order random variable. Let us now consider two pth-order random variables, ζ and η. We define their pth-order fractional correlation as [3]

$$< \zeta, \eta >_p \equiv \mathcal{E}\{\zeta(\eta)^{(p-1)}\}, \tag{2}$$

where

$$(\cdot)^{(p-1)} \equiv |\cdot|^{(p-1)} \operatorname{sgn}(\cdot) \tag{3}$$

for real-valued random variables and

$$(\cdot)^{(p-1)} \equiv |\cdot|^{(p-2)}\overline{(\cdot)} \tag{4}$$

for complex-valued random variables. In Eqs.(3) and (4), sgn$(\cdot)$ denotes the signum function, while the overbar denotes complex conjugation, respectively.

The above definitions are clearly seen to reduce to the usual SOS and HOS in the cases where those exist and can be easily extended to include random processes and their corresponding fractional correlation sequences. For example, if $\{X_k\}$, $k = 1, 2, 3, \ldots$, is a discrete-time random process, we can define its fractional, pth-order correlation sequence as

$$\rho_p(n,m) = < X_n, X_m >_p = \mathcal{E}\{X_n(X_m)^{(p-1)}\}, \tag{5}$$

which, for $p = 2$, gives the usual autocorrelation sequence.

A pth-order random process $\{X_k\}$, $k = 1, 2, 3, \ldots$, will be called *pth-order stationary* if its corresponding pth-order correlation sequence $\rho_p(n,m)$ in Eq.(5) depends only on the difference $l = m - n$ of its arguments. Sample averages can be used to define the FLOS of an *ergodic* stationary observed time series $\{X_k\}$, $k = 1, 2, 3, \ldots$, similarly to ensemble averages:

$$r_p(l) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{k=-N}^{N} X_k(X_{k+l})^{(p-1)}, \tag{6}$$

The basic properties of the FLOS of SαS processes are summarized as

P.1 For any ζ_1 and ζ_2, we have

$$< a_1\zeta_1 + a_2\zeta_2, \eta >_p = a_1 < \zeta_1, \eta >_p + a_2 < \zeta_2, \eta >_p \tag{7}$$

P.2 If ζ and η are independent, then

$$< \zeta, \eta >_p = 0, \tag{8}$$

while the converse is *not* true.

P.3 If η_1 and η_2 are independent, then

$$< a_1\eta_1 + a_2\eta_2, a_1\eta_1 + a_2\eta_2 >_p = \tag{9}$$
$$|a_1|^p < \eta_1, \eta_1 >_p + |a_2|^p < \eta_2, \eta_2 >_p \tag{10}$$

P.4 For a stationary pth-order random process $\{X_k\}$, $k = 1, 2, 3, \ldots$, its pth-order correlation and the corresponding sample average satisfy

$$\rho_p(l) \le \rho_p(0), \tag{11}$$
$$l = 0, \pm 1, \pm 2, \ldots.$$
$$r_p(l) \le r_p(0), \tag{12}$$

3. Moment Methods for Random Signal Detection

In this section, we look at moment-based methods for random signal detection and classification. As an illustrative example, we consider the detection of a stochastic, FIR signal in clutter. More specifically, we consider the hypothesis testing problem:

$$H_0 \quad : \quad x_l = w_l \qquad l = 0, 1, 2, \ldots, M \quad (13)$$
$$H_1 \quad : \quad x_l = \sum_{k=0}^{q} s_k u_{l-k} + w_l,$$

where $\{u_k\}$ is a sequence of iid SαS random variables, $\{s_k\}$, $k = 0, 1, 2, \ldots, q$, is a known signal sequence, and $\{w_k\}$ is a sequence of SαS random noise variables independent of the FIR signal. Finally, we are going to assume that $M > q$. For the dependence structure of the signal and the noise, we are not making any assumptions beyond those stated above.

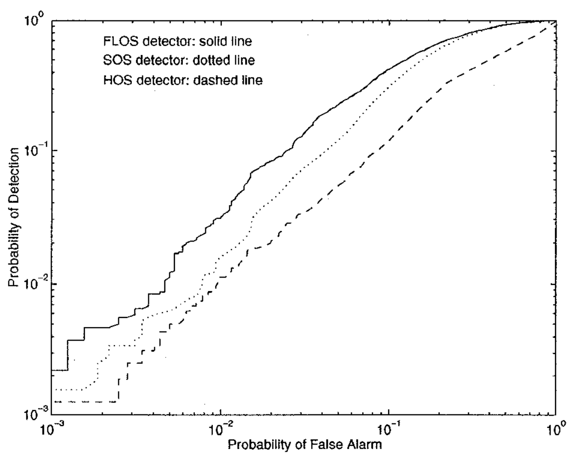

Figure 1: ROC of FLOS- (solid line), SOS- (dotted line), and HOS- (dashed line) based detector.

We propose [20] a detection rule that consists of computing the test statistic

$$r_p = \frac{1}{M} \sum_{n=0}^{M} |y_n|^p, \qquad p < \alpha/2 \qquad (14)$$

where $y_n = \sum_{l=0}^{q} s_{q-l} x_{n-l}$ is the observed sequence filtered with a filter matched to $\{s_k\}$, $k = 0, 1, 2, \ldots, q$. If the test statistics exceeds a threshold, hypothesis H_1 is declared, otherwise hypothesis H_0 is declared. This proposed test statistic is a direct generalization of the SOS-based energy detector [12] and HOS-based detectors [4].

The performance of our proposed FLOS-based detector relative to its SOS- and HOS-based counterparts is illustrated with the following example. The test signal is the stochastic FIR signal $x_l = 0.3u_l + 0.2u_{l-1} - 0.1u_{l-2} + 0.1u_{l-3}$, where the variables $\{u_l\}$ are iid, Laplace-distributed random variables of unit variance and the sequence $\{w_l\}$ are sea-clutter samples. We chose $M = 100$ samples per block, a FLOS of order $p = 1$, and a HOS statistic based on fourth-order cumulants [4]. The ROCs of the three detectors are shown in Fig. 1. Clearly, the performance of the fourth-order cumulant-based detector is the lowest of the three, while the proposed FLOS-based detector gives the highest performance.

4. Summary, Conclusions, and Possible Future Research

In this paper, we evaluated the performance of new algorithms that we recently proposed on the real data and examined the case of detection of random signals using moment-based methods. We found that the new algorithms outperformed existing ones, especially at the low probabilities of false alarm that any realistic radar would operate.

Thus, further research seems to be due in the design of new radar systems, in which the signal processors are designed on the basis of the alpha-stable models. Other specific radar signal processing issues that need to be addressed within the framework of alpha-stable distributions and fractional, lower-order statistics include beamforming and bearing estimation [18, 17], time-frequency distributions, application of alpha-stable fractals in radar, and radar image processing. Another avenue that also needs to be explored is that of developing robust classifiers based on existing and new features extracted from radar signals [2]. This and similar research is currently underway and its results will be announced shortly.

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