ROBUST ADAPTIVE BEAMFORMING IN ALPHA-STABLE NOISE ENVIRONMENTS

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ABSTRACT

We develop an new adaptive beamforming technique based on fractional lower-order moment theory. The proposed adaptive beamformer adjusts the array response to a desired signal while discriminating against impulsive interference modeled as a stable process. Simulation results show that the new technique performs better in localizing a target both in space and Doppler, and thus offers the potential for improved airborne radar performance in space-time adaptive processing (STAP) applications.

1. INTRODUCTION

Spikes due to clutter sources such as ocean waves, and glints due to reflections from large flat surfaces such as buildings or vehicles are usually present in radar returns. A statistical model of impulsive interference has been proposed, which is based on the theory of symmetric alpha-stable ($S\alpha S$) random processes [1]. The model is of a statistical-physical nature and has been shown to arise under very general assumptions and to describe a broad class of impulsive interference. In particular, it was shown in [1] that the first order distribution of the amplitude of the radar return follows a SαS law, while the first-order joint distribution of the quadrature components of the envelope of the radar return follows an isotropic stable law. This model is mathematically more appealing than existing models for impulsive interference.

Recently, we developed robust techniques for source detection and localization in the presence of noise modeled as a complex isotropic stable process [2]. First, we presented optimal, maximum likelihood-based approaches and we introduced the Cauchy beamformer [3]. Also, we developed subspace methods based on fractional lower-order statistics, for radar applications where reduced computational cost is a crucial design parameter [4]. Source localization can be considered as the important initial step in adaptive array processing. Once the desired signal and interference directions are determined, adaptive statistically optimum beamforming maximizes the signal-to-noise ratio of the array output for optimal signal detection. The multiple sidelobe canceller (MSC) is perhaps the earliest statistically optimum beamformer introduced by Applebaum *et al.* [5]. Widrow and associates studied the adaptive antenna problem by minimizing the mean square error between the beamformer output and a reference signal [6]. Reed *et al.* concentrated on adaptive radar applications. They developed an adaptive processor which maximizes the probability of detection for a fixed false-alarm rate [7]. They also introduced a direct method of adaptive weight computation, based on the sample covariance matrix of the noise field, which provides rapid convergence [8].

In this paper, we introduce a new adaptive beamformer based on a constrained least mean p-norm (LMP) algorithm. The new LMP beamformer is designed to iteratively adapt the weights of a space-time sensor array so as to minimize the pth-order moment ($p < 2$) of the array output while maintaining a specific response in the look direction. The algorithm is applicable to array processing problems where the existing interference is impulsive in nature and can be modeled as an alpha-stable process. In Section 2, we present some necessary preliminaries on α-stable processes. In Section 3, we formulate the space-time adaptive processing (STAP) problem for airborne radar. In Section 4, we present second- and lower-order solutions to the adaptive beamforming problem. Finally, in Section 5, the improved performance of the proposed adaptive beamforming method in the presence of impulsive noise is demonstrated via simulation experiments.

2. MATHEMATICAL PRELIMINARIES

In this section, we introduce the statistical model that will be used to describe the additive noise. The model is based on the class of *isotropic* $S\alpha S$ distributions, and is well-suited for describing impulsive noise processes [1].

A complex random variable (r.v.) $X = X_1 + jX_2$ is isotropic $S\alpha S$ if X_1 and X_2 are jointly $S\alpha S$ and have a symmetric distribution. The characteristic function of X is

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given by

$$\varphi(\omega) = \mathcal{E}\{\exp(\jmath\Re[\omega X^*])\} = \exp(-\gamma|\omega|^\alpha), \quad (1)$$

where $\omega = \omega_1 + \jmath\omega_2$. The *characteristic exponent* α is restricted to the values $0 < \alpha \leq 2$ and it determines the shape of the distribution. The smaller the characteristic exponent α, the heavier the tails of the density. The *dispersion* γ ($\gamma > 0$) plays a role analogous to the role that the variance plays for second-order processes. Namely, it determines the spread of the probability density function around the origin.

Stable processes satisfy the stability property which states that linear combinations of jointly stable variables are indeed stable. They arise as limiting processes of sums of independent, identically-distributed random variables via the generalized central limit theorem. They are described by their characteristic exponent α, taking values $0 < \alpha \leq 2$. Gaussian processes are stable processes with $\alpha = 2$. Stable distributions have heavier tails than the normal distribution, possess finite pth order moments only for $p < \alpha$, and are appropriate for modeling noise with outliers.

3. STAP PROBLEM FORMULATION

Space-time adaptive processing (STAP) refers to multidimensional adaptive algorithms that simultaneously combine the signals from the elements of an array antenna and the multiple pulses of a coherent radar waveform, to suppress interference and provide target detection [9, 10, 11].

Consider a uniformly spaced linear array radar antenna consisting of N elements, which transmits a coherent burst of M pulses at a constant pulse repetition frequency (PRF) f_r and over a certain range of directions of interest. The array receives signals generated by q narrow-band moving targets which are located at azimuth angles $\{\theta_k;\ k = 1,\ldots,q\}$ and have relative velocities with respect to the radar $\{\nu_k;\ k = 1,\ldots,q\}$ corresponding to Doppler frequencies $\{f_k;\ k = 1,\ldots,q\}$. Since the signals are narrow-band, the propagation delay across the array is much smaller than the reciprocal of the signal bandwidth, and it follows that, by using a complex envelop representation, the array output can be expressed as [9]:

$$\mathbf{x}(t) = \mathbf{V}(\Theta, \varpi)\mathbf{s}(t) + \mathbf{n}(t), \quad (2)$$

where

- $\mathbf{x}(t) = [x_1(t), \ldots, x_{MN}(t)]^T$ is the array output vector (N: number of array elements, M: number of coherent pulses, and t may refer to the number of the coherent processing intervals (CPI's) available at the receiver);
- $\mathbf{s}(t) = [s_1(t), \ldots, s_q(t)]^T$ is the signal vector emitted by the sources as received at the reference sensor 1 of the array;
- $\mathbf{V}(\Theta, \varpi) = [\mathbf{v}(\vartheta_1, \varpi_1), \ldots, \mathbf{v}(\vartheta_q, \varpi_q)]$ is the space-time steering matrix, where $\vartheta_k = \frac{d}{\lambda_0}\cos(\theta_k)$ and $\varpi_k = \frac{f_k}{f_r}$ are the spatial and normalized Doppler frequencies, respectively;
- $\mathbf{v}(\vartheta_k, \varpi_k) = \mathbf{b}(\varpi_k) \otimes \mathbf{a}(\vartheta_k)$ is the space-time steering vector, where $\otimes$ denotes the Kronecker product and
  - $\mathbf{a}(\vartheta_k) = [1, e^{\jmath 2\pi\vartheta_k}, \ldots, e^{\jmath(N-1)2\pi\vartheta_k}]^T$ is the spatial steering vector;
  - $\mathbf{b}(\varpi_k) = [1, e^{\jmath 2\pi\varpi_k}, \ldots, e^{\jmath(M-1)2\pi\varpi_k}]^T$ is the temporal steering vector;
- $\mathbf{n}(t) = [n_1(t), \ldots, n_r(t)]^T$ is the noise vector.

Our objective is to adaptively adjust the array weights in real time to respond to a desired target (ϑ_0, ϖ_0) while discriminating against noise and interfering signals.

4. ADAPTIVE BEAMFORMING

In the following, we first describe an array weight adaptation algorithm which minimizes the noise power at the array output while maintaining a chosen frequency response in the angle-Doppler values of interest [12]. Then, we propose an adaptation process based on the minimization of the dispersion of the array output for applications where impulsive clutter and jamming signals can be modeled as stable processes. The use of the minimum dispersion criterion is justified because it has been shown that minimizing the dispersion is equivalent to minimizing the average magnitude and the probability of large estimation errors [13].

4.1. Second-Order Statistics Formulation

Among the methods of statistically optimum beamforming, the Constrained Least Mean-Squares (LMS) adaptive beamformer introduced by Frost [12] is of special interest. The basic idea is to constrain the response of the beamformer so that signals from the direction of interest are passed undistorted. The array weights $\mathbf{w}$ are chosen to minimize the output variance subject to appropriate linear constraints. The problem is formulated as follows:

$$\min\ E\{|y|^2\} \quad \text{such that} \quad \mathbf{C}^H\mathbf{w} = \mathbf{f}, \quad (3)$$

where $y(t)$ is the space-time filter output:

$$y(t) = \mathbf{w}^H\mathbf{x}(t). \quad (4)$$

By using Lagrange multipliers, the optimum space-time filter is shown to be [12]:

$$\mathbf{w}_{\text{opt}} = \mathbf{R}_x^{-1}\mathbf{C}[\mathbf{C}^H\mathbf{R}_x^{-1}\mathbf{C}]^{-1}\mathbf{f} \quad (5)$$

Direct application of (5) implies knowledge of the correlation matrix $\mathbf{R}_x$ of the array output vector. In most practical situations, the operating environment is non-stationary and an estimate of the weight vector $\mathbf{w}$ must be recomputed periodically. The beamformer weights adaptation procedure proposed in [12] is based on a gradient-descent constrained LMS algorithm:

$$\mathbf{w}(0) = \mathbf{F}$$
$$\mathbf{w}(t+1) = \mathbf{P}[\mathbf{w}(t) - \mu y^*(t)\mathbf{x}(t)] + \mathbf{F}, \quad (6)$$

where

$$\mathbf{P} = \mathbf{I} - \mathbf{C}[\mathbf{C}^H\mathbf{C}]^{-1}\mathbf{C}^H,$$

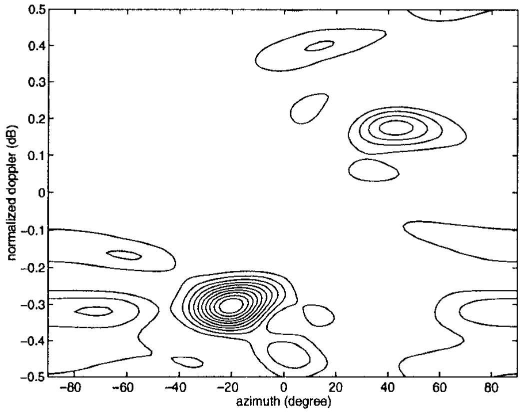

Figure 1: Adapted pattern of the LMS beamformer.

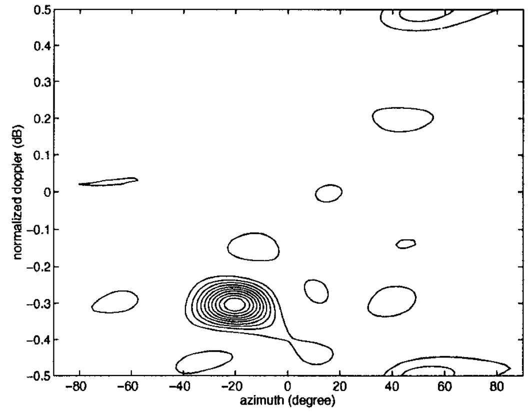

Figure 2: Adapted pattern of the LMP beamformer.

$$\mathbf{F} = \mathbf{C}[\mathbf{C}^H\mathbf{C}]^{-1}\mathbf{f},$$

and μ is a small positive step-size parameter which controls the rate of change of the weight vector $\mathbf{w}(t)$. Discussion on the convergence performance of the constrained LMS algorithm to the optimum weight vector can be found in [12].

4.2. Lower-Order Statistics Formulation

Since $S\alpha S$ processes do not possess finite pth order moments for $p \geq \alpha$, traditional beamforming techniques employing second- and higher-order moments cannot be applied in impulsive noise environments modeled under the stable law. Instead, properties of fractional lower-order moments (FLOM's) and covariations should be used. Hence, when we consider SαS processes of infinite variance, the above problem can be reformulated so that the pth-order moment of the array output is minimized subject to a set of linear constraints:

$$\min \; E\{|y|^p\} \quad \text{such that} \quad \mathbf{C}^H\mathbf{w} = \mathbf{f} \quad (p < \alpha). \qquad (7)$$

Unlike the optimization problem described in the previous section, there is no closed form solution for the problem shown in (7). Therefore, an iterative scheme based on a stochastic gradient method can be used to adapt the array weights. We will refer to the beamformer described in (7) as the *Least Mean p-Norm (LMP)* beamformer. The adaptation procedure for the LMP adaptive beamformer weights can be derived to be as follows:

$$\mathbf{w}(0) = \mathbf{F}$$

$$\mathbf{w}(t+1) = \mathbf{P}[\mathbf{w}(t) - \mu|y(t)|^{p-2}y^*(t)\mathbf{x}(t)] + \mathbf{F}, \qquad (8)$$

where

$$\mathbf{P} = \mathbf{I} - \mathbf{C}[\mathbf{C}^H\mathbf{C}]^{-1}\mathbf{C}^H,$$

$$\mathbf{F} = \mathbf{C}[\mathbf{C}^H\mathbf{C}]^{-1}\mathbf{f}.$$

When $p = 1$, the above algorithm is the well-known signed LMS algorithm. The L_p norm has been used in the past within several contexts such as deconvolution problems in seismic applications [14, 15] and spectral estimation [16]. In radar and sonar applications it can offer better localization results in certain impulsive noise types, as shown in the next section.

5. EXPERIMENTAL RESULTS

In this section, we present preliminary comparison results between the two beamforming techniques described in the paper. The array is linear with five sensors spaced a half-wavelength apart ($N = 5$). The number of transmitted pulses is $M = 10$. The target of interest is located at angle $\theta_S = -20^o$ and has normalized Doppler $\varpi_S = -0.3$. An alpha-stable ($\alpha = 1.5$) distributed interfering signal is present at angle $\theta_I = 40^o$ with normalized Doppler $\varpi_I = 0.2$. The noise follows the bivariate isotropic stable distribution with characteristic exponent $\alpha = 1.5$ ($SNR = -5dB$). Both beamformers are constrained to have unit response at the direction and Doppler of the target of interest:

$$\mathbf{C}^H\mathbf{w} = \mathbf{f} \;\leadsto\; \mathbf{v}^H(\vartheta_S, \varpi_S)\mathbf{w} = 1.$$

Both methods are initialized at the space-time steering vector of the interference target

$$\mathbf{w}(0) = \mathbf{v}(\vartheta_I, \varpi_I).$$

In Figures 1 and 2, we plot isosurfaces of the adapted patterns:

$$P\mathbf{w}(\vartheta_k, \varpi_k) = |\mathbf{w}^H\mathbf{v}(\vartheta_k, \varpi_k)|^2. \qquad (9)$$

The two-dimensional angle-Doppler frequency responses of the two methods are shown for the 200th iteration. The principal plane cuts at target Doppler ($\varpi_S = -0.3$) are given in Figures 3 and 4 for several iterations. As we can see, the LMP beamformer ($p = 1$) places a strong null at the location of the impulsive interference.

6. CONCLUSIONS

A new adaptive beamformer based on lower-order statistics has been introduced to adjust an array response to a desired

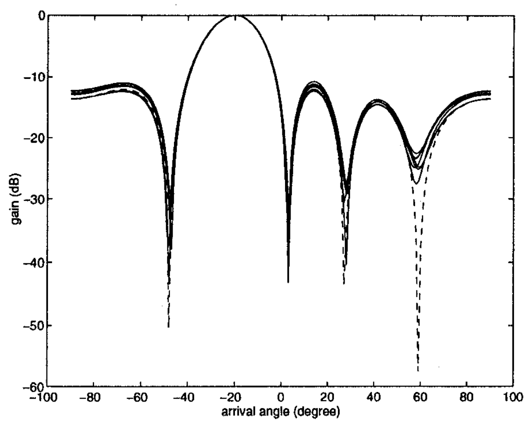

Figure 3: Principal plane cuts at target Doppler for the LMS beamformer.

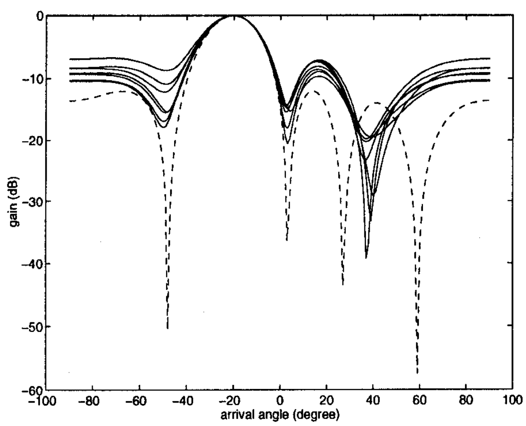

Figure 4: Principal plane cuts at target Doppler for the LMP beamformer.

signal while discriminating against impulsive interference. Simulation results showed that the new technique performs better in localizing a target both in space and Doppler, and thus offers the potential for improved airborne radar performance.

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