SAR IMAGE DENOISING: A MULTISCALE ROBUST STATISTICAL APPROACH

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Abstract: Synthetic aperture radar (SAR) images are inherently affected by multiplicative speckle noise, which is due to the coherent nature of the scattering phenomenon. It appears sensible to reduce speckle in SAR images, provided that the structural features and textural information are not lost. We present a novel speckle removal algorithm within the framework of wavelet analysis. First, we show that the subband decompositions of logarithmically transformed SAR images are best described by alpha-stable distributions, a family of heavy-tailed densities. Consequently, we design a *maximum a posteriori* (MAP) estimator that exploits this *a priori* information. We use the alpha-stable model to develop a blind speckle-suppression processor that performs a non-linear operation on the data, and we relate this non-linearity to the degree of non-Gaussianity of the data. Finally, we compare our proposed method to current state-of-the-art soft thresholding technique applied on an aerial image and we quantify the achieved performance improvement.

1. INTRODUCTION

Speckle phenomena affect all coherent imaging systems including laser, medical ultrasound, and SAR imagery. In particular, speckle filtering is an important pre-processing step to improve the overall performance of automatic target detection and recognition algorithms based on SAR images. Recently, there has been considerably interest in using the wavelet transform as a powerful tool for recovering SAR images from noisy data [1, 2]. Basically, all wavelet-based methods involve as a first step the use of a logarithmic transform to separate the noise from the original image. Then, different wavelet shrinkage approaches are adopted, which are based on Donoho's pioneering work [3]. In [2] the authors perform a comparative study between a complex wavelet coefficient shrinkage filter and several standard speckle filters that are largely used by SAR imaging scientists, and show that the wavelet-based approach is among the best for speckle removal.

SAR images filtering also requires a good preservation of textural features. In a recent work [4], we have shown that a successful imaging algorithm can achieve both noise reduction and feature preservation if it takes into consideration the true statistics of the signal and noise components. Specifically, we have shown that the subband decompositions of ultrasound images have significantly non-Gaussian statistics that are best described by families of heavy-tailed distributions such as the alpha-stable and consequently we designed a Bayesian estimator that exploits these statistics. The approach presented here is similar to the method reported in [4]. The differences are that: (i) we select a different cost function for the design of the Bayesian processor, which gives raise to slightly different shapes of the nonlinearities applied to the noisy wavelet coefficients, and (ii) we use a different method for estimating the parameters of the alpha-stable distribution from noisy observations, which is based on Koutrouvelis' [5] regression method.

2. STATISTICAL CHARACTERIZATION OF WAVELET SUBBAND COEFFICIENTS

As a first step of our approach (see Figure 1), and similarly to existing techniques [1, 2, 4], the logarithm of the image is decomposed into several scales through a multiresolution analysis employing the 2-D wavelet transform [6]. Parametric Bayesian processing presupposes proper modeling for the prior probability density function (PDF) of the resulting signal and noise wavelet coefficients. We model the signal component of the wavelet coefficients using a two-parameter symmetric alpha-stable ($S\alpha S\tilde{)}$ distribution, while we use a zero-mean Gaussian model for the noise component. The choice of these models is motivated in the following.

2.1. Statistical properties of speckle noise

The statistical properties of speckle noise were studied by Goodman [7]. He has shown that, if the number of scatterers per resolution cell is large, a fully developed speckle pattern can be modeled as the magnitude of a complex Gaussian field with independent and identically distributed (i.i.d.) real and imaginary components. Arsenault and April [8] have shown that when the image intensity is logarithmically transformed, the speckle noise is approximately

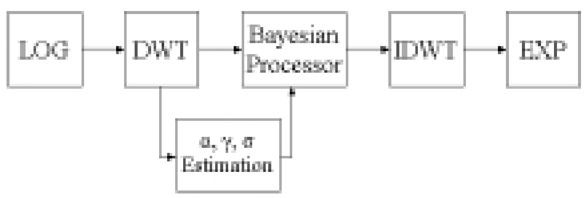

Figure 1. *Block diagram of the speckle suppression algorithm.*

Gaussian additive noise, and it tends to a normal probability much faster than the intensity distribution. Consequently, we use the *log-normal* distribution as a speckle noise model : If X follows the log-normal distribution with certain mean and variance values, then $\ln X$ follows the normal distribution with the same mean and the same variance. A log-normal random variable can be generated using:

$$X_{log-normal} = \exp\left(X_{normal}\sqrt{2\,log\frac{M}{m}} + ln\,m\right) \quad (1)$$

where M and m are the mean and the median values of the distribution, respectively, and X_{normal} is a standard zero-mean, unit-variance Gaussian random variable. There is a straightforward equivalence between the Equivalent Number of Looks (ENL) in a speckle image and the parameter m in the above expression [2].

2.2. Alpha-stable statistical model

The signal components of the wavelet decomposition in various scales are modelled as $S\alpha S$ processes. The $S\alpha S$ distribution is best defined by its characteristic function:

$$\phi(\omega) = \exp(\jmath\delta\omega - \gamma|\omega|^{\alpha}), \quad (2)$$

where α is the *characteristic exponent*, taking values $0 < \alpha \leq 2$, δ $(-\infty < \delta < \infty)$ is the *location parameter*, and γ $(\gamma > 0)$ is the *dispersion* of the distribution. The $S\alpha S$ model is suitable for describing signals that have highly non-Gaussian statistics and its parameters can be estimated from noisy observations. To justify its use for this particular application we modelled a series of SAR images from the MSTAR Public Clutter dataset [1]. The data set contains X-band images with 1784 x 1476 pixels and 1 ft x 1 ft resolution at 15^o depression angles.

As a starting point, we check whether the data is in the stable domain of attraction by estimating the characteristic exponent, α, directly from the data. This is done using the maximum likelihood (ML) method described by Nolan in [9]. We iterate three times the separable wavelet decomposition (as described in [6]) using Daubechies' Symmlet 8 basis wavelet. Than, we model the coefficients of each subband by using the $S\alpha S$ family. The results are summarized in Table 1, which shows the ML estimates of the characteristic exponent α corresponding to a cropped version (512 x 512 pixels) of the *HB061589* image from the data set. It can be observed that all subbands exhibit distinctly non-Gaussian characteristics, with values of α away from the Gaussian point of $\alpha = 2$. As further stability diagnostic, we employ the *amplitude probability density* (APD) plot that gives a good indication of whether the $S\alpha S$ fit matches the data near the mode and on the tails of the distribution. Figure 2 shows an example of modelling the vertical subband at the first level of decomposition of the SAR image under study. In particular, the plot proves that the class of $S\alpha S$ distributions is superior to generalized Laplacian densities [6, 10] because it provides a better fit to both the mode and the tails of the empirical density of the actual data.

[1] http://www.mvlab.wpafb.af.mil/public/sdms/

Table 1. *Modeling wavelet subband coefficients of a SAR image using $S\alpha S$ distributions. The tabulated key parameter α defines the degree of non-Gaussianity as deviations from the value $\alpha = 2$.*

Level	Image Subbands		
	Horizontal	Vertical	Diagonal
1	1.239	1.283	1.302
2	1.418	1.125	1.295
3	1.286	1.019	1.380

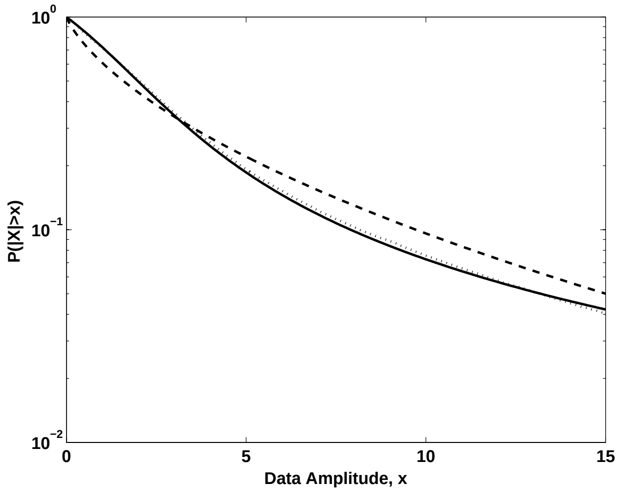

Figure 2. *APD plot of vertical wavelet subband coefficients at first level of decomposition of a SAR image. The empirical APD (dotted line) is fitted with a $S\alpha S$ model (solid line) and the "generalized" Laplacian function (dashed line).*

3. DESIGN OF A MAP PROCESSOR FOR SPECKLE MITIGATION

In order to be able to construct a MAP processor, first one should estimate the parameters of the prior distributions of the signal and noise components of the measurements. The output of the **DWT** block in Figure 1 is a mixture of $S\alpha S$ signal and Gaussian noise. We observe that the PDF of the measured coefficients (d) is the convolution between the PDFs of the signal (s) and noise components (ξ). Consequently, the associated characteristic function of the measurements is given by the product of the characteristic functions of the signal and noise:

$$\Phi_d(\omega) = \exp\left(-\gamma_s|\omega|^{\alpha_s}\right)\cdot\exp\left(-\frac{\sigma^2}{2}|\omega|^2\right) \quad (3)$$

At this point, instead of directly fitting the Fourier transform of the empirical PDF of the measured coefficients with the function $\Phi_d(\omega)$ as we did in [4], we observe that (3) implies:

$$\log[-(\log|\Phi_d(\omega)|^2 + \sigma^2\omega^2)] = \log(2\gamma_s) + \alpha_s\log|\omega| \quad (4)$$

First, we estimate the level of noise σ as in [3], then we find the parameters α_s and γ_s by regressing

$$y = \log[-(\log|\Phi_d(\omega)|^2 + \sigma^2\omega^2)]$$

on $w = \log |\omega|$ in the model

$$y_k = \mu + \alpha\, w_k + \epsilon_k \tag{5}$$

where $\mu = \log(2\gamma)$, ϵ_k denotes an error term, and $(\omega_k, k = 1, \ldots, K)$ is an appropriate set of real numbers. We should note here that Koutrouvelis [5] used a similar approach to estimate the parameters of alpha-stable distributions. He proved that its regression method gives very good results in terms of consistency, bias, and efficiency.

Naturally, the remaining issue is the design of a formal Bayesian estimator that exploits all the a priori information outlined above. Our goal is to find the Bayes risk estimator $\hat{s}$ that minimizes the conditional risk, which is the loss averaged over the conditional distribution of s, given the noisy observation, d:

$$\hat{s}(d) = \arg\min_{\hat{s}} \int L[s, \hat{s}(d)]\, P_{s|d}(s\,|d)\, ds \tag{6}$$

Selecting the uniform cost function:

$$L[s, \hat{s}(d)] = \begin{cases} 0, & \text{for } |s - \hat{s}| < \epsilon \\ 1, & \text{otherwise} \end{cases} \tag{7}$$

the MAP estimator can be easily derived as being:

$$\hat{s}(d) = \arg\max_{\hat{s}} P_{s|d}(s\,|d) \tag{8}$$

It is important to underline at this point that under the loss function in (7), expression (6) is well defined for all $S\alpha S$ random variables (with characteristic exponent α taking values in the whole range $0 < \alpha \leq 2$).

Bayes' theorem gives the *a posteriori* PDF of s based on the measured data:

$$P_{s|d}(s\,|d) = \frac{P_{d|s}(d\,|s)\, P_s(s)}{P_d(d)}, \tag{9}$$

where $P_s(s)$ is the *prior* PDF of the signal component of the measurements and $P_{d|s}(d\,|s)$ is the *likelihood* function. Substituting (9) in (8), we get:

$$\hat{s}(d) = \arg\max_{\hat{s}} P_{d|s}(d\,|s)\, P_s(s)$$
$$= \arg\max_{\hat{s}} P_\xi(d - s) P_s(s) \;=\; \arg\max_{\hat{s}} P_\xi(\xi) P_s(s) \tag{10}$$

Figure 3 depicts the numerically computed MAP input-output curves for five different values of the signal characteristic exponent, α, namely, $\alpha = 2$ (Gaussian data), $\alpha = 1.95$ (slightly non-Gaussian data), $\alpha = 1.5$, $\alpha = 1$, and $\alpha = 0.5$ (considerably heavy-tailed data). Apart from the case $\alpha = 2$, all curves correspond to a nonlinear "coring" operation, i.e., large-amplitude observations are essentially preserved while small-amplitude values are suppressed. This is expected since small measurement values are assumed to come from signal values close to zero. On inspecting Figure 3 it can be observed that for a given ratio γ/σ, the amount of shrinkage decreases as α decreases. The intuitive explanation for this behavior is that the smaller the value of α, the heavier the tails of the signal PDF and the greater the probability that the measured value is due to the signal.

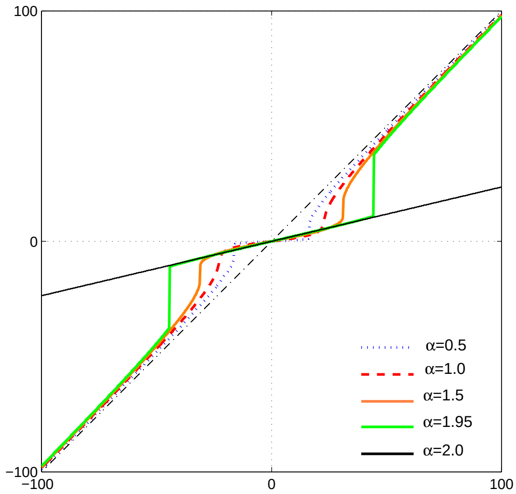

Figure 3. *MAP processor input-output curves for alpha-stable signal ($0.5 \leq \alpha \leq 2$) and Gaussian noise prior distributions. The dash-dotted line indicates the identity function.*

4. SIMULATION RESULTS

In this section, we show simulation results obtained by processing an aerial image, obtained by cropping the *westaerialconcorde* image, which can be found in the Matlab's Image Processing Toolbox. The original image is shown in Figure 4(a). The simulated speckle image, contaminated with synthetic noise with $ENL = 9.4$ is shown in Figure 4(b). We compared the results of our approach with the wavelet shrinkage denoising using soft thresholding. Both schemes were developed using Daubechies' Symmlet 8 mother wavelet. Moreover, in order to minimize the effect of pseudo-Gibbs phenomena, we have embedded both methods into the cycle spinning algorithm [11]. The maximum number of wavelet decompositions we used was 5. The following quality measures have been used for performance comparison, which are summarized in Table 2:

- MSE: Mean-square error between the denoised image and the original speckle-free data.
- β: A correlation measure, which should be close to unity for an optimal effect of edge preservation (see e.g. [4]).
- s/m: Standard-deviation-to-mean ratio used as a measure of image speckle in homogeneous regions.

Table 2. *Image enhancement measures obtained using the 2 wavelet-based methods.*

Measure	MSE	β	s/m
Without Filtering	43.82	0.25	0.49
Soft Thresholding	16.72	0.43	0.34
Bayesian Denoising	16.27	0.45	0.35

From Table 2 it can be seen that our proposed Bayesian approach exhibits the best performance according to all met-

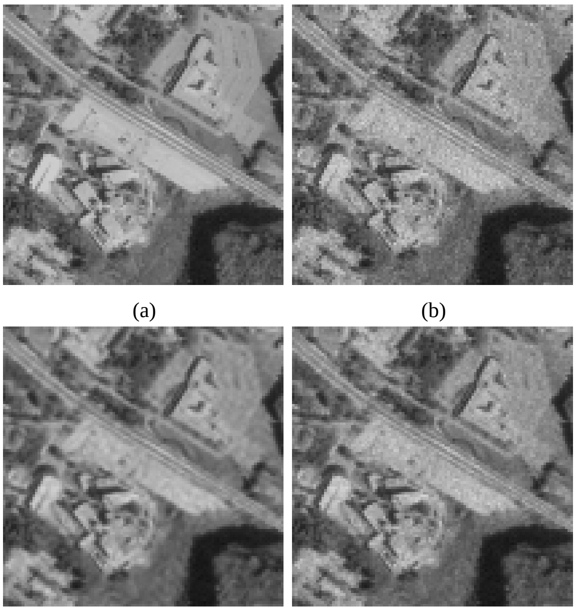

Figure 4. *Results of various speckle suppressing methods. (a) Original image. (b) Simulated speckle image* $(ENL = 9.4)$. *(c) Soft thresholding. (d) Bayesian denoising.*

rics. In Figure 4 we show for visual comparison the results from the processing of our test image. Although it achieves a good speckle suppression performance, the image processed by soft thresholding is oversmoothed (Figure 4(c)). It seems that the Bayesian processor performs like a feature detector, retaining the features that are clearly distinguishable in the speckled data but cutting out anything which is assumed to be constituted by noise (Figure 4(d)).

5. DISCUSSION

We introduced a new statistical representation for wavelet coefficients of SAR images. We designed and tested a MAP processor which relies on this representation and we found it more effective than traditional wavelet shrinkage methods, both in terms of speckle reduction and signal detail preservation. Our processor is based on solid statistical theory, and it does not depend on the use of any *ad hoc* thresholding parameter. The method proposed in Section 3 for choosing the "coring" nonlinearity could be thus considered as a principled way of shrinking noisy data, relying on the true statistics of the signal and noise wavelet coefficient.

Naturally, our approach is more computationally expensive due to the fact that the prior distribution parameters need to be estimated at each scale of interest. However, this is not a serious problem for off-line processing.

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