A NEW PHASE GRADIENT AUTOFOCUS TECHNIQUE FOR HIGH RESOLUTION IMAGE FORMATION BASED ON FRACTIONAL LOWER-ORDER STATISTICS

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ABSTRACT

This paper addresses the problem of autofocusing in Synthetic Aperture Radar (SAR) imagery by introducing techniques that achieve robust image restoration in the presence of severe heavy-tailed clutter and noise. We extend the current state-of-the-art Phase Gradient Autofocus (PGA) method by employing Fractional Lower-Order Statistics (FLOS) of the phase history data. The introduced FLOS-based PGA method mitigates the effects of impulsive additive clutter in the measurements and achieves high-resolution SAR image formation. The proposed approach is compared to conventional processing by using actual radar imagery data.

1. INTRODUCTION

In many coherent radar signal processing applications it is assumed that certain relative motions exist between the radar system and its target [1]. For example, in strip-mapping SAR the radar's aircraft platform is assumed to travel in a perfectly straight line at constant velocity. In practice, the motion between the radar and the target does not meet this ideal. Even under the best circumstances, turbulence and vibration will cause the aircraft's flight path to deviate arbitrarily from the assumed coordinates. As a result, motion compensation mechanisms are required to alleviate the effects of the radar's deviation from the assumed ideal path.

Motion compensation is a critical function that helps to achieve high resolution in SAR imagery. More specifically, increasing system resolution rapidly increases motion compensation accuracy requirements. As pointed out in [2], uncompensated motion results in serious image defocus caused by the presence of quadratic and higher-order phase errors. Autofocus techniques use the SAR data itself to estimate and remove the phase errors from the same data. Accurate extraction of motion-induced phase errors from SAR data is dependent on the specific scene imaged and on the particular autofocus algorithm used. The majority of the existing autofocus algorithms assume that the phase errors are space-invariant in azimuth and apply the same correction vector to all scatteres in the image. Most importantly, their underlying mathematical methodology is entirely based on the Fourier transform and on the second-order statistics of the data.

The Phase Gradient Autofocus algorithm, which is the current state-of-the-art SAR autofocus method, estimates the entire phase error function by locating the peak amplitude function within each azimuth over all compressed range bins and by computing its first derivative using the derivative property of the Fourier transform [3, 4]. The integration of the phase gradient provides the phase error function estimate. The PGA averages the estimates over many range bins in a weighted least-squares sense. The PGA algorithm is not model-based, i.e., implementation does not require a polynomial expression for the phase error function. However, it may be sensitive to the presence of clutter/noise and may suffer from aliasing effects due to the derivative computation with the Fourier transform. To address these problems, a maximum likelihood version of the PGA was introduced, based on formal optimal estimation theory [5].

Existing autofocus techniques for estimating Doppler shifts and/or pulse delays from echoed signals are based on second-order statistics and cross-correlation operations. A serious problem arises when the autofocusing has to be performed in the presence of heavy-tailed clutter. This paper shows that second-order based autofocus methods operating in severe clutter backgrounds provide Doppler shifts and/or pulse delay estimates that may be severely biased. In the presence of severe clutter, the frequency shifts of two adjacent azimuth spectra can be estimated more accurately by methods that have inherent capability of enhancing output signal-to-clutter ratio and of being robust in the presence of mismatches in the assumed conditions of clutter.

In this paper, we extend the current state-of-the-art PGA method by employing Fractional Lower-Order Statistics of the phase history data. The paper is organized as follows: In Section 2, we present some necessary preliminaries on α-stable theory. In Section 3, we present the FLOS-based extension of the PGA method. Finally, in Section 4, we demonstrate the benefits of the proposed approach by means of simulations and we compare the new FLOS-based method to conventional processing with actual SAR data.

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2. ALPHA-STABLE MODELS

It is recognized that effective phase error correction of SAR imagery in the presence of background clutter can be achieved only on the basis of appropriate statistical modeling [5]. The probabilistic models currently employed are based almost exclusively on the Gaussian distribution and, in many real life applications, they fail to provide reasonable agreement between the theoretical and the experimentally observed statistics [6, 7]. Therefore, SAR autofocusing signal processing design cannot be reliably based exclusively on Gaussian models but rather on families of possibly heavy-tailed distributions.

The class of symmetric alpha-stable ($S\alpha S$) distributions, a natural generalization of the Gaussian distribution, has some important characteristics that make it very attractive for modeling a wide range of signal and noise environments. This class of distributions is best defined by its characteristic function:

$$\varphi(\omega) = \exp(\jmath\delta\omega - \gamma|\omega|^{\alpha}), \tag{1}$$

where α is the *characteristic exponent* restricted to the values $0 < \alpha \leq 2$, δ is the *location parameter*, and γ ($\gamma > 0$) is the *dispersion* of the distribution. The dispersion parameter γ determines the spread of the distribution while the characteristic exponent α determines the shape of the distribution and it is an index of non-Gaussianity. The smaller the α, the less Gaussian the data ($\alpha = 2$ coincides with the Gaussian distribution). Alpha-stable densities obey two important properties which further justify their role in data modeling: They satisfy the stability property, which states that linear combinations of jointly stable variables are indeed stable, and they arise as limiting processes of sums of independent, identically-distributed random variables via the generalized central limit theorem.

An important difference between the Gaussian and the other distributions of the alpha-stable family is that only moments of order less than α exist for the non-Gaussian alpha-stable family members. Since alpha-stable processes do not possess finite p-th order moments for $p \geq \alpha$, covariances do not exist on the space of alpha-stable random variables. Instead, a quantity called *covariation* plays an analogous role for statistical signal processing problems involving stable processes to the role played by covariance in the case of second-order processes. The covariation of X and Y is defined by

$$[X, Y]_\alpha = \frac{E\{XY^{<p-1>}\}}{E\{|Y|^p\}}\gamma_Y, \tag{2}$$

for every $1 \leq p < \alpha$ and where we use throughout the convention $Y^{<\beta>} = |Y|^{\beta-1}Y^*$. The covariation of jointly $S\alpha S$ r.v.'s X and Y is not generally symmetric and has the following properties [6]. For any complex constants a, b, and c,

P1 If X_1, X_2 and Y are jointly $S\alpha S$, then

$$[aX_1 + bX_2, Y]_\alpha = a[X_1, Y]_\alpha + b[X_2, Y]_\alpha$$

P2 If Y_1 and Y_2 are independent and Y_1, Y_2 and X are jointly $S\alpha S$, then

$$[aX_1, bY_1 + cY_2]_\alpha = ab^{<\alpha-1>}[X_1, Y_1]_\alpha + ac^{<\alpha-1>}[X_1, Y_2]_\alpha$$

P3 If X and Y are independent $S\alpha S$, then $[X, Y]_\alpha = 0$.

3. THE FLOS-BASED PHASE GRADIENT AUTOFOCUS METHOD (PGA-FLOS)

The PGA algorithm consists of several critical steps that include: (i) center (circular) shifting, (ii) windowing, (iii) phase difference estimation, and (iv) iterative correction. The center shifting and windowing operations are common to all PGA versions [5]. First, PGA selects the strongest target on each range line and circularly shifts it to the scene center. This operation preserves the effects of the phase error on the selected target while simultaneously removing any linear phase component associated with the target. The shifting operation creates a new image where all the targets are aligned and stacked in the center of the scene. The intent of the windowing step is to preserve the phase error information contained in the center-shifted targets while at the same time rejecting information from all other surrounding clutter and targets. After windowing, a Fourier transform is applied in the cross-range dimension on each range line.

The heart of the PGA method is the phase error estimation function. Consider samples of range-compressed data for which there are N range lines and M aperture positions, so that a total of $M \times N$ samples are used. In our model, the real and imaginary (I and Q) components of the complex reflectivity of a point target on each range line are treated as alpha-stable ($S\alpha S$) random variables. In addition, the clutter reflectivity I and Q values are also modeled as independent $S\alpha S$ random variables. All clutter and target components are mutually independent across all values of range and cross range.

In the range compressed domain, the phase errors are modeled such that between the lth and first aperture positions there exists a phase difference, ψ_l, that is constant across all range lines. The model for the center-shifted, windowed and transformed data at the kth range bin is then taken to be:

$$\begin{aligned} g_{k1} &= a_k + n_{k1} \\ g_{k2} &= a_k e^{j\psi_2} + n_{k2} \\ &\vdots \\ g_{kM} &= a_k e^{j\psi_M} + n_{kM} \end{aligned} \tag{3}$$

where $k = 1, \ldots, N$ and g_{kl} represents the sample aperture position l on the kth range bin. The phase at the first aperture position is arbitrarily assigned the value of zero. A direct approach to the phase estimation problem is to use all $N \times M$ data points to derive a maximum-likelihood (ML) estimate of ψ_i. A simpler approach that can be implemented in real SAR systems, is to use data on two adjacent pulses at a time to estimate the *phase difference* between them. These differences may then be integrated to obtain an estimate for the entire ψ_i. The introduced PGA-FLOS algorithm, is based on the *covariation* of the sample at the ith aperture position, g_{ki}, with the sample at the jth aperture position, g_{kj}.

Considering the covariation of g_{ki} with g_{kj} and by using properties $P1$ to $P3$ we get:

$$\begin{aligned} [g_{ki}, g_{kj}]_\alpha &= [a_k e^{j\psi_i} + n_{ki}, a_k e^{j\psi_j} + n_{kj}]_\alpha \\ &= e^{j\psi_i}[a_k, a_k e^{j\psi_j} + n_{kj}]_\alpha + [n_{ki}, a_k e^{j\psi_j} + n_{kj}]_\alpha \\ &= e^{j\psi_i}(e^{j\psi_j})^{<\alpha-1>}[a_k, a_k]_\alpha + e^{j\psi_i}[a_k, n_{kj}]_\alpha + \\ &\quad (e^{j\psi_j})^{<\alpha-1>}[n_{ki}, a_k]_\alpha + [n_{ki}, n_{kj}]_\alpha \\ &= \gamma_{a_k} e^{j(\psi_i - \psi_j)} + \delta_{ij}\gamma_{n_k} \end{aligned} \tag{4}$$

where γ_{a_k} is the dispersion of a_k, γ_{n_k} is the clutter dispersion of n_k, and δ_{ij} is the Kronecker delta function. Hence, by using two adjacent-pulse samples (i.e., for $i = m$ and $j = m - 1$) and all the available ranges, the covariation-based estimator of the difference of the phase errors, $\Delta\psi_m \equiv \psi_m - \psi_{m-1}$ is

$$\Delta\psi_m^{(PGA-FLOS)} = \arg(\sum_{k=1}^{N}[g_{k,m-1}, g_{k,m}]_\alpha) \qquad (5)$$

where $\arg(x)$ denotes the principal value of the angle of the complex quantity x, computed in the interval $[-\pi, \pi]$.

In practice, we have to estimate the covariation in (5) from the two adjacent-pulse samples. One such covariation estimator, based on the fractional lower-order moments (FLOM's) of the stable process, is given by [7]:

$$\hat{\Delta\psi}_m^{(PGA-FLOS)} = \arg(\sum_{k=1}^{N}(g_{k,m-1})^{<p_1>}(g_{k,m}^*)^{<p_2>}), \qquad (6)$$

where $0 \leq p_1 < \frac{\alpha}{2}$ and $0 \leq p_2 < \frac{\alpha}{2}$. Hence, the proposed PGA-FLOS estimator is really a class of FLOS-based phase estimator kernels parameterized by p_1 and p_2. The different kernels can be used in place of the original PGA kernel within the same algorithmic structure of the PGA method. As a result, considerable flexibility can be achieved, a fact that can be used for optimization purposes in the presence of different clutter operational environments. Once the estimate for $\hat{\Delta\psi}_m$ is obtained for all m, the entire aperture phase error is estimated by integrating the $\hat{\Delta\psi}_m$ values:

$$\hat{\psi}_m = \sum_{l=2}^{m}\hat{\Delta\psi}_l, \quad \hat{\psi}_1 = 0. \qquad (7)$$

The conventional PGA-MLG method [5] based on a Gaussian assumption for the data statistics, employs a second-order cross-correlation to the data, and is a special member of the introduced family of FLOS-based PGA kernels: PGA-MLG results from (6) when $p_1 = p_2 = 1$. Finally, for purposes of easy reference, the original PGA kernel is given by

$$\hat{\dot{\Delta\psi}}^{(PGA)}(t) = \frac{\sum_{k=1}^{N}\Im[\dot{g}_k(t)g_k^*(t)]}{\sum_{k=1}^{N}|g_k(t)|}, \qquad (8)$$

where $\Im[x]$ denotes the imaginary part of a complex number x, $\dot{f}(t)$ denotes the derivative of $f(t)$ with respect to t, and instead of a discrete measurement of the aperture position l, the continuous parameter t is used. As we can see in (6) and (8), all three methods use only data on adjacent pulses in the range-compressed space to estimate a single value of the phase between the two pulses. Then, these phase differences can be summed to produce an estimate of the phase error function across the entire aperture.

4. ALGORITHMIC ASSESSMENT

In this section, we assess the performance of the PGA-FLOS algorithm in comparison with the performance of the second-order based PGA algorithm using real SAR imagery. We apply a sinewave plus third-order polynomial phase error, shown in Figure 1(c) to the SAR image in Figure 1(a). In addition, a clutter component at SCR=7dB degrades the image. The resulting blurred and noisy image is depicted in Figure 1(b). We focus the degraded SAR image by applying iteratively the original PGA, the PGA-MLG, and PGA-FLOS methods. After 4 iterations, the phase compensated SAR images obtained by the three methods are shown in Figure 2. It is evident from this experiment that the SAR images obtained by the PGA-MLG and PGA-FLOS methods are much sharper than the SAR image focused by the original PGA method. This is due primarily to a significant bias term that is inherent in the original PGA phase estimator. The source of this bias lies in an assumption of high target-to-clutter ratio in the derivation of the original PGA kernel in (8), a condition that is often not met in real SAR imagery [5]. The experiment demonstrates that the PGA-FLOS method achieves a better estimate of the phase error and manages to better focus the noisy image. The PGA-MLG has a comparable performance while the original PGA kernel fails to satisfactory focus the SAR image.

In conclusion, we have developed, tested, and validated a novel autofocus method based on the fractional lower-order statistics (FLOS) of the data. The new method performs a non-linear transformation of the measurements and it includes the conventional PGA algorithm as a special case. Thus, it provides flexibility that can be useful for optimization purposes when operating in the presence of changing clutter/noise backgrounds.

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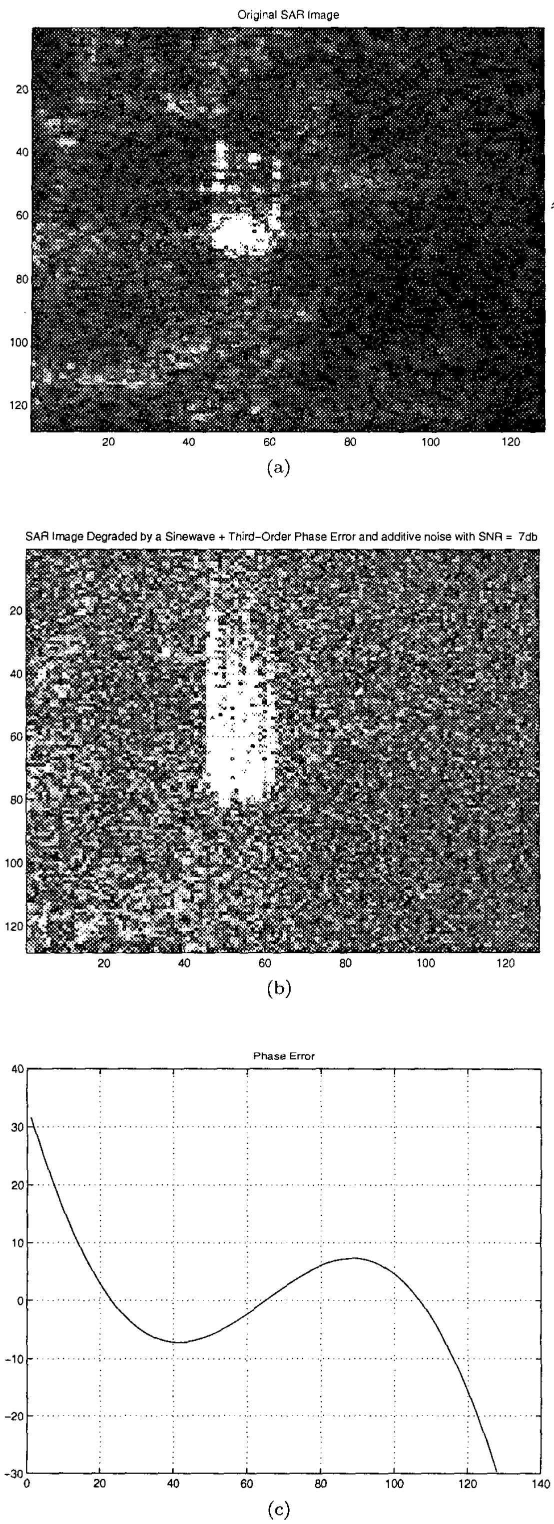

Figure 1: (a) Original SAR image; (b) SAR image degraded by sinewave plus third-order phase error shown in (c) and additive clutter with SNR = 7 dB.

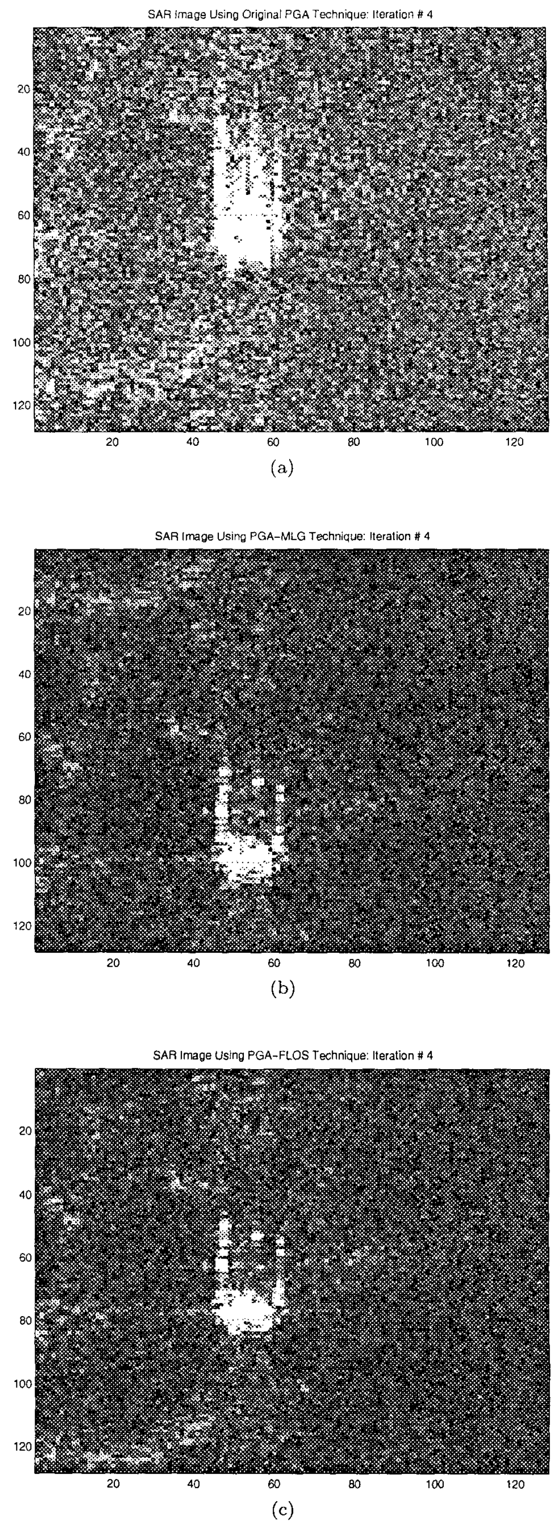

Figure 2: Phase compensated SAR images by the original PGA (a), PGA-MLG (b), and PGA-FLOS (c) methods after 4 iterations.