Space-Time Adaptive Processing in Stable Impulsive Interference*

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Abstract

This paper studies methods for Space-Time Adaptive Processing-based (STAP-based) parameter estimation in the presence of impulsive noise backgrounds. *Towards this goal, the theory of alpha-stable random processes provides an elegant and mathematically tractable framework for the solution of the detection and parameter estimation problems in the presence of impulsive radar clutter. We develop joint target angle and Doppler, maximum likelihood-based estimation techniques from radar measurements retrieved in the presence of impulsive noise (thermal, jamming, or clutter) modeled as an alpha-stable, complex random process. We derive the Cramér-Rao bounds for the additive Cauchy interference scenario to assess the best-case estimation accuracy which can be achieved. The results are of great importance in the study of space-time adaptive processing (STAP) for airborne pulse Doppler radar arrays operating in impulsive interference environments.*

1 Introduction

Future advanced airborne radar systems must be able to detect, identify, and estimate the parameters of a target in severe interference backgrounds. As a result, the problem of clutter and jamming suppression has been the focus of considerable research in the radar engineering community [1, 2, 3]. It is recognized that effective clutter suppression can be achieved only on the basis of appropriate statistical modeling. Most of the theoretical work in detection and estimation for radar applications has focused on the case where clutter is assumed to follow the Gaussian model. The Gaussian assumption is frequently motivated by the physics of the problem and it often leads to mathematically tractable solutions. Recently, experimental results have been reported where clutter returns are impulsive in nature [4]. In addition, a statistical model of impulsive interference has been proposed, which is based on the theory of symmetric alpha-stable ($S\alpha S$) random processes [5]. The model is of a statistical-physical nature and has been shown to arise under very general assumptions and to describe a broad class of impulsive interference. In addition, the theory of *complex isotropic alpha-stable* random processes provides an elegant and mathematically tractable framework for the solution of the detection and parameter estimation problems in the presence of impulsive radar clutter.

Space Time Adaptive Processing (STAP) has been introduced as a generalization of displaced-phase-center antenna (DPCA) processing and has been recognized as the technology which will enable long-range detection of increasingly smaller targets in the presence of severe clutter and jamming. STAP refers to adaptive antenna processors that simultaneously combine the signals received on multiple elements of an antenna array and from multiple pulse repetition periods of a radar coherent processing interval [6]. In other words, the STAP processor can be viewed as a two-dimensional (2-D) filter which performs both beamforming (spatial filtering) and Doppler (temporal) filtering to suppress interference and achieve target detection and parameter estimation. As a result, several researchers have concentrated on the theoretical advancement and understanding of the STAP discipline [7, 8, 9, 10].

As mentioned in [7], much of the work reported for

*The work in this paper was supported by the Office of Naval Research under Contract N00014-92-J-1034 and by Rome Laboratory under Contract F30602-95-1-0001.

radar systems has concentrated on *target detection* in Gaussian or non-Gaussian backgrounds [2, 11, 12, 13]. In this paper, we address the *target parameter estimation problem* through the use of STAP radar array sensor data retrieved in the presence of impulsive interference. First, we formulate the space-time adaptive processing (STAP) problem for airborne radar. Then, we present the Cramér-Rao analysis and we derive bounds on the variances of the spatial and temporal frequency estimates. In particular, we derive Cramér-Rao bounds on angle and Doppler estimator accuracy for the case of additive Cauchy noise. Finally, we present some experimental results on the localization capabilities of the maximum likelihood method based on the additive Cauchy noise assumption.

2 STAP Problem Formulation

Space-time adaptive processing (STAP) refers to multidimensional adaptive algorithms that simultaneously combine the signals from the elements of an array antenna and the multiple pulses of a coherent radar waveform to suppress interference and provide target detection [6, 7].

Consider a uniformly spaced linear radar array antenna consisting of N elements, which transmits a coherent burst of M pulses at a constant pulse repetition frequency (PRF) $f_r = 1/T_r$ and over a certain range of directions of interest. The array receives signals generated by q narrow-band moving targets which are located at azimuth angles $\{\phi_k;\ k = 1, \ldots, q\}$ and have relative velocities with respect to the radar $\{\nu_k;\ k = 1, \ldots, q\}$ corresponding to Doppler frequencies $\{f_k;\ k = 1, \ldots, q\}$. Since the signals are narrowband, the propagation delay across the array is much smaller than the reciprocal of the signal bandwidth, and it follows that, by using a complex envelop representation, the array output can be expressed as [6]:

$$\mathbf{x}(t) = \mathbf{V}(\boldsymbol{\psi}, \boldsymbol{\omega})\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where

- $\mathbf{x}(t) = [x_1(t), \ldots, x_{MN}(t)]^T$ is the array output vector (N: number of array elements, M: number of pulses, t may refer to the number of the coherent processing intervals (CPI's) available at the receiver);
- $\mathbf{s}(t) = [\beta_1(t), \ldots, \beta_q(t)]^T$ is the signal vector emitted by the sources as received at the reference sensor 1 of the array;
- $\mathbf{V}(\boldsymbol{\psi}, \boldsymbol{\omega}) = [\mathbf{v}(\psi_1, \omega_1), \ldots, \mathbf{v}(\psi_q, \omega_q)]$ is the *space-time steering matrix* where $\psi_k = 2\pi\frac{d}{\lambda_0}\sin(\phi_k)$ is the normalized angle and $\omega_k = 2\pi\frac{f_k}{f_r}$ is the normalized Doppler;
- $\mathbf{v}(\psi_k, \omega_k) = \mathbf{b}(\omega_k) \otimes \mathbf{a}(\psi_k)$ is the *space-time steering vector*, where $\otimes$ denotes the Kronecker matrix product, and
  - $\mathbf{a}(\psi_k) = [1, e^{-j\psi_k}, \ldots, e^{-j(N-1)\psi_k}]^T$ is the *spatial steering vector*;
  - $\mathbf{b}(\omega_k) = [1, e^{-j\omega_k}, \ldots, e^{j(M-1)\omega_k}]^T$ is the *temporal steering vector*.
- $\mathbf{n}(t) = [n_1(t), \ldots, n_{MN}(t)]^T$ is the noise vector whose components are modeled as Cauchy-distributed random variables with pdf given by

$$\chi_\gamma(r) = \frac{\gamma}{2\pi(r^2 + \gamma^2)^{3/2}}, \tag{2}$$

  where the *dispersion* γ ($\gamma > 0$) plays a role analogous to the role that the variance plays for second-order processes. Namely, it determines the spread of the probability density function around the origin.

Assuming the availability of S coherent processing intervals (CPI's) $t_1, \ldots, t_S$, the data can be expressed as

$$\mathbf{X} = \mathbf{V}(\boldsymbol{\psi}, \boldsymbol{\omega})\mathbf{S} + \mathbf{N}, \tag{3}$$

where $\mathbf{X}$ and $\mathbf{N}$ are the $MN \times S$ matrices

$$\mathbf{X} = [\mathbf{x}(t_1), \ldots, \mathbf{x}(t_S)], \tag{4}$$

$$\mathbf{N} = [\mathbf{n}(t_1), \ldots, \mathbf{n}(t_S)], \tag{5}$$

and $\mathbf{S}$ is the $q \times S$ matrix

$$\mathbf{S} = [\mathbf{s}(t_1), \ldots, \mathbf{s}(t_S)]. \tag{6}$$

Our objective is to jointly estimate the directions-of-arrival $\{\phi_k;\ k = 1, \ldots, q\}$ and the Doppler frequencies $\{f_k;\ k = 1, \ldots, q\}$ of the source targets.

3 The Cramér-Rao Bound for Cauchy Noise

The Cramér-Rao bound for the error variance of an unbiased estimator $\hat{\boldsymbol{\Theta}}$ satisfies

$$\mathbf{C}_{\hat{\boldsymbol{\Theta}}} - \mathbf{J}^{-1}(\boldsymbol{\Theta}) \geq 0 \tag{7}$$

where $\mathbf{C}_{\hat{\boldsymbol{\Theta}}}$ is the covariance matrix of $\hat{\boldsymbol{\Theta}}$, $\mathbf{J}(\boldsymbol{\Theta})$ is the Fisher information matrix, and $\mathbf{T} \geq 0$ is interpreted as meaning that the matrix $\mathbf{T}$ is semidefinite positive. Then, the following theorem holds for the case of complex isotropic Cauchy noise.

Theorem 1 *The CRB for $\boldsymbol{\psi}$, $\boldsymbol{\omega}$ and γ is given by*

$$CRB(\boldsymbol{\psi},\boldsymbol{\omega}) = \left[\sum_{t=1}^{S}\left(\boldsymbol{\Sigma} - \boldsymbol{\Upsilon}^T(t)\boldsymbol{\Xi}\boldsymbol{\Upsilon}(t)\right)\right]^{-1}, \quad (8)$$

and

$$CRB(\gamma) = \frac{5}{4}\frac{\gamma^2}{MNS} \quad (9)$$

where

$$\boldsymbol{\Sigma} = \frac{3}{5\gamma^2}\cdot\begin{bmatrix} \Re\left\{\sum_{t=1}^{S}\boldsymbol{\beta}^H(t)\mathbf{D}_{b\otimes d_a}^H\mathbf{D}_{b\otimes a}\boldsymbol{\beta}(t)\right\} & \Re\left\{\sum_{t=1}^{S}\boldsymbol{\beta}^H(t)\mathbf{D}_{b\otimes d_a}^H\mathbf{D}_{d_b\otimes a}\boldsymbol{\beta}(t)\right\} \\ \left[\Re\left\{\sum_{t=1}^{S}\boldsymbol{\beta}^H(t)\mathbf{D}_{b\otimes d_a}^H\mathbf{D}_{d_b\otimes a}\boldsymbol{\beta}(t)\right\}\right]^T & \Re\left\{\sum_{t=1}^{S}\boldsymbol{\beta}^H(t)\mathbf{D}_{d_b\otimes a}^H\mathbf{D}_{b\otimes a}\boldsymbol{\beta}(t)\right\} \end{bmatrix}$$

$$\boldsymbol{\Upsilon}(t) = \frac{3}{5\gamma^2}\cdot\begin{bmatrix} \Re\left\{\boldsymbol{\beta}^H(t)\mathbf{D}_{b\otimes d_a}^H\mathbf{D}_{b\otimes a}\right\} & \Im\left\{\boldsymbol{\beta}^H(t)\mathbf{D}_{b\otimes d_a}^H\mathbf{D}_{b\otimes a}\right\} \\ \Re\left\{\boldsymbol{\beta}^H(t)\mathbf{D}_{d_b\otimes a}^H\mathbf{D}_{b\otimes a}\right\} & \Im\left\{\boldsymbol{\beta}^H(t)\mathbf{D}_{d_b\otimes a}^H\mathbf{D}_{b\otimes a}\right\} \end{bmatrix} \quad t = 1,\cdots,S$$

$$\boldsymbol{\Xi} = \frac{5\gamma^2}{3}\cdot\begin{bmatrix} \left[\Re\left\{\mathbf{D}_{b\otimes a}^H\mathbf{D}_{b\otimes a}\right\}\right]^{-1} & \left[\Im\left\{\mathbf{D}_{b\otimes a}^H\mathbf{D}_{b\otimes a}\right\}\right]^{-1} \\ -\left[\Re\left\{\mathbf{D}_{b\otimes a}^H\mathbf{D}_{b\otimes a}\right\}\right]^{-1} & \left[\Re\left\{\mathbf{D}_{b\otimes a}^H\mathbf{D}_{b\otimes a}\right\}\right]^{-1} \end{bmatrix}$$

and

$$\mathbf{D}_{b\otimes a} = [\mathbf{b}(\omega_1)\otimes\mathbf{a}(\psi_1),\cdots,\mathbf{b}(\omega_q)\otimes\mathbf{a}(\psi_q)],$$
$$\mathbf{D}_{d_b\otimes a} = [\mathbf{d}_b(\omega_1)\otimes\mathbf{a}(\psi_1),\cdots,\mathbf{d}_b(\omega_q)\otimes\mathbf{a}(\psi_q)],$$
$$\mathbf{D}_{b\otimes d_a} = [\mathbf{b}(\omega_1)\otimes\mathbf{d}_a(\psi_1),\cdots,\mathbf{b}(\omega_q)\otimes\mathbf{d}_a(\psi_q)],$$

where $\mathbf{d}_a(\psi_j) = [d_1^a(\psi_j)\cdots d_N^a(\psi_j)] = [\frac{\partial a_1(\psi_j)}{\partial\psi_j}\cdots\frac{\partial a_N(\psi_j)}{\partial\psi_j}]$, $\mathbf{d}_b(\omega_j) = [d_1^b(\omega_j)\cdots d_M^b(\omega_j)] = [\frac{\partial b_1(\omega_j)}{\partial\omega_j}\cdots\frac{\partial b_M(\omega_j)}{\partial\omega_j}]$ *and $\boldsymbol{\beta}(\mathbf{t})$ is given by*

$$\begin{aligned}\boldsymbol{\beta}(t) &= diag[\beta_1(t)\cdots\beta_q(t)] \\ &= diag[\beta_{\Re,1}(t)\cdots\beta_{\Re,q}(t)] + \\ &\quad j\cdot diag[\beta_{\Im,1}(t)\cdots\beta_{\Im,q}(t)] \\ &= \boldsymbol{\beta}_{\Re} + j\cdot\boldsymbol{\beta}_{\Im} \qquad t = 1,\cdots,S\end{aligned}$$

where $\beta_{\Re,i}(t) = \Re\{\beta_i(t)\}$, $\beta_{\Im,i}(t) = \Im\{\beta_i(t)\}$, and $\Re\{\cdot\}$ and $\Im\{\cdot\}$ are the real and imaginary part operators, respectively.

Proof Given in [14].

We should note that the above bound can be achieved only when there exist unbiased estimators for all the models parameters γ, $\boldsymbol{\beta}(t)$, $\boldsymbol{\psi}$ and $\boldsymbol{\omega}$. A useful insight on the CRB can be gained if we consider the case of a single target ($q = 1$) located in $\psi = \frac{2\pi d}{\lambda_0}\sin\phi$, with normalized Doppler shift $\omega = 2\pi f T_r$ and constant target amplitude β. In this case, we have

$$CRB(\phi) = \frac{\gamma^2}{S|\beta|^2}\frac{\lambda_0^2}{(2\pi d)^2}\cdot\frac{5N(\|\mathbf{d}_b\|^2 - \delta_b^2/M)}{3\xi}\cdot\frac{1}{\cos^2(\phi)} \quad (10)$$

$$CRB(f) = \frac{\gamma^2}{S|\beta|^2}\frac{1}{(2\pi T_r)^2}\cdot\frac{5M(\|\mathbf{d}_a\|^2 - \delta_a^2/N)}{3\xi}, \quad (11)$$

where

$$\delta_a = \sum_{i=1}^{N}|d_i^a|,\quad \delta_b = \sum_{i=1}^{M}|d_i^b|,\quad \rho = \sum_{i=1}^{MN}|d_{g(i)}^a||d_{f(i)}^b|,$$

and

$$\xi = (M\|\mathbf{d}_a\|^2 - \frac{M}{N}\delta_a^2)(N\|\mathbf{d}_b\|^2 - \frac{N}{M}\delta_b^2) - (\delta_a\delta_b - \rho)^2.$$

Finally, if we consider an airborne radar system that utilizes a uniform linear array antenna and a waveform with a uniform pulse repetition interval, the bounds (10) and (11) are given by:

$$CRB(\phi) = \frac{\gamma^2}{S|\beta|^2}\cdot\frac{\lambda_0^2}{(2\pi d)^2}\cdot\frac{20}{M^2N^2(N^2-1)}\cdot\frac{1}{\cos^2(\phi)} \quad (12)$$

and

$$CRB(f) = \frac{\gamma^2}{S|\beta|^2}\cdot\frac{1}{(2\pi T_r)^2}\cdot\frac{20}{N^2M^2(M^2-1)}. \quad (13)$$

As can be seen in (10) and (11), target angle accuracy is a function of Doppler frequency and vice-versa. In addition, the bounds are functions of the generalized SNR function given by $\frac{S|\beta|^2}{\gamma^2}$, similarly to the Gaussian case where the bounds are functions of the SNR. The larger the dispersion γ of the noise, the higher the CRB.

4 Experimental Results

In this section, we show preliminary results on the localization capability of two likelihood functions. Namely, the likelihood function based on the Cauchy additive noise assumption (MLC) and the likelihood function based on the Gaussian additive noise assumption (MLG). The array is linear with five sensors

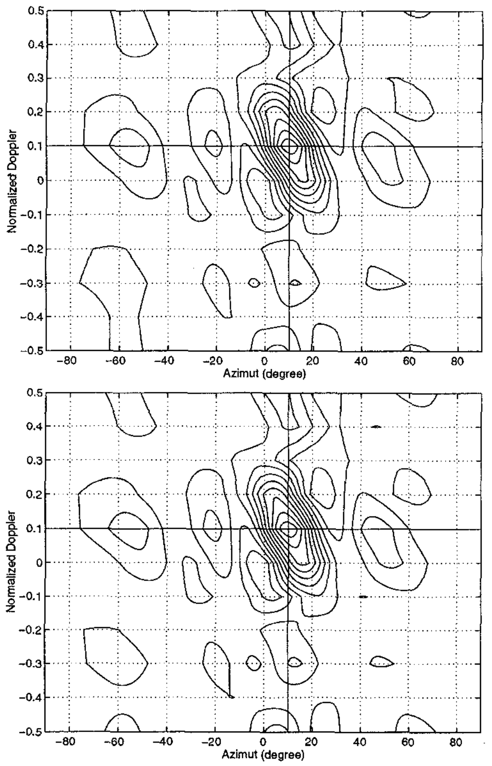

Figure 1: MLC (top) and MLG (bottom) angle-Doppler spectra ($N = 5$, $M = 4$, $\phi = 10^o$, $fT_r = 0.1$, number of snapshots $S = 50$). Additive stable noise with $\alpha = 2$ and $\gamma = 20$ (GSNR = 4dB).

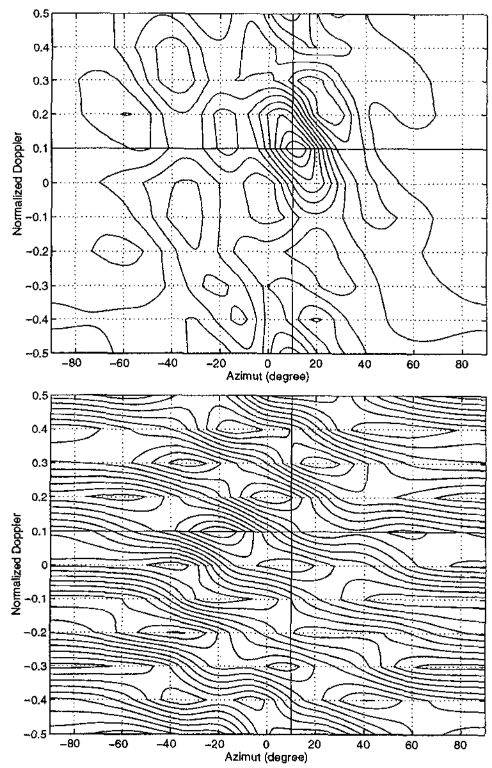

Figure 2: MLC (top) and MLG (bottom) angle-Doppler spectra ($N = 5$, $M = 4$, $\phi = 10^o$, $fT_r = 0.1$, number of snapshots $S = 50$). Additive stable noise with $\alpha = 1.1$ and $\gamma = 20$ (GSNR = 4dB).

spaced half wavelength apart ($N = 5$). The number of transmitted pulses is $M = 4$. A single moving target has parameters $[\{\theta, \varpi\}] = [\{10^o, 0.1\}]$. The number of snapshots available to the likelihood functions is $S = 50$. The additive noise follows the bivariate isotropic stable distribution.

In Figures 1 and 2, we show isosurfaces of the two likelihood functions (space-time spectral estimates) MLC and MLG. Two types of alpha stable noise corresponding to two values of the characteristic exponent $\alpha = 1.1$ and $\alpha = 2.0$ (Gaussian) were used. We can see that the MLG likelihood function cannot localize the target when $\alpha = 1.1$. On the other hand, the MLC likelihood function exhibits better localization capabilities for non-Gaussian additive noise environments ($\alpha = 1.1$) and at the same time, performs well in Gaussian interference.

Figure 3 shows the resulting MSE curves as functions of the characteristic exponent α. The number of snapshots available to the algorithms is $S = 10$. As we can clearly see, the estimation accuracy of the MLC method is fairly robust to the changes of α. On the other hand, the MLG algorithm exhibits very large mean square estimation errors for non-Gaussian environments. Note that when $\alpha = 2$, i.e., for the Gaussian noise case, the MLG method has the least MSE, as expected.

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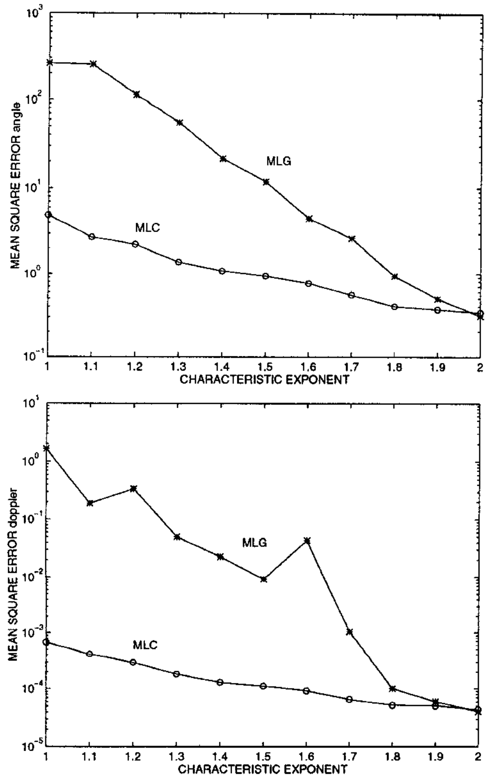

Figure 3: MSE of the estimated angle (top) and Doppler (bottom) as a function of the characteristic exponent α.

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