Radar CFAR Thresholding in Heavy-Tailed Clutter and Positive Alpha-Stable Measurements*

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Abstract

This paper shows how to apply Rohling's order-statistics constant false alarm rate (OS CFAR) algorithm, developed for a Rayleigh background, to the case of heavy-tailed clutter background. In particular, we study the performance of the OS CFAR processor when the output measurements of the square-law detector can be modeled as Positive Alpha-Stable ($P\alpha S$) random variables with a shape parameter (characteristic exponent) equal to 0.5. We derive the exact expressions for the detection and false alarm probabilities of the OS and cell averaging (CA) CFAR detectors, and we compare their performance by means of their corresponding receiver operating characteristics.

1. Introduction

In radar systems, target detection involves the comparison of the absolute value (linear detector) or the squared magnitude (square-law detector) of the coherent receiver output signals with a certain threshold. It is extremely important for the detection radar to be able to operate in nonstationary background noise environments with a predetermined constant level of performance. In signal processing terms, the goal is to maintain a constant false alarm rate (CFAR) when the background noise level fluctuates.

It is impossible to maintain CFAR performance in nonstationary environments with a detection scheme that employs a fixed threshold. Hence, CFAR detectors have been designed that set the threshold adaptively according to local information on the background noise. More specifically, CFAR detectors estimate characteristics of the noise, such as its shape and scale, by processing a window of reference cells surrounding the cell under test in range and/or frequency.

The cell averaging (CA) approach is such an adaptive procedure. The CA CFAR detector uses the maximum likelihood estimate of the noise power to set the threshold adaptively on the assumption that the background noise amplitude samples are independent, identically distributed (iid) random variables with a Rayleigh probability density function (pdf). The CA CFAR detector is the optimum CFAR detector in terms of detection probability in homogeneous noise background with Rayleigh statistics [3].

However, the assumption of a uniform clutter situation within the reference window is hardly ever maintained in practice due to transitions in clutter characteristics, clutter areas of small extensions, and interfering target echoes occurring within the reference window of the radar test cell. Because the performance of the CA CFAR detector degrades considerably in non-homogeneous situations, Rohling modified the common CA CFAR technique by replacing the arithmetic averaging estimator of clutter power with a new module based on order-statistics (OS) [7]. The OS CFAR processor determines the detection threshold based on the ranked reference cells, a procedure that protects against non-homogeneous situations caused by interfering targets and clutter edges.

The performances of the various CFAR procedures in a homogeneous noise background are compared based on the corresponding relative "additional detectability loss" or "CFAR loss." Naturally, the false alarm probability is a function of the threshold but it also depends on the assumed statistics of the underlying noise. In many situations the

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Gaussian distribution cannot appropriately model the clutter returns, whose amplitude distribution is far more impulsive that the one predicted by the Rayleigh law. Today, a number of studies suggest that clutter modeling is more accurately achieved by considering families of density functions, which depend on a scale parameter ruling the clutter "power" and a shape parameter ruling the behavior of the distribution in the high-amplitude tail [9, 4, 2]. Hence, it is important to ensure CFAR performance against variations of both parameters, even at a cost of higher CFAR loss than the single-parameter techniques.

The design and performance evaluation of biparametric CFAR processors for Weibull background has been studied in the past. Weber and Haykin have proposed a two-parameter algorithm in which the threshold is obtained from two ranked background samples [10]. To achieve a lower CFAR loss algorithm, Ravid and Levanon used the maximum likelihood method to estimate the shape and scale parameters of the Weibull distribution [6].

In this paper, we study the design and performance of CFAR processors for the case of positive alpha-stable ($P\alpha S$) measurements. $P\alpha S$ processes have been shown to be related to the power or energy flow in many physical processes including radar sea clutter modulation [5]. We consider OS CFAR processing for the case of the Pearson (or Lévy) distribution. We show that the OS processor gives indeed rise to a CFAR detector for Pearson-distributed, heavy tailed output signals and we study the false alarm probability of the resulting system.

2. Order Statistics CFAR (OS CFAR) for Heavy-Tailed Measurements

In a typical radar detection scheme, the decision is realized by the following thresholding operation:

$$e(Y) = \begin{cases} \text{target present}, & \text{if } Y \geq S \\ \text{target absent}, & \text{if } Y < S \end{cases} \quad (1)$$

The task of the CFAR system is to provide in an adaptive and systematic manner the threshold value needed. Various CFAR systems are distinguished by the way this threshold is obtained. In existing CFAR systems, target detection is performed by using the sliding window technique. When calculating the threshold, two aspects must be considered. One is the average clutter power level in the reference window and the other is the required false alarm probability, P_{fa}. Accordingly, the threshold S is calculated as the product

$$S = TZ \quad (2)$$

where Z is the estimate of the average clutter power and T is a scaling factor used to achieve a certain P_{fa}. Different CFAR procedures are characterized by the method

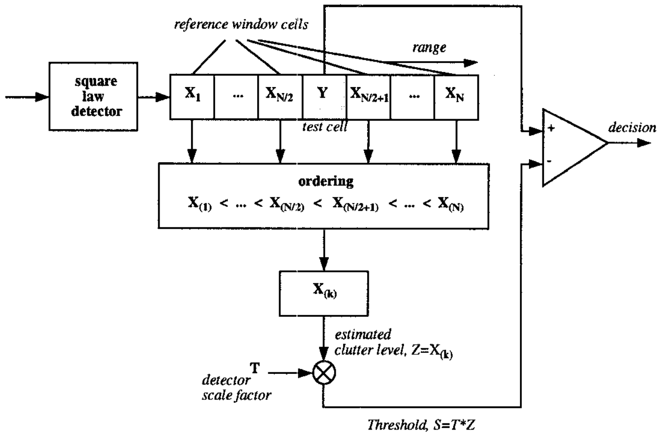

Figure 1. Block diagram of the OS CFAR detector structure.

used to estimate the clutter level. In the following, we first summarized the OS CFAR results for the case of Rayleigh clutter. Then, we study the OS CFAR processor for the case of Pearson-distributed data.

2.1. OS CFAR for Pearson-Distributed Data

Rohling has proposed a CFAR algorithm based on order statistics that exhibits reduced sensitivity to non-homogeneous environments as compared to the cell averaging CFAR method. The procedure that takes place in an OS CFAR system is shown in Figure 1. The data in a reference window around the test cell are used to calculate the decision threshold. The first step is to obtain a measure of the clutter level Z. The N reference cells are ordered according to their magnitude

$$X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(N-1)} \leq X_{(N)} \quad (3)$$

and Z is taken to be equal to $X_{(k)}$, the kth largest sample. (In the case of the cell averaging CFAR processor, Z is the average of the reference window values). Then, Z is multiplied by a scaling factor T which depends on the desired false alarm probability for a given window of size N when the background noise is homogeneous. The resulting product TZ is directly used as the threshold value. In practice, the measurements within a certain number of guard cells, directly adjacent to either side of the test cell, are not taken into consideration when estimating Z because signal energy can spill over into the adjacent range cells and may affect the procedure.

In the following, we perform the analysis of a detector based on order statistics when the cell samples follow the Pearson distribution. We demonstrate that the OS processor in Figure 1 is indeed a CFAR processor for Pearson-distributed data by showing that the false alarm probability P_{fa} is independent of the dispersion γ of the measurements.

2.1.1 Probability of False Alarm P_{fa}

Assume that $X_1, \ldots, X_N$ follow the Pearson distribution with probability density function (pdf) [8]

$$p_{X_i}(x) = \begin{cases} \frac{\gamma}{\sqrt{2\pi}} \frac{1}{x^{3/2}} e^{-\gamma^2/2x}, & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

and cumulative density function (cdf)

$$P_{X_i}(x) = Pr\{X_i \leq x\} = \begin{cases} 2(1 - \Phi(\frac{\gamma}{\sqrt{x}})), & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where γ is the dispersion or scale parameter of the distribution. The dispersion determines the spread of the density, much in the same way that the standard deviation determines the spread of the Rayleigh density. $\Phi(x)$ denotes the cumulative density function of the standard Gaussian distribution, $N(0, 1)$.

With these assumptions, for any given P_{fa} the decision threshold S as well as the scaling factor T can be derived as follows. P_{fa} indicates the probability that a noise random variable Y_0 is interpreted as target echo during the thresholding decision (1). This probability is given by

$$P_{fa} = Pr\{Y_0 \geq TZ\}. \tag{6}$$

In order to calculate P_{fa} according to (6), both the pdfs of Y_0 and of Z must be known. Since $Z = X_{(k)}$ is an ordered statistic value, its pdf can be determined based on the pdf, $p_{X_i}(x)$, and the cdf, $P_{X_i}(x)$, of the samples according to [1]

$$p_{X_{(k)}}(x) = \frac{[1 - P_X(x)]^{N-k} \, [P_X(x)]^{k-1} \, p_X(x)}{B(k, N - k + 1)} \tag{7}$$

where $B(a, b)$ is the *beta function*. Hence, by using (4), (5), and (7), the pdf of the kth value of the ordered statistic for Pearson-distributed random variables $X_1, \ldots, X_N$ is given by

$$\begin{aligned} p_{X_{(k)}}(x) &= \frac{1}{B(k, N-k+1)} [2\Phi(\gamma/\sqrt{x}) - 1]^{N-k} \, 2^{k-1} \\ & [1 - \Phi(\gamma/\sqrt{x})]^{k-1} \frac{\gamma}{\sqrt{2\pi}} \frac{1}{x^{3/2}} e^{-\gamma^2/2x}. \end{aligned} \tag{8}$$

Now, the P_{fa} can be calculated for a fixed factor T

$$\begin{aligned} P_{fa} &= Pr\{Y_0 \geq TX_{(k)}\} \\ &= \int_0^\infty Pr\{Y_0 \geq Tx\} \, p_k(x) \, dx \end{aligned} \tag{9}$$

By using (8) and the substitution $y = \gamma/\sqrt{x}$, the false alarm probability can also be expressed as

$$\begin{aligned} P_{fa}^{OS} = \sqrt{\frac{2}{\pi}} \frac{1}{B(k, N-k+1)} \int_0^\infty \text{erf}(y/\sqrt{2T}) \\ [\text{erf}(y/\sqrt{2})]^{N-k} \, [\text{erfc}(y/\sqrt{2})]^{k-1} \, e^{-y^2/2} \, dy \end{aligned} \tag{10}$$

where

$$\text{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2} \, dt \tag{11}$$

is the *error function*, $\text{erfc}(y) = 1 - \text{erf}(y)$ is the complementary error function.

The important conclusion from (10) is that the false alarm probability is controlled by the scaling factor T and it does not depend on the dispersion parameter γ of the Pearson-distributed parent population. As a consequence, the OS method may actually be considered a CFAR method for Pearson background. Naturally, the use of order statistics does not define a single CFAR method but rather a family of CFAR methods parameterized by the rank order index k. For a given order statistic $X_{(k)}$, a distinct CFAR processor is established.

2.1.2 Probability of Detection P_d

We now compute the detection probability P_d of the OS CFAR detector. We will consider the case of a Rayleigh fluctuating target in a heavy-tailed background noise scenario when the CFAR processor is preceded by a square-law detector, as in Figure 1.

For a Rayleigh fluctuating target with parameter σ_s^2, the output of a square-law detector has an exponential pdf. Hence, for the case of the "target present" hypothesis, the test-cell measurement is exponentially distributed:

$$p_{Y_1}(y) = \begin{cases} \frac{1}{2\sigma_s^2} e^{-y/2\sigma_s^2}, & y \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

The corresponding cumulative density function (cdf) is

$$P_{Y_1}(y) = \begin{cases} 1 - e^{-y/2\sigma_s^2}, & y \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

Hence, the probability of the random variable Y_1 exceeding the threshold is

$$P_d = Pr\{Y_1 \geq TX_{(k)}\} \tag{14}$$

Assuming the presence of heavy-tailed clutter that leads to Pearson-distributed reference-cell data $X_1, \ldots, X_N$, the density of the kth-order statistic is given by expression (8), and therefore P_d can be written as:

$$P_d^{OS} = \int_0^\infty Pr\{Y_1 \geq Tx\} \, p_k(x) \, dx$$

$$= \sqrt{\frac{2}{\pi}} \frac{1}{B(k, N-k+1)} \int_0^{\infty} e^{-\frac{\gamma^2}{\sigma_s^2}\frac{T}{2y^2}}$$
$$[\mathrm{erf}(y/\sqrt{2})]^{N-k} \, [\mathrm{erfc}(y/\sqrt{2})]^{k-1} \, e^{-y^2/2} \, dy \quad (15)$$

As we can see in (15), the detection probability is a function of the ratio of the clutter dispersion γ over the power parameter of the Rayleigh fluctuating target σ_s. To obtain the detection probability for a given probability of false alarm, one first has to solve (10) with respect to the required threshold, then substitute this value into (15) and numerically integrate the expression.

2.1.3 CA CFAR for Pearson-Distributed Data

The OS CFAR processor has been introduced as an improvement on the cell averaging (CA) CFAR method, which selects the average of the reference cell values instead of the kth-order statistic as a measure of the clutter level Z, i.e, $Z = (1/N)\sum_{i=1}^{N} X_i$. It is of interest to compare the P_{fa} of the two processors for the case of Pearson-distributed measurements. When using cell averaging, Z is a Pearson-distributed random variable since it is the average sum of N Pearson-distributed random variables. The dispersion of Z is equal to $\gamma_Z = \sqrt{N}\gamma_{X_i}$. Hence, the pdf of Z is given by

$$p_Z(z) = \begin{cases} \frac{\sqrt{N}\gamma}{\sqrt{2\pi}} \frac{1}{z^{3/2}} e^{-N\gamma^2/2z}, & z \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

Then, using again (6), the P_{fa} can be expressed as

$$\begin{aligned} P_{fa}^{CA} &= Pr\{Y_0 \geq TZ\} \\ &= \int_0^{\infty} Pr\{Y_0 \geq Tz\} \, p_Z(z) \, dz \\ &= \sqrt{\frac{2N}{\pi}} \int_0^{\infty} \mathrm{erf}(y/\sqrt{2T}) \;\; e^{-Ny^2/2} \, dy. \quad (17) \end{aligned}$$

From (17) we see that cell averaging is also a CFAR method for Pearson background. The corresponding probability of detection for the case of a Rayleigh target can be expressed as:

$$\begin{aligned} P_d^{CA} &= Pr\{Y_1 \geq TZ\} \\ &= \int_0^{\infty} Pr\{Y_1 \geq Tz\} \, p_Z(z) \, dz \\ &= \sqrt{\frac{2N}{\pi}} \int_0^{\infty} e^{-\frac{\gamma^2}{\sigma_s^2}\frac{T}{2y^2}} \; e^{-Ny^2/2} \, dy. \quad (18) \end{aligned}$$

We see again that the detection probability is a function of the ratio of the clutter dispersion γ over the power parameter of the Rayleigh fluctuating target σ_s.

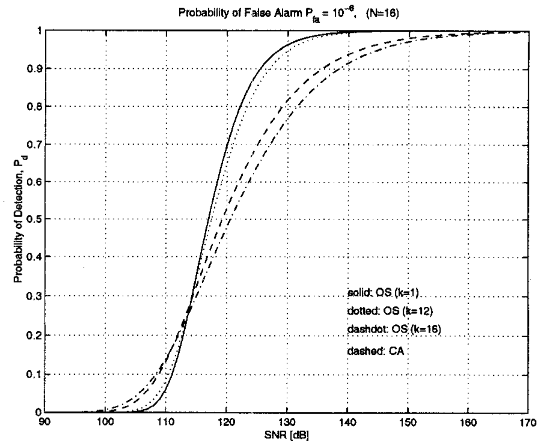

Figure 2. Detection probability of OS and CA CFAR processors in Pearson background as a function of the SNR. The probability of false alarm is equal to $P_{fa} = 10^{-6}$.

2.2. Results and Discussion

In this section, the detection performance of the OS CFAR processor in Pearson-distributed data in homogeneous situations is shown and compared with the performance of the CA CFAR detector. The detection performances are obtained by varying the number of the reference cells N and, for the case of the OS CFAR processor, the rank order index k. For a certain P_{fa} value, we used expressions (10) and (17) to obtain the appropriate threshold scaling factors T for the OS and CA detectors, respectively. Then, we numerically integrated (15) and (18) to calculate the corresponding P_d values.

In Figure 2, the probability of detection P_d is plotted versus the signal to noise ratio (SNR) for a probability of false alarm $P_{fa} = 10^{-6}$. The SNR is defined as

$$\mathrm{SNR} = 20 \log \frac{\sigma_s}{\gamma}, \quad (19)$$

where σ_s is the parameter of the Rayleigh fluctuating target and γ is the dispersion parameter of the Pearson-distributed output of the square-law detector due to the background heavy-tailed clutter.

As it can be seen in Figure 2, for high SNR values, i.e., for SNR higher than approximately 115dB, the detection performance of the OS CFAR processor is improved for smaller values of the rank order index k. On the other hand, for low SNR values, the detection performance of the OS CFAR processor is better for larger values of the rank order index k. As a middle ground rank order, the value of $k = 3N/4$ seems to result in only a negligible additional

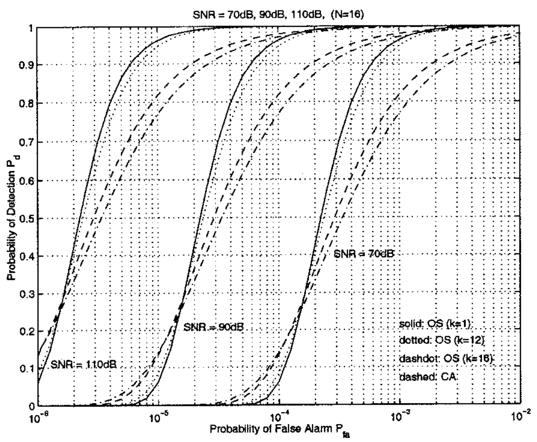

Figure 3. Receiver Operating Characteristics: Detection probability of OS and CA CFAR processors in Pearson background versus the probability of false alarm.

CFAR loss. The detection performance of the CA CFAR processor is also shown in the figure with a dashed line. It is apparent that for an appropriate choice of the rank order index k, the corresponding OS CFAR processor outperforms CA CFAR for the case of homogeneous Pearson-distributed background clutter.

Figure 3 depicts the receiver operating characteristic (ROC) curves for the OS and CA CFAR processors employing $N = 16$ reference cells and operating in SNR equal to 70dB, 90dB, and 110dB. Observing Figure 3 we see that, for an SNR value of 70dB and for the range of P_{fa} values between 1.5×10^{-4} to 10^{-2}, the OS CFAR processor employing the first-order statistic (minimum) of the reference cells exhibits the best performance. On the other hand, for P_{fa} values less than 1.5×10^{-4}, the other extremum, i.e., the last-order statistic (maximum) of the reference cells has the best P_d. Similar observations can be made for the case of the other operating SNR values for the corresponding P_{fa}. Again, the performance curve of the CA CFAR processor falls in between the OS family of curves, and a good compromise order statistic corresponds to a value of k equal to $3N/4$. In Figure 3 we see that as the SNR value decreases, the shape of the ROC curves stays the same but the performance curves move to the right towards higher P_{fa} values, as expected.

2.3. Future Work

In this work, we derived the theoretical analysis on the performance of the OS and CA CFAR processors for the case of Pearson-distributed data. We showed that the two processors are indeed CFAR with respect to the scale (dispersion) parameter γ, for the case of $P\alpha S$ distributed measurements with a shape parameter α known *a priori* to be equal to 0.5.

When the characteristic exponent of the $P\alpha S$ distribution is unknown, we need to introduce a two-parameter estimation CFAR algorithm. The adaptive threshold will effectively be based on the estimation of the scale (dispersion) and shape (characteristic exponent) parameters using the method of negative-order moments proposed by Pierce in [5]. The method will be tested and the corresponding CFAR loss will be related to the variance of the estimated parameters.

To reduce the variance and the associated CFAR loss, a CFAR algorithm in which the parameters are estimated using the maximum likelihood (ML) method will be developed. Naturally, the ML algorithm will be more computationally intensive than the approaches based on moments or order-statistics. But even if it is not efficient enough to be implemented, the expected performance of the ML-based CFAR method can serve as a comparative reference for the simpler algorithms.

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