WAVELET-BASED ULTRASOUND IMAGE DENOISING USING AN ALPHA-STABLE PRIOR PROBABILITY MODEL

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ABSTRACT

Ultrasonic images are generally affected by multiplicative speckle noise, which is due to the coherent nature of the scattering phenomenon. Speckle filtering is thus a critical pre-processing step in medical ultrasound imagery, provided that the features of interest for diagnosis are not lost. We present a novel speckle removal algorithm within the framework of wavelet analysis. First, we show that the subband decompositions of logarithmically transformed ultrasound images are best described by alpha-stable distributions, a family of heavy-tailed densities. Consequently, we design a Bayesian estimator that exploits this *a priori* information. Using the alpha-stable model we develop a noise-removal processor that performs a non-linear operation on the data. Finally, we compare our proposed technique to current state-of-the-art speckle reduction methods. Our algorithm effectively reduces speckle, it preserves step edges, and it enhances fine signal details, better than existing methods.

1. INTRODUCTION

Speckle phenomena affect all coherent imaging systems including laser, SAR imagery, and medical ultrasound. In particular, it is important to reduce speckle in medical images since its presence affects the tasks of human interpretation and diagnosis.

Several methods have been proposed in the past for removing speckle. Among them, the classical Wiener filter is not adequate, since it is designed mainly for additive noise suppression. To address this issue, Jain [1] developed a homomorphic approach, which by taking the logarithm of the image, converts the multiplicative into additive noise, and consequently applies the Wiener filter. Also, the adaptive weighted median filter, introduced in [2], can effectively suppress speckle but it fails to preserve many useful details, being merely a low-pass filter.

Over the past decade, there has been considerably interest in using the wavelet transform as a powerful tool for recovering signals from noisy data [3, 4, 5]. For the purpose of speckle reduction, Zong *et al.* [5] use the above described homomorphic approach to separate the noise from the original image. Subsequently, they adopt regularized soft thresholding (wavelet shrinkage) to remove noise energy within the finer scales and nonlinear processing of feature energy for contrast enhancement.

Simoncelli *et al.* [4] developed non-linear estimators, based on formal Bayesian theory, which outperform classical linear processors and simple thresholding estimators in removing noise from visual images. They used a generalized Laplacian model for the subband statistics of the signal and developed a noise-removal algorithm, which performs a "coring" operation to the data. In a recent work, Tsakalides *et al.* [6] showed that alpha-stable distributions, a family of heavy-tailed densities, are sufficiently flexible and rich to appropriately model wavelet coefficients of images in coding applications.

Our approach for ultrasound image denoising consists of two major modules: (i) a subband representation function that utilizes the wavelet transform, and (ii) a Bayesian denoising algorithm based on an alpha-stable *prior* for the signal. The outlines of the proposed method are given in the following.

2. STATISTICAL CHARACTERIZATION OF WAVELET SUBBAND COEFFICIENTS

As a first step of our approach (see Fig. 1), and similarly to existing techniques [5, 7], the logarithm of the image is decomposed into several scales through a multiresolution analysis employing the 2-D wavelet transform [8]. This step guarantees that the speckle is transformed from multiplicative into additive and its characteristics are differentiated from the signal characteristics in each decomposition level.

Parametric Bayesian processing presupposes proper modeling for the prior probability density function (PDF) of the resulting signal and noise wavelet coefficients. Taking into account the homomorphic approaches in [5] and [7], it seems adequate to use the *log-normal* distribution as a speckle noise model : If X follows the log-normal distribution with certain parameters μ and σ, then $\ln X$ follows the normal distribution with mean μ and variance σ. The log-normal PDF is given by:

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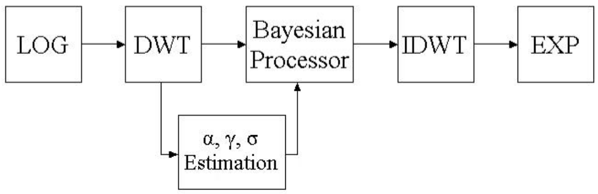

Fig. 1. Block diagram of the speckle suppression algorithm.

$$y = \frac{1}{x\sigma\sqrt{2\pi}} \exp(-\frac{(\ln x - \mu)^2}{2\sigma^2}) \quad (1)$$

The signal components of the wavelet decomposition in various scales are modeled as Symmetric alpha-stable ($S\alpha S$) processes. The $S\alpha S$ distribution is best defined by its characteristic function:

$$\phi(\omega) = \exp(\jmath\delta\omega - \gamma|\omega|^{\alpha}), \quad (2)$$

where α is the *characteristic exponent*, taking values $0 < \alpha \leq 2$, δ ($-\infty < \delta < \infty$) is the *location parameter*, and γ ($\gamma > 0$) is the *dispersion* of the distribution.

The $S\alpha S$ model is suitable for describing signals that have highly non-Gaussian statistics and its parameters can be estimated from noisy observations. To justify its use for this particular application we modeled a series of 44 abdominal ultrasound images (DICOM format, 4-MHz transducer, 256 gray-level resolution). Because of limited space, we only describe here the modeling of a single image, a kidney ultrasound, randomly chosen from that series.

As a starting point, we check whether the data is in the stable domain of attraction by estimating the characteristic exponent, α, directly from the data. This is done using the maximum likelihood (ML) method described by Nolan in [9], which gives reliable estimates and provides the most tight confidence intervals. First, we iterate three times the separable wavelet decomposition (as described in [8]) using Daubechies' Symmlet 4 basis wavelet. Than, we model the coefficients of each subband by using the $S\alpha S$ family. The results are summarized in Table 1, which shows the ML estimates of the characteristic exponent α. It can be observed that all subbands exhibit distinctly non-Gaussian characteristics, with values of α varying between 1 and 1.7, away from the Gaussian point of $\alpha = 2$.

As further stability diagnostic, we employ the *amplitude probability density* (APD) plot that gives a good indication of whether the $S\alpha S$ fit matches the data near the mode and on the tails of the distribution. Fig. 2 shows a highly accurate stable fit. The plot corresponds to the vertical subband at the first level of decomposition of the image analyzed. In particular, the plot proves that the class of $S\alpha S$ distributions is superior to generalized Laplacian densities [4, 8] because it provides a better fit to both the mode and the tails of the empirical density of the actual data.

These results clearly point to the need for the design of Bayesian processors that take into consideration the non-Gaussian heavy-tailed character of the data to achieve close to optimal speckle suppression performance.

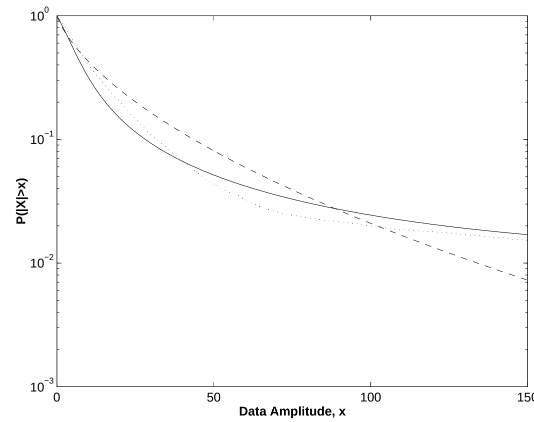

Fig. 2. Comparison between $S\alpha S$ and "generalized Laplacian" APDs depicted in solid and dashed lines, respectively. $S\alpha S$ has parameters $\alpha = 1.156$ and $\gamma = 7.908$, while the estimated parameters of the "generalized Laplacian" distribution are $p = 0.461$, and $s = 2.004$.

Table 1. $S\alpha S$ modeling of wavelet subband coefficients corresponding to a kidney ultrasound image. The tabulated key parameter α defines the degree of non-Gaussianity as deviations from the value $\alpha = 2$.

Level	Image Subbands		
	Horizontal	Vertical	Diagonal
1	1.481	1.156	1.255
2	1.678	1.340	1.417
3	1.224	1.114	1.025

3. PARAMETER ESTIMATION AND DESIGN OF THE BAYESIAN PROCESSOR

The results presented in the previous section motivate the use of a two-parameter $S\alpha S$ model for the signal component while we use a zero-mean Gaussian model for the noise component of the wavelet coefficients. In other words, the observed signal (the output of the **DWT** block in Fig. 1) is a mixture of $S\alpha S$ signal and Gaussian noise. Because of the lack of closed-form expressions for the general $S\alpha S$ PDF, we propose a method that is based on characteristic functions.

Specifically, since the PDF of the measured coefficients (d) is the convolution between the PDFs of the signal (s) and noise components (ξ), the associated characteristic function of the measurements is given by the product of the characteristic functions of the signal and noise:

$$\Phi_d(\omega) = \Phi_s(\omega) \cdot \Phi_\xi(\omega), \quad (3)$$

We estimate the parameters α_s, γ_s, and σ by fitting the Fourier transform of the empirical PDF of the measured coefficients with the function $\Phi_d(\omega)$. In practice, we first estimate the level of noise σ as in [3], and we perform the optimization only with respect to the $S\alpha S$ parameters α_s and γ_s.

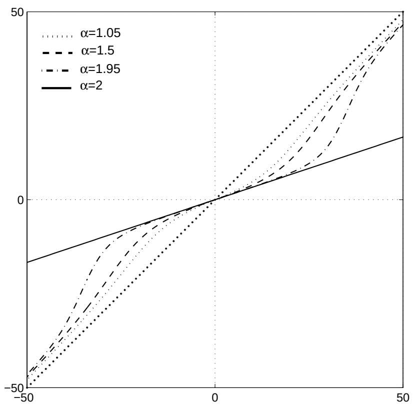

Fig. 3. Bayesian processor input-output curves for alpha-stable signal ($1 < \alpha \leq 2$) and Gaussian noise prior distributions.

Naturally, the remaining issue is the design of a formal Bayesian estimator that exploits all the a priori information outlined above. The Bayes risk estimator under a quadratic cost function minimizes the mean-square error (MSE) and is given by the conditional mean of s, given d:

$$\hat{s}(d) = \int s\, P_{s|d}(s\,|d) \cdot ds = \frac{\int P_{\xi}(d-s)P_s(s)s \cdot ds}{\int P_{\xi}(d-s)P_s(s) \cdot ds} \qquad (4)$$

In this work, the signal component of the wavelet coefficients is modeled as an alpha-stable random variable that does not have finite second-order statistics and thus the MSE is not defined.

For this reason we consider instead the Bayesian estimator that minimizes the mean absolute error and that can be shown to be the conditional median of s, given d. But, since the conditional density $P_{s|d}(s\,|d)$ is symmetric around zero, the conditional median coincides with the conditional mean. Hence, the Bayesian estimator for the absolute error cost function is again given by equation (4).

Figure 3 illustrates the processor dependency on the parameter α of the signal prior PDF. Specifically, for a given ratio γ_s/σ, the amount of shrinkage decreases as α decreases. The intuitive explanation for this behavior is that the smaller the value of α, the heavier the tails of the signal PDF and the greater the probability that the measured value is due to the signal.

4. SIMULATION RESULTS

We compared the results of our approach with the homomorphic Wiener filtering, median filtering, and wavelet shrinkage denoising using soft thresholding. All these techniques where applied to the image modeled in Section 2, for three different levels of unit-mean log-normal multiplicative noise. The soft thresholding scheme was developed using Daubechies' Symmlet 8 mother wavelet as suggested in [5]. Moreover, in order to minimize the effect of pseudo-Gibbs phenomena, we have embedded both wavelet-based methods into the cycle spinning algorithm [10]. The maximum number of wavelet decompositions we used was 5.

Table 2. Image enhancement measures obtained by the 3 denoising methods. The S/MSE is given in dB.

Without Filtering	4.14	9.75	16.94
Homomorphic Wiener	10.16	12.20	17.32
Median Filtering	9.87	11.89	17.07
Soft Thresholding	10.70	14.33	17.98
Bayesian Denoising	11.69	15.44	20.06

To quantify the achieved performance improvement the standard signal to noise ratio (SNR) is not adequate due to the multiplicative nature of speckle noise. Instead, a common way to achieve this in coherent imaging is to calculate the signal-to-MSE (S/MSE) ratio, defined as [7]:

$$S/MSE = 10\log_{10}\left(\sum_{i=1}^{K} S_i^2 / \sum_{i=1}^{K} (\hat{S}_i - S_i)^4\right) \qquad (5)$$

Note that this measure corresponds to the classical SNR for the case of additive noise. The results are summarized in Table 2. From the table it can be easily seen that our proposed Bayesian approach exhibits the best speckle mitigation performance.

Remember however, that in ultrasound imaging, we are interested in suppressing speckle noise while at the same time preserving the edges of the original image that often constitute features of interest for diagnosis. Having in mind this observation, in Fig. 4 we show for visual comparison a result from the processing of our test image. Although they achieve a good speckle suppression performance, the homomorphic Wiener filter and the median filter loose many of the signal details and the resulting images are blurred (see Fig. 4(c, d)). On the other hand, the image processed by soft thresholding is oversmoothed (cf. Fig. 4(e)). It seems that the Bayesian processor performs like a feature detector, retaining the features that are clearly distinguishable in the speckled data but cutting out anything which is assumed to be constituted by noise (Fig. 4(f)).

5. CONCLUSION

We introduced a new statistical representation for the wavelet coefficients of medical ultrasound images. We designed and tested a Bayesian processor which relies on this representation and found it more effective than existing methods, both in terms of speckle reduction and signal detail preservation. Our proposed algorithm could be easily adapted for the purpose of denoising other types of medical images where the noise can be modeled as additive Gaussian and signal-independent.

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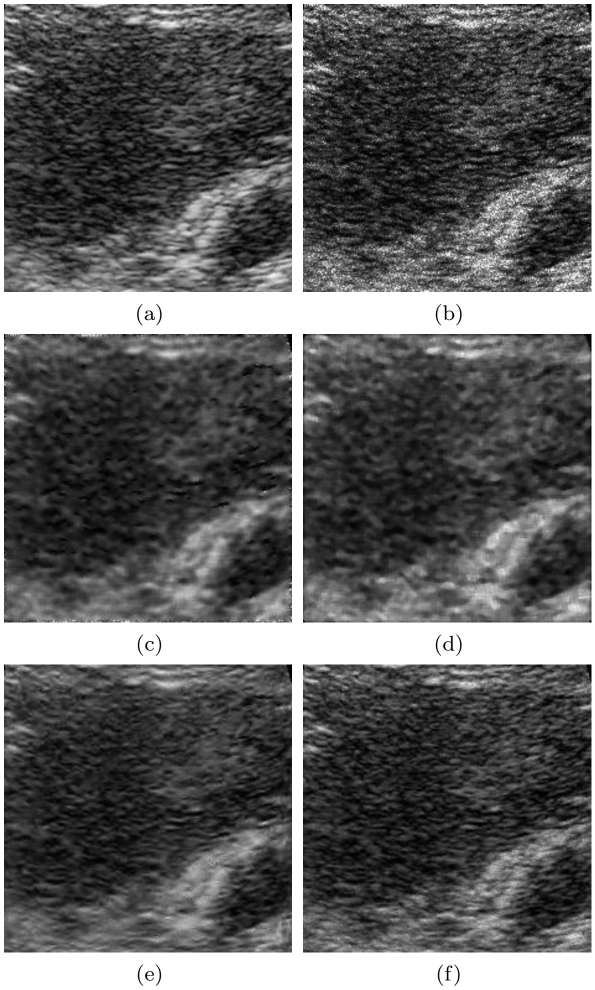

Fig. 4. Results of various speckle suppressing methods: (a) Original image; (b) Image degraded with simulated speckle noise ($S/MSE = 9.75dB$); (c) Homomorphic Wiener filtering; (d) Median filtering; (e) Soft thresholding; (f) Bayesian denoising.

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