JOINT TARGET ANGLE AND DOPPLER ESTIMATION IN INTERFERENCE MODELED AS A STABLE PROCESS

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ABSTRACT

We describe new methods on the modeling of the amplitude statistics of airborne radar clutter by means of alpha-stable distributions. We develop target angle and Doppler, maximum likelihood-based estimation techniques from radar measurements retrieved in the presence of impulsive noise (thermal, jamming, or clutter) modeled as a multivariate sub-Gaussian random process. We derive the Cramér-Rao bounds for the additive sub-Gaussian interference scenario to assess the best-case estimation accuracy which can be achieved. Finally, we introduce a new joint spatial- and doppler-frequency high-resolution estimation technique based on the fractional lower-order statistics of the measurements of a radar array. The results are of great importance in the study of space-time adaptive processing (STAP) for airborne pulse Doppler radar arrays operating in impulsive interference environments.

1. INTRODUCTION

Future advanced airborne radar systems must be able to detect, identify, and estimate the parameters of a target in severe interference backgrounds. As a result, the problem of clutter and jamming suppression has been the focus of considerable research in the radar engineering community. It is recognized that effective clutter suppression can be achieved only on the basis of appropriate statistical modeling. Recently, experimental results have been reported where clutter returns are impulsive in nature. In addition, a statistical model of impulsive interference has been proposed, which is based on the theory of symmetric alpha-stable ($S\alpha S$) random processes [1]. The model is of a statistical-physical nature and has been shown to arise under very general assumptions and to describe a broad class of impulsive interference. In addition, the theory of *multivariate sub-Gaussian* random processes provides an elegant and mathematically tractable framework for the solution of the detection and parameter estimation problems in the presence of impulsive correlated radar clutter.

The work of the first and third authors was supported by Rome Laboratory under Contract F30602-95-1-0001.

As mentioned in [2], much of the work reported for radar systems has concentrated on target detection in Gaussian or Non-Gaussian backgrounds [3, 4, 5, 6]. In this paper, we address the *target parameter estimation problem* through the use of radar array sensor data retrieved in the presence of impulsive interference. In particular, we derive Cramér-Rao bounds on angle and Doppler estimator accuracy for the case of additive sub-Gaussian noise. Initially, we consider the case of additive multivariate Cauchy noise, assuming knowledge of the *underlying matrix* of the distribution. In addition, we present a new subspace-based method for joint spatial- and doppler-frequency high-resolution estimation in the presence of impulsive noise. The results obtained here can be viewed as generalizations of the work done in [7, 8] to the 2-D frequency estimation problem in impulsive interference backgrounds.

In Section 2, we present necessary preliminaries on α-stable processes and results on the modeling of real clutter data by means of $S\alpha S$ distributions. In Section 3, we formulate the space-time adaptive processing (STAP) problem for airborne radar. In Section 4, we present the Cramér-Rao analysis, we form the maximum likelihood function, and we derive bounds on the variances of the spatial and temporal frequency estimates. In Section 5, we define the covariation matrix of the space-time radar sensor output snapshot and we show that eigendecomposition-based methods, such as the MUSIC algorithm, can be applied to the sample covariation matrix to extract the angle/Doppler information from the measurements. Finally, in Section 6, we demonstrate via Monte Carlo experiments the improved performance of the proposed angle and Doppler target localization methods in the presence of a wide range of impulsive noise environments.

2. MATHEMATICAL PRELIMINARIES

In this section, we introduce the statistical model that will be used to describe the additive noise. The model is based on the class of *isotropic* $S\alpha S$ distributions, and is well-suited for describing impulsive noise processes [1].

Stable processes satisfy the stability property which states that linear combinations of jointly stable variables are indeed stable. They arise as limiting processes of sums of independent, identically-distributed random variables via the generalized central limit theorem. They are described by their characteristic exponent α, taking values $0 < \alpha \leq 2$. Gaussian processes are stable processes with $\alpha = 2$. Stable distributions have heavier tails than the normal distribution, possess finite pth order moments only for $p < \alpha$, and are appropriate for modeling noise with outliers.

A complex random variable (r.v.) $X = X_1 + \jmath X_2$ is isotropic $S\alpha S$ if X_1 and X_2 are jointly $S\alpha S$ and have a symmetric distribution. The characteristic function of X is given by

$$\varphi(\omega) = \mathcal{E}\{\exp(\jmath\Re[\omega X^*])\} = \exp(-\gamma|\omega|^\alpha), \quad (1)$$

where $\omega = \omega_1 + \jmath\omega_2$. The *characteristic exponent* α is restricted to the values $0 < \alpha \leq 2$ and it determines the shape of the distribution. The smaller the characteristic exponent α, the heavier the tails of the density. The *dispersion* γ ($\gamma > 0$) plays a role analogous to the role that the variance plays for second-order processes. Namely, it determines the spread of the probability density function around the origin.

Several complex r.v.'s are jointly $S\alpha S$ if their real and imaginary parts are jointly $S\alpha S$. When X and Y are jointly $S\alpha S$ with $1 < \alpha \leq 2$, the *covariation* of X and Y is defined by

$$[X,Y]_\alpha = \frac{\mathcal{E}\{XY^{<p-1>}\}}{\mathcal{E}\{|Y|^p\}}\gamma_Y, \quad 1 \leq p < \alpha, \quad (2)$$

where $\gamma_Y = [Y,Y]_\alpha$ is the dispersion of the r.v. Y, and we use throughout the convention $Y^{<p>} = |Y|^{p-1}Y^*$. Also, the *covariation coefficient* of X and Y is defined by

$$\lambda_{X,Y} = \frac{[X,Y]_\alpha}{[Y,Y]_\alpha}, \quad (3)$$

and by using (2), it can be expressed as

$$\lambda_{X,Y} = \frac{E\{XY^{<p-1>}\}}{E\{|Y|^p\}}, \qquad \text{for } 1 \leq p < \alpha. \quad (4)$$

The covariation of complex jointly $S\alpha S$ r.v.'s is not generally symmetric and has the following properties:

P1 If X_1, X_2 and Y are jointly $S\alpha S$, then for any complex constants a and b,

$$[aX_1 + bX_2, Y]_\alpha = a[X_1,Y]_\alpha + b[X_2,Y]_\alpha;$$

P2 If Y_1 and Y_2 are independent and X_1, X_2 and Y are jointly $S\alpha S$, then for any complex constants a, b and c,

$$[aX_1, bY_1 + cY_2]_\alpha = ab^{<\alpha-1>}[X_1,Y_1]_\alpha + ac^{<\alpha-1>}[X_1,Y_2]_\alpha;$$

P3 If X and Y are independent $S\alpha S$, then $[X,Y]_\alpha = 0$.

Figure 1 shows results on the modeling of the amplitude statistics of real radar clutter by means of $S\alpha S$ distributions obtained by Ma and Nikias. The estimation of the parameters of the stable distribution from the real clutter data was achieved by methods based on fractional lower-order

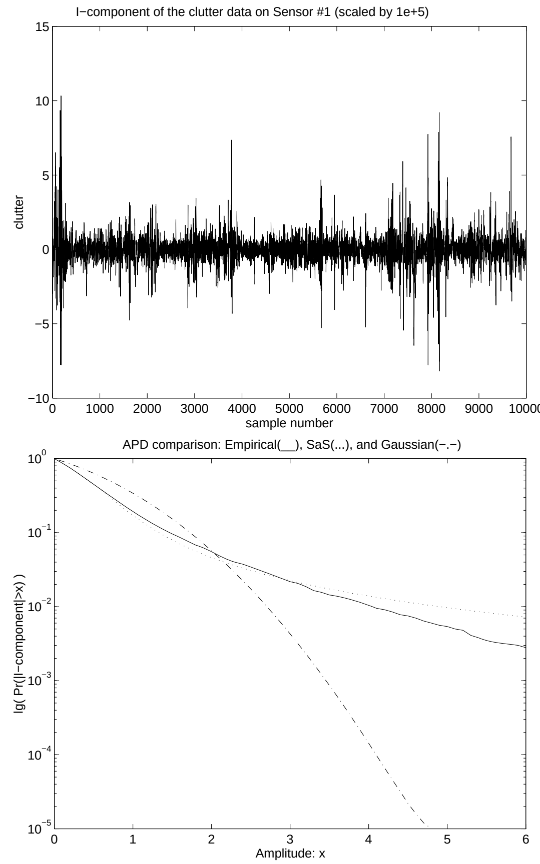

Figure 1: The in-phase component of the clutter time series (top) and the corresponding amplitude probability density (APD) curves (Empirical: solid, $S\alpha S$: dotted, Gaussian: dash-dotted) (bottom).

and negative-order moments, as described in [9]. For the particular case shown here, the characteristic exponent of the distribution which best fits the data was calculated to be $\alpha = 1.5$. The impulsive nature of the clutter data is obvious in Figure 1 which demonstrates that the $S\alpha S$ distribution is superior to the Gaussian distribution for modeling the particular radar clutter data under study.

3. STAP PROBLEM FORMULATION

Space-time adaptive processing (STAP) refers to multidimensional adaptive algorithms that simultaneously combine the signals from the elements of an array antenna and the multiple pulses of a coherent radar waveform, to suppress interference and provide target detection [2, 10, 11].

Consider a uniformly spaced linear array radar antenna consisting of N elements, which transmits a coherent burst of M pulses at a constant pulse repetition frequency (PRF) f_r and over a certain range of directions of interest. The array receives signals generated by q narrow-band moving targets which are located at azimuth angles $\{\theta_k;\ k = 1,\ldots,q\}$ and have relative velocities with respect to the radar $\{\nu_k;\ k=1,\ldots,q\}$ corresponding to Doppler frequencies $\{f_k;\ k=1,\ldots,q\}$. Since the signals are narrow-band, the propagation delay across the array is much smaller than the reciprocal of the signal bandwidth, and it follows that, by using a complex envelop representation, the array output can be expressed as [10]:

$$\mathbf{x}(t)=\mathbf{V}(\Theta,\varpi)\mathbf{s}(t)+\mathbf{n}(t), \tag{5}$$

where

- $\mathbf{x}(t)=[x_1(t),\ldots,x_{MN}(t)]^T$ is the array output vector (N: number of array elements, M: number of pulses, t may refer to the number of the coherent processing intervals (CPI's) available at the receiver);
- $\mathbf{s}(t)=[s_1(t),\ldots,s_q(t)]^T$ is the signal vector emitted by the sources as received at the reference sensor 1 of the array;
- $\mathbf{V}(\Theta,\varpi)=[\mathbf{v}(\vartheta_1,\varpi_1),\ldots,\mathbf{v}(\vartheta_q,\varpi_q)]$ is the *space-time steering matrix* ($\varpi_k=\frac{f_k}{f_r}$);
- Space-Time steering vector: $\mathbf{v}(\vartheta_k,\varpi_k)=\mathbf{b}(\varpi_k)\otimes\mathbf{a}(\vartheta_k)$;
  - $\mathbf{a}(\vartheta_k)=[1,e^{\jmath 2\pi\vartheta_k},\ldots,e^{\jmath(N-1)2\pi\vartheta_k}]^T$ is the spatial steering vector ($\vartheta_k=\frac{d}{\lambda_0}\cos(\theta_k)$);
  - $\mathbf{b}(\varpi_k)=[1,e^{\jmath 2\pi\varpi_k},\ldots,e^{\jmath(M-1)2\pi\varpi_k}]^T$ is the temporal steering vector.
- $\mathbf{n}(t)=[n_1(t),\ldots,n_{MN}(t)]^T$ is the noise vector.

Assuming the availability of P coherent processing intervals (CPI's) $t_1,\ldots,t_P$, the data can be expressed as

$$\mathbf{X}=\mathbf{V}(\Theta,\varpi)\mathbf{S}+\mathbf{N}, \tag{6}$$

where $\mathbf{X}$ and $\mathbf{N}$ are the $MN\times P$ matrices

$$\mathbf{X}=[\mathbf{x}(t_1),\ldots,\mathbf{x}(t_P)], \tag{7}$$

$$\mathbf{N}=[\mathbf{n}(t_1),\ldots,\mathbf{n}(t_P)], \tag{8}$$

and $\mathbf{S}$ is the $q\times P$ matrix

$$\mathbf{S}=[\mathbf{s}(t_1),\ldots,\mathbf{s}(t_P)]. \tag{9}$$

Our objective is to jointly estimate the directions-of-arrival $\{\vartheta_k;\ k=1,\ldots,q\}$ and the Doppler frequencies $\{f_k;\ k=1,\ldots,q\}$ of the source targets.

4. CRAMÉR-RAO BOUND ANALYSIS

Given a single snapshot containing a target at angle ϕ and Doppler frequency f, the space-time snapshot can be written as [10]

$$\mathbf{x}=\alpha\mathbf{v}(\phi,f)+\mathbf{n}. \tag{10}$$

where α is the target's complex amplitude given by

$$\alpha=x+jy. \tag{11}$$

The vector $\mathbf{v}$ is an $NM\times 1$ vector called the *space-time steering vector*. It may be expressed as

$$\mathbf{v}(\phi,f)=\mathbf{b}(f)\otimes\mathbf{a}(\phi) \tag{12}$$

where $\mathbf{a}(\phi)$ is the $N\times 1$ *spatial steering vector* containing the interelement phase shifts for a target at ϕ, and $\mathbf{b}(\mathrm{f})$ is the $M\times 1$ *temporal steering vector* that contains the interpulse phase shifts for a target with Doppler f. It is assumed that the functional form of $\mathbf{v}(\phi,f)$ is known. In addition, we can write

$$v_i(\phi,f)=b_{(\lfloor i/N\rfloor)}(f)\cdot a_{(i-\lfloor i/N\rfloor N)}(\phi)=b_{f(i)}\cdot a_{g(i)} \tag{13}$$

where $f(i)=\lfloor i/N\rfloor$ and $g(i)=i-\lfloor i/N\rfloor N$.

The snapshot contains a noise component $\mathbf{n}$ which includes clutter, jamming, thermal noise, and any other undesired signals. We model $\mathbf{n}$ as a multivariate Cauchy process with pdf given by

$$f(\mathbf{n})=\frac{c||\mathbf{R}||^{-1/2}}{[1+\mathbf{n}^T\mathbf{R}^{-1}\mathbf{n}]^{(MN+1)/2}}, \tag{14}$$

where $\mathbf{R}$ is a positive-definite matrix which models the statistical dependence of the impulsive noise process, and $c=\frac{1}{\pi^{(MN+1)/2}}\Gamma(\frac{MN+1}{2})$ with $\Gamma()$ being the Gamma function. As a first approximation to the problem, we will assume that the noise present at the array is statistically independent both along the array sensors and along time. In this case, each component of the noise vector is modeled as a complex isotropic Cauchy process with marginal pdf given by

$$\chi_\gamma(r)=\frac{\gamma}{2\pi(r^2+\gamma^2)^{3/2}} \tag{15}$$

Under the independence assumption it follows from (10) and (12) that the joint density function for the case of a single snapshot is given by [7]

$$f(\mathbf{n})=\prod_{i=1}^{MN}f(n_i)=\frac{\gamma^{MN}}{(2\pi)^{MN}\prod_{i=1}^{MN}(\gamma^2+|x_i-\alpha v_i|^2)^{3/2}} \tag{16}$$

In the following, it will be convenient to work with the normalized spatial and temporal frequency variables:

$$\psi=\frac{2\pi d}{\lambda_0}\sin\phi\quad,\quad\omega=2\pi fT_r. \tag{17}$$

The estimation problem involves four real valued parameters. We arrange them to form a 4×1 parameter vector

$$\mathbf{\Theta}=[\theta_1\quad\theta_2\quad\theta_3\quad\theta_4]=[\psi\quad\omega\quad x\quad y]. \tag{18}$$

Then, given a single snapshot $\mathbf{x}$, the likelihood function $L(\mathbf{\Theta})$, ignoring the constant terms, is given by

$$L(\mathbf{\Theta})=-\frac{3}{2}\sum_{i=1}^{NM}\log\left(\gamma^2+|x_i-\beta v_i(\psi,\omega)|^2\right). \tag{19}$$

The Cramér-Rao bound for the error variance of an unbiased estimator $\hat{\mathbf{\Theta}}$ satisfies

$$\mathbf{C}_{\hat{\mathbf{\Theta}}}-\mathbf{J}(\mathbf{\Theta})\geq 0 \tag{20}$$

where $\mathbf{C}_{\hat{\mathbf{\Theta}}}$ is the covariance matrix of $\hat{\mathbf{\Theta}}$ and ≥ 0 is interpreted as meaning that the matrix is semidefinite positive. The matrix $\mathbf{J}(\mathbf{\Theta})$ is the Fisher information matrix given by

$$\mathbf{J}(\mathbf{\Theta})=E\{[\partial L(\mathbf{\Theta})/\partial\mathbf{\Theta}][\partial L(\mathbf{\Theta})/\partial\mathbf{\Theta}]^T\}. \tag{21}$$

First, we calculate the derivatives of the log-likelihood function given in (19) with respect to the components of $\mathbf{\Theta}$. We have that

$$\frac{\partial L}{\partial \psi} = 3 \sum_{i=1}^{MN} \frac{\Re\{\beta^* b^*_{f(i)} (d^a_{g(i)})^* n_i\}}{\gamma^2 + |n_i|^2} \tag{22}$$

where $d_i^a = \partial a_i / \partial \psi, \quad i = 1, \cdots, N$. In addition

$$\frac{\partial L}{\partial \omega} = 3 \sum_{i=1}^{MN} \frac{\Re\{\beta^* a^*_{g(i)} (d^b_{f(i)})^* n_i\}}{\gamma^2 + |n_i|^2} \tag{23}$$

where $d_i^b = \partial b_i / \partial \omega, \quad i = 1, \cdots, M$. Additionally,

$$\frac{\partial L}{\partial x} = 3 \sum_{i=1}^{MN} \frac{\Re\{a^*_{g(i)} b^*_{f(i)} n_i\}}{\gamma^2 + |n_i|^2} \tag{24}$$

and

$$\frac{\partial L}{\partial y} = -3 \sum_{i=1}^{MN} \frac{\Im\{a^*_{g(i)} b^*_{f(i)} n_i\}}{\gamma^2 + |n_i|^2}. \tag{25}$$

By performing the second derivatives and expectations in a similar way, the Fisher information matrix $\mathbf{J}(\mathbf{\Theta})$ is derived to be

$$\mathbf{J}(\mathbf{\Theta}) = \frac{3}{5\gamma^2} \cdot \begin{bmatrix} M|\beta|^2 \parallel \mathbf{d}_a \parallel^2 & |\beta|^2 \rho & yM\delta_a & xM\delta_a \\ |\beta|^2 \rho & N|\beta|^2 \parallel \mathbf{d}_b \parallel^2 & yN\delta_b & xN\delta_b \\ yM\delta_a & yN\delta_b & MN & 0 \\ xM\delta_a & xN\delta_b & 0 & MN \end{bmatrix},$$

where

$$\delta_a = \sum_{i=1}^{N} |d_i^a|, \quad \delta_b = \sum_{i=1}^{M} |d_i^b|, \quad \rho = \sum_{i=1}^{MN} |d^a_{g(i)}||d^b_{f(i)}|,$$

and $\mathbf{d}_a = [d_1^a \ \cdots \ d_n^a]$, $\mathbf{d}_b = [d_1^b \ \cdots \ d_n^b]$. Since target angle and Doppler are the two parameters of primary interest, we shall focus on the upper left 2×2 block of the Fisher information matrix $\mathbf{J}_{2\times 2}$. The inverse of matrix $\mathbf{J}_{2\times 2}$ is obtained by applying the partitioned matrix inversion lemma. The result is

$$\mathbf{J}^{-1}_{2\times 2}(\mathbf{\Theta}) = \frac{1}{\xi} \cdot \frac{5\gamma^2}{3|\beta|^2} \cdot \begin{bmatrix} N(\parallel \mathbf{d}_b \parallel^2 - \frac{1}{M}\delta_b^2) & \delta_a\delta_b - \rho \\ \delta_a\delta_b - \rho & M(\parallel \mathbf{d}_a \parallel^2 - \frac{1}{N}\delta_a^2) \end{bmatrix}, \tag{26}$$

where $\xi = (M \parallel \mathbf{d}_a \parallel^2 - \frac{M}{N}\delta_a^2)(N \parallel \mathbf{d}_b \parallel^2 - \frac{N}{M}\delta_b^2) - (\delta_a\delta_b - \rho)^2$. The Cramér-Rao bounds of the resulting spatial and temporal frequency estimates are obtained from (26) as

$$CRB(\psi) = \frac{\gamma^2}{|\beta|^2} \cdot \frac{5N(\parallel \mathbf{d}_b \parallel^2 - \delta_b^2/M)}{3\xi} \tag{27}$$

and

$$CRB(\omega) = \frac{\gamma^2}{|\beta|^2} \cdot \frac{5M(\parallel \mathbf{d}_a \parallel^2 - \delta_a^2/N)}{3\xi}. \tag{28}$$

A useful insight on the CRB can be gained if we consider the case of a linear array whose sensors are spaced a half-wavelength apart, and of a waveform with a uniform pulse repetition interval. The spatial and temporal steering vectors for such a system are:

$$\mathbf{a}(\psi) = \begin{bmatrix} 1 \\ e^{-j\psi} \\ \vdots \\ e^{-j(N-1)\psi} \end{bmatrix}, \quad \mathbf{b}(\omega) = \begin{bmatrix} 1 \\ e^{-j\omega} \\ \vdots \\ e^{-j(M-1)\omega} \end{bmatrix}. \tag{29}$$

In this case, it follows from (27) and (28) that

$$CRB(\psi) = \frac{\gamma^2}{|\beta|^2} \cdot \frac{20}{M^2 N^2 (N^2 - 1)} \tag{30}$$

and

$$CRB(\omega) = \frac{\gamma^2}{|\beta|^2} \cdot \frac{20}{M^2 N^2 (M^2 - 1)}. \tag{31}$$

The term $\gamma^2/|\beta|^2$ in the above expressions for the CRB can be viewed as the inverse of a quantity analogous to the signal-to-noise ratio (SNR) for the Gaussian case, i.e., a generalized SNR, so to speak. The larger the dispersion γ of the noise, the higher the CRB.

5. THE ARRAY COVARIATION MATRIX

In this section, we will assume that the q signal waveforms are non-coherent, statistically independent, complex isotropic $S\alpha S$ ($1 < \alpha \leq 2$) random processes with zero location parameter and covariation matrix $\mathbf{\Gamma}_S = \text{diag}(\gamma_{s_1}, \ldots, \gamma_{s_q})$. Also, the noise vector $\mathbf{n}(t)$ is a complex isotropic $S\alpha S$ random process with the same characteristic exponent α as the signals. The noise is assumed to be independent of the signals with covariation matrix $\mathbf{\Gamma}_N = \gamma_n \mathbf{I}$.

Now, we define the *covariation matrix*, $\mathbf{\Gamma}_X$, of the observation vector process $\mathbf{x}(t)$ as the matrix whose elements are the covariations $[x_i(t), x_j(t)]_\alpha$ of the components of $\mathbf{x}(t)$. We obtain the following expression for the covariations of the sensor measurements:

$$[x_i(t), x_j(t)]_\alpha = \sum_{k=1}^{q} v_i(\vartheta_k, \varpi_k) v_j^{<\alpha-1>}(\vartheta_k, \varpi_k)\gamma_{s_k} + \gamma_n \delta_{i,j} \quad i, j = 1, \ldots, MN. \tag{32}$$

In matrix form, (32) gives the following expression for the covariation matrix of the observation vector:

$$\mathbf{\Gamma}_X \triangleq [\mathbf{x}(t), \mathbf{x}(t)]_\alpha = \mathbf{V}(\Theta, \varpi)\mathbf{\Gamma}_S \mathbf{V}^{<\alpha-1>}(\Theta, \varpi) + \gamma_n \mathbf{I}, \tag{33}$$

where the (i,j)th element of matrix $\mathbf{V}^{<\alpha-1>}(\Theta, \varpi)$ results from the (j,i)th element of $\mathbf{V}(\Theta, \varpi)$ according to the operation

$$[\mathbf{V}^{<\alpha-1>}(\Theta, \varpi)]_{i,j} = [\mathbf{V}(\Theta, \varpi)]^{<\alpha-1>}_{j,i} \tag{34}$$

Clearly, when $\alpha = 2$, i.e., for Gaussian distributed signals and noise, the expression for the covariation matrix is identical to the well-known expression for the covariance matrix:

$$\mathbf{R}_X = \mathbf{V}(\Theta, \varpi)\mathbf{\Sigma}\mathbf{V}^H(\Theta, \varpi) + \sigma^2 \mathbf{I}, \tag{35}$$

where $\mathbf{\Sigma}$ is the signal covariance matrix.

When the amplitude response of the sensors equals unity, it follows that

$$[\mathbf{V}^{<\alpha-1>}(\Theta,\varpi)]_{i,j} = [\mathbf{V}(\Theta,\varpi)]^*_{j,i}, \quad (36)$$

and thus the covariation matrix can be written as

$$\mathbf{\Gamma}_X = \mathbf{V}(\Theta,\varpi)\mathbf{\Gamma}_S\mathbf{V}^H(\Theta,\varpi) + \gamma_n\mathbf{I}. \quad (37)$$

Observing (37), we conclude that standard subspace techniques can be applied to the covariation or the covariation coefficient matrices of the observation vector to extract the bearing information. In practice, we have to estimate the covariation matrix from a finite number of array sensor measurements. A proposed estimator for the covariation coefficient $\lambda_{x_i(t),x_j(t)}$ is called the *fractional lower order (FLOM) estimator* and is given by [8]

$$\hat{\lambda}_{x_i(t),x_j(t)} = \frac{\sum_{t=1}^n x_i(t)x_j^{<p-1>}(t)}{\sum_{t=1}^n |x_j(t)|^p} \quad (38)$$

for some $0 \leq p < \alpha$. We will refer to the new algorithm resulting from the eigendecomposition of the array covariation coefficient matrix as the **2-D Robust Covariation-Based MUSIC** or **2-D ROC-MUSIC**.

6. EXPERIMENTAL RESULTS

In this section, we show preliminary results on the resolution capability of 2-D ROC-MUSIC versus 2-D MUSIC as a function of the noise characteristic exponent α and the target separation. Complete simulation comparison results can be found in [12]. The array is linear with five sensors spaced half wavelength apart ($N = 5$). The number of transmitted pulses is $M = 10$. Three moving targets impinge on the array from directions $\mathbf{\Theta} = [-20^\circ, -40^\circ, 40^o]$ and they have Doppler values $\mathbf{D} = [-0.3, -0.2, 0.3]$). The number of snapshots available to the algorithms is $P = 1000$. The noise follows the bivariate isotropic stable distribution.

Since the alpha-stable family for $\alpha < 2$ determines processes with infinite variance, we define an alternative signal-to-noise ratio. Namely, we define the *Generalized SNR (GSNR)* to be the ratio of the signal power over the noise dispersion γ:

$$GSNR = 10\log(\frac{1}{\gamma M}\sum_{t=1}^{M}|s(t)|^2). \quad (39)$$

The GSNR is 22.3 dB ($\gamma = 1$). The characteristic exponent α of the additive noise is unknown to the 2-D ROC-MUSIC algorithm. The parameter p in the estimation of the covariation matrix (cf. (38)): was set equal to $p = 0.8$. Clearly, MUSIC can be thought as a special case of 2-D ROC-MUSIC with $p = 2$.

In Figures 2 and 3, space-time spectral estimates are shown for the 2-D ROC-MUSIC and 2-D MUSIC algorithms. Two types of alpha stable noise corresponding to two values of the characteristic exponent $\alpha = 1.5$ and $\alpha = 2.0$ (Gaussian) were used. We can see that the 2-D MUSIC method exhibits high-resolution performance only for the case of Gaussian additive noise while it cannot resolve the two closely moving targets when the additive noise is alpha-stable with $\alpha = 1.5$. On the other hand, the 2-D ROC-MUSIC method exhibits better resolution capabilities for non-Gaussian additive noise environments ($\alpha = 1.5$) and at the same time, performs well in Gaussian interference.

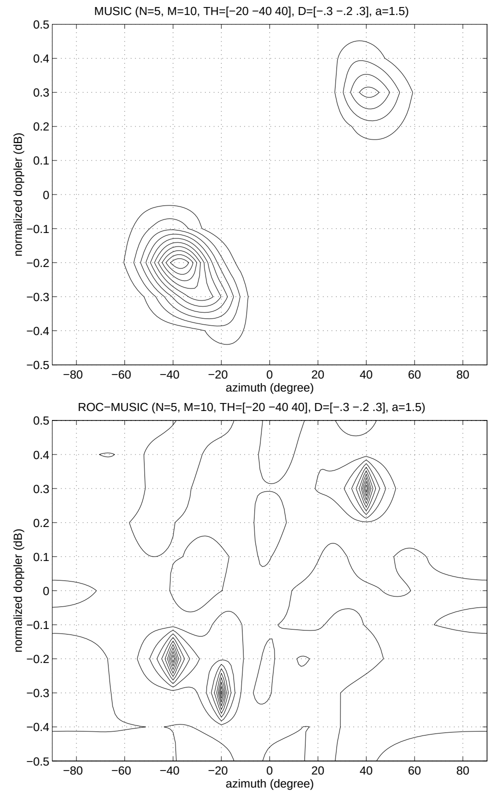

Figure 2: 2-D MUSIC and 2-D ROC-MUSIC angle-Doppler spectra ($N = 5$, $M = 10$, $\mathbf{\Theta} = [-20^\circ, -40^\circ, 40^o]$ $\mathbf{D} = [-0.3, -0.2, 0.3]$). Additive stable noise ($\alpha = 1.5$, $\gamma = 4$).

Figure 4 illustrates the variation of the algorithmic performance with respect to the spatial angle separation of the two closely spaced incoming targets for GSNR= 22.3 dB, ($\alpha = 1.5$). As expected, the resolution capability of both algorithms improves with increased angle separation between the two targets. But for a given probability of resolution, the 2-D ROC-MUSIC algorithm requires a lower angle separation threshold than the 2-D MUSIC algorithm.

7. CONCLUSIONS

We considered the problem of target-angle and Doppler estimation with an airborne radar employing space-time adaptive processing. We derived Cramér-Rao bounds on angle and Doppler estimator accuracy for the case of additive multivariate Cauchy interference of known underlying matrix. We introduced a new joint spatial- and doppler-frequency high-resolution estimation technique based on the fractional lower-order statistics of the measurements of a radar array.

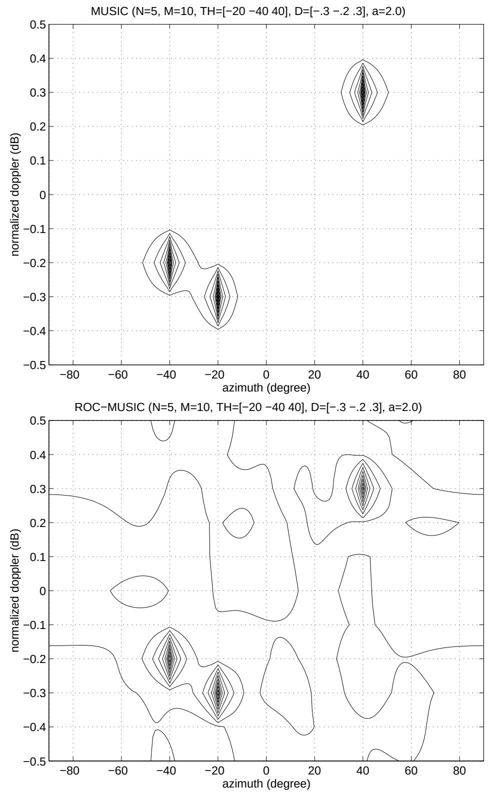

Figure 3: 2-D MUSIC and 2-D ROC-MUSIC angle-Doppler spectra ($N = 5$, $M = 10$, $\mathbf{\Theta} = [-20^o, -40^o, 40^o]$ $\mathbf{D} = [-0.3, -0.2, 0.3]$). Additive Gaussian noise ($\alpha = 2.0$, $\gamma = 4$).

We showed that the proposed 2-D ROC-MUSIC algorithm provides better angle/Doppler estimates than the 2-D MUSIC in a wide range of impulsive interference environments and it can be used in STAP radar applications.

8. ACKNOWLEDGMENTS

The authors would like to thank Dr. Jack Ma who provided the results on the modeling of real clutter data, shown in Figure 1.

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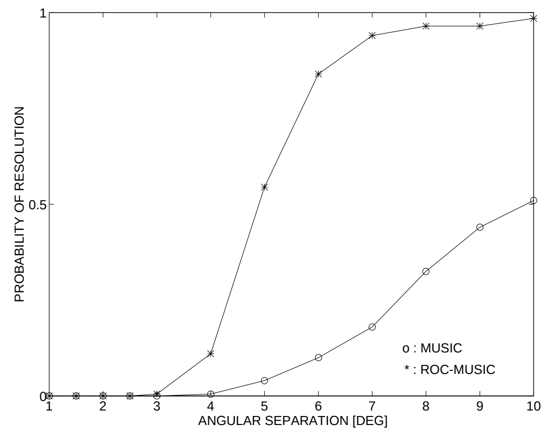

Figure 4: Probability of resolution as a function of the source angular separation, $\alpha = 1.5$.

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