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CHARACTERIZATION OF AN UNDERWATER ACOUSTIC SIGNAL USING THE STATISTICS OF THE WAVELET SUBBAND COEFFICIENTS

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A novel statistical scheme for the characterization of underwater acoustic signals is tested in a shallow water environment for the classification of the bottom properties. The scheme is using the statistics of the 1-D wavelet coefficients of the transformed signal. For geoacoustic inversions based on optimization procedures, an appropriate norm is defined, based on the Kullback-Leibler divergence (KLD), expressing the difference between two statistical distributions. Thus the similarity of two environments is determined by means of an appropriate norm expressing the difference between two acoustical signals. The performance of the proposed inversion method is studied using synthetic acoustic signals generated in a shallow water environment over a fluid bottom.

1. Introduction

Recently, a new method for the classification of the underwater acoustic signals has been proposed by the authors, aiming at the definition of an alternative set of "observables" to be used for geoacoustic inversions [1]. The study was motivated by the fact that it is not always possible to obtain a set of identifiable and measurable properties of the acoustic signal to be used in the framework of an inversion process. As the efficiency of an inversion procedure is directly related to the character of the observables, a major task on a specific physical problem is to define observables which will be more sensitive to changes of the environmental parameters and easily identified in noisy conditions. In previous works [1,2] it was shown that the modelling of the statistics of the wavelet subband coefficients of the measured signal, provides an alternative way for obtaining a set of observables which is easily calculated and has the necessary sensitivity in changes of the environmental parameters, so that its use for inversions to be well justified. Here, this method is tested in shallow water environments for the recovery of the bottom parameters. The inversion is based on an optimization scheme utilizing the Kullback-Leibler divergence to measure the similarity between the observed signal and a signal calculated using a candidate set of bottom parameters.

2. The classification scheme

In the framework of the proposed approach, an acoustic signal is classified using the statistics of the subband coefficients of its 1-D wavelet transform. In particular, the measured signal is decomposed into several scales by employing a multilevel 1-D Discrete Wavelet Transform (DWT) [3]. This transform works as follows: starting from the given signal $s(t)$, two sets of coefficients are computed at the first level of decomposition, (i) approximation coefficients A1 and (ii) detail coefficients D1. These vectors are obtained by convolving $s(t)$ with a low-pass filter for approximation and with a high-pass filter for detail, followed by dyadic decimation. At the second level of decomposition, the vector A1 of the approximation coefficients, is decomposed in two sets of coefficients using the same approach replacing $s(t)$ by A1 and producing A2 and D2. This procedure continues in the same way, namely at the k-th level of decomposition we filter the vector of the approximation coefficients computed at the (k-1)-th level.

2.1. *Derivation of the statistics of the wavelet subband coefficients*

The Feature Extraction (FE) step is motivated by previous works on image processing [4,5,6]. The signal is first decomposed into several scales by employing a 1-D DWT as described above. The next step is based on the accurate modelling of the tails of the marginal distribution of the wavelet coefficients at each subband by adaptively varying the parameters of a suitable density function. The extracted features of each subband are the estimated parameters of the corresponding model. For the acoustical signals studied, the wavelet subband coefficients are modelled as symmetric alpha-Stable ($S\alpha S$) random variables.

The $S\alpha S$ distribution is best defined by its characteristic function [7,8]:

$$\phi(\omega) = \exp(\imath\delta\omega - \gamma^{\alpha}|\omega|^{\alpha}), \tag{1}$$

where α is the *characteristic exponent*, taking values $0 < \alpha \leq 2$, δ $(-\infty < \delta < \infty)$ is the *location parameter*, and γ $(\gamma > 0)$ is the *dispersion* of the distribution. The characteristic exponent is a shape parameter, which controls the "thickness" of the tails of the density function. The smaller the value of α, the heavier the tails of the $S\alpha S$ density function. The dispersion parameter determines the spread of the distribution around its location parameter, similar to the variance of the Gaussian.

In general, no closed-form expressions exist for the $S\alpha S$ density functions. Two important special cases of $S\alpha S$ densities with closed-form expressions are the Gaussian ($\alpha = 2$) and the Cauchy ($\alpha = 1$). Unlike the Gaussian density, which has exponential tails, stable densities have tails following an algebraic rate of decay ($P(X > x) \sim Cx^{-\alpha}$, as $x \to \infty$, where C is a constant depending on the model parameters), hence random variables following $S\alpha S$ distributions with small α values are highly impulsive.

2.2. *Feature Extraction*

After the implementation of the 1-D wavelet transform, the marginal statistics of the coefficients at each decomposition level are modelled via a $S\alpha S$ distribution. Then, to extract the features, we simply estimate the (α, γ) pairs at each subband.

Thus, a given acoustic signal S, decomposed in L levels, is associated with the set of the $L+1$ pairs of the estimated parameters:

$$S \mapsto \{(\alpha_1, \gamma_1), (\alpha_2, \gamma_2), \ldots, (\alpha_{L+1}, \gamma_{L+1})\}, \tag{2}$$

where (α_i, γ_i) are the estimated model parameters of the i-th subband. Note that we follow the convention that $i = 1$ corresponds to the detail subband at the first decomposition level, while $i = L+1$ corresponds to the approximation subband at the L-th level. The total size of the above set equals $2(L+1)$ which means that the content of an acoustic signal can be represented by only a few parameters, in contrast with the large number of the transform coefficients.

As it has already been mentioned, the FE step becomes an estimator of the model parameters. The desired estimator in our case is the maximum likelihood (ML) estimator. The estimation of the $S\alpha S$ model parameters is performed using the consistent ML method described by Nolan [9], which provides estimates with the most tight confidence intervals.

2.3. *Similarity Measurement*

In the proposed classification scheme, the similarity measurement between two distinct acoustic signals was carried out by employing the Kullback-Leibler divergence (KLD) [10]. As there is no closed-form expression for the KLD between two general $S\alpha S$ densities which are not Cauchy or Gaussian, numerical methods should be employed for the computation of the KLD between two numerically approximated $S\alpha S$ densities.

In order to avoid the increased computational complexity of a numerical scheme, we first transform the corresponding characteristic functions into valid probability density functions and then the KLD is applied on these normalized versions of the characteristic functions. Due to the one-to-one correspondence between a $S\alpha S$ density and its associated characteristic function, it is expected that the KLD between normalized characteristic functions will be a good similarity measure between the acoustic signals.

If $\phi(\omega)$ is a characteristic function corresponding to a $S\alpha S$ distribution, then the function

$$\hat{\phi}(\omega) = \frac{\phi(\omega)}{c} \tag{3}$$

is a valid density function when

$$c = \int_{-\infty}^{\infty} \phi(\omega)\, d\omega.$$

For the parameterization of the $S\alpha S$ characteristic function given by Eq. (1) and assuming that the densities are centered at zero, that is $\delta = 0$, which is true in the case of wavelet subband coefficients since the average value of a wavelet is zero, the normalization factor is given by

$$c = \frac{2\Gamma\left(\frac{1}{\alpha}\right)}{\alpha\, \gamma}. \tag{4}$$

By employing the KLD between a pair of normalized $S\alpha S$ characteristic functions, the following closed form expression is obtained [4]:

$$D(\hat{\phi}_1 \| \hat{\phi}_2) = \ln\left(\frac{c_2}{c_1}\right) - \frac{1}{\alpha_1} + \left(\frac{\gamma_2}{\gamma_1}\right)^{\alpha_2} \cdot \frac{\Gamma\left(\frac{\alpha_2+1}{\alpha_1}\right)}{\Gamma\left(\frac{1}{\alpha_1}\right)} \tag{5}$$

where (α_i, γ_i) are the estimated parameters of the characteristic function $\phi_i(\cdot)$ and c_i is its normalizing factor. It can be shown that D is the appropriate cost function for our application as $D(\hat{\phi}_1 \| \hat{\phi}_2) \geq 0$ with equality if and only if $(\alpha_1, \gamma_1) = (\alpha_2, \gamma_2)$.

Thus, the implementation of an L-level DWT on each underwater acoustic signal results in its representation by $L+1$ subbands, $(D_1, D_2, \ldots, D_L, A_L)$, where D_i, A_i denote the i-th level detail and approximation subband coefficients, respectively. Assuming that the wavelet coefficients belonging to different subbands are independent, Eq. (5) yields the following expression for the overall distance between two acoustic signals S_1, S_2:

$$D(S_1 \| S_2) = \sum_{k=1}^{L+1} D(\hat{\phi}_{S_1,k} \| \hat{\phi}_{S_2,k}). \tag{6}$$

3. Study of the sensitivity of the KLD for geoacoustic inversions

In order to validate the proposed classification scheme, first we need to study the sensitivity of the proposed cost function, which measures the similarity with respect to changes of the environmental parameters. Our previous efforts in this respect were mainly oriented towards the sensitivity of the KLD with respect to small changes of the sound speed profile in the water column. First we observed that the set of statistical parameters of the subband coefficients of a specific signal, propagated through the water column, change significantly when the sound speed profile varies and eventually that the KLD is a suitable tool for monitoring the corresponding model parameters' variation [1]. In the last paper, we have also shown that similar conclusions can be derived for the sensitivity of the KLD when applied to signals measured after interacting with different types of ocean beds. The classification parameters of the sea-bed are typically the compressional and shear velocities, the densities of the various layers, the attenuation coefficients and the thickness of the sedimentary layers. In order to simplify the study we had chosen to simulate experiments by assuming semi-infinite fluid bottoms, thus restricting our environmental parameters to compressional velocity and bottom density only.

Here, we extend this study by adding the case of an ocean bottom consisting of two layers of fluid material. The properties of the substrate are considered constant, while we allow the compressional velocity and density of the sedimentary layer to vary within prescribed limits. We simulate the propagation of an acoustic pulse of central frequency $f_0 = 200\ Hz$ and bandwidth $\Delta f = 50\ Hz$ over a distance of $R = 5\ km$ and we apply the proposed classification scheme to the simulated measurements for a source and receiver pair placed at mid-depth of the water column. The environmental parameters for the reference environment appear in Table I. We estimated the statistics of the four subband coefficients of the 3-level 1-D wavelet transform applied to each one of the synthetic signals and the KLD between a reference signal and the set of simulated signals that are obtained by changing the sediment compressional velocity from the value of $c_b = 1550\ m/sec$ to the value of $c_b = 1650\ m/sec$ in steps of 5 m/sec and the density from $\rho_b = 1180\ kg/m^3$ to $\rho_b = 1220\ kg/m^3$ in steps of 1 kg/m^3. We have chosen to restrict our analysis to narrow limits of the two parameters in order to focus on small variations of the geoacoustic parameters. It should also be pointed out that we have chosen to study a two layered bottom in order to assess the performance of the KLD in cases where a small part of the bottom changes.

Table 1. The shallow water environment.

Water Depth (H)	200 m
Range (R)	5 km
Central Frequency (f_0)	200 Hz
Bandwidth (Δf)	50 Hz
Source/Receiver depth	100 m
Sound speed profile in the water:	
$c_w(0)$	1500 m/sec
$c_w(50)$	1490 m/sec
$c_w(200)$	1515 m/sec
Sediment layer :	
c_b	1550 m/sec
ρ_b	1200 kg/m^3
Semi-infinite substrate:	
c_{sb}	1800 m/sec
ρ_{sb}	1500 kg/m^3

The simulated data are calculated using the Normal-Mode program MODE1 developed at FO.R.T.H. These data are provided as input to the inverse discrete Fourier transform to yield the signals in the time domain.

Each of the time-domain signals is decomposed by implementing a 3-level 1-D DWT using the db2 and db4 wavelets. The reference signal is that corresponding to the reference environment.

Fig. 1 displays the KLD between the reference signal and each signal corresponding to the geoacoustic parameters indicated at the axes of the diagram. In order to be consistent with previous studies, we have included two plots in the figure, the first of which corresponds to the case where both the approximation and detail subbands are considered and the second corresponding to the case where only the detail subbands are taken into account. The star in the two plots corresponds to zero KLD, that is, its coordinates are equal to the geoacoustic parameters of the reference signal.

It can be seen that the inclusion of the approximation subband only affects the discrim-

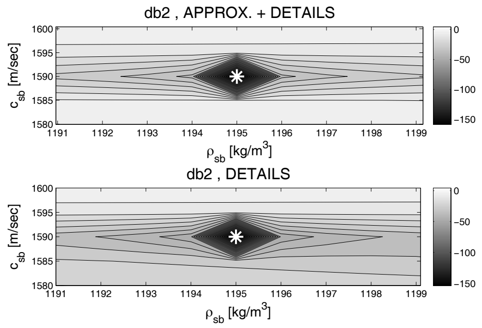

Fig. 1. KLD between the reference signal and signals corresponding to different values of the sediment compressional velocity and density, decomposed with the db2 wavelet, using (a) all wavelet subbands and (b) only the details

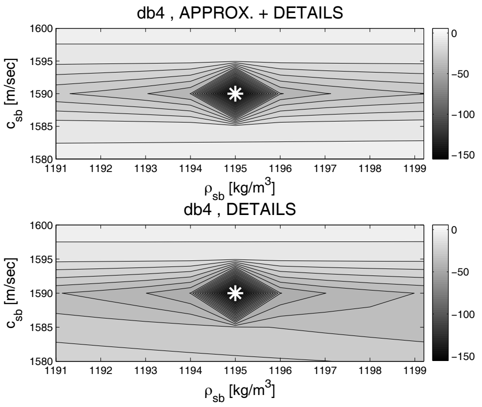

Fig. 2. KLD between the reference signal and signals corresponding to different values of the sediment compressional velocity and density, decomposed with the db4 wavelet, using (a) all wavelet subbands and (b) only the details

ination power of the KLD between the reference signal and the signals which are already "far" from it.

Fig. 2 presents the KLD between the reference signal and each signal corresponding to different geoacoustic parameters when all the signals are decomposed using the db4 wavelet. As we can see, there is an improvement in comparison with the results provided by the db2 wavelet, which is a good indication that the db4 wavelet can be used with confidence for performing the proposed classification process.

Fig. 3 presents the variation of the KLD between a reference signal corresponding to a

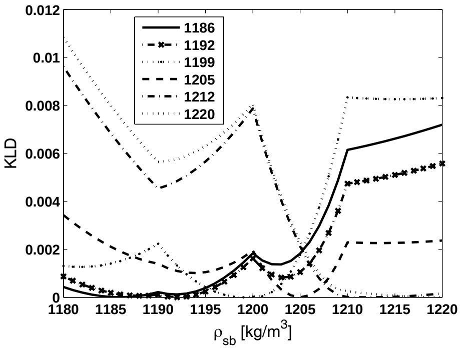

Fig. 3. KLD between each one of the signals corresponding to the specific densities of the sediment layer and signals corresponding to different values of the sediment density, decomposed with the db4 wavelet.

sediment density other than that of the reference environment and the signals corresponding to sediment layers of different densities within the limits adopted in the previous study. The compressional velocity is considered constant (=1600 m/sec) for each one of these signals. The purpose of the study illustrated in Fig. 3 is to assess the performance of the classification scheme for small variations of the bottom parameter which is known to be the less accurately estimated by any inversion scheme applied to acoustical data, namely the bottom density, for a class of different reference values. Although the reference values are chosen within the prescribed limits, they can be used for the derivation of more general conclusions with respect to the performance of the proposed classification scheme. We observe that, for each one of the reference signals, the correct value of the bottom density is obtained with confidence limits that are narrow enough to ensure a reliable estimation of the parameter. This observation is again consistent with that of the preliminary studies presented in [1].

4. Conclusions

The purpose of the present paper was to provide additional evidence of the reliability and good behavior of an acoustic signal classification scheme based on a $S\alpha S$ modelling of the coefficients of a 1-D wavelet decomposition. The scheme was originally developed for the classification of the acoustical signals to be used for tomographic applications and has illustrated its efficiency through simulations corresponding to shallow water environments. At the present stage, it is the performance of the KLD, being used so far for signal similarity measurements, which is systematically studied. This is considered to be the first necessary step before proceeding to the inversions. Here, the sensitivity of the KLD with respect to variations of the sediment parameters is studied as an additional step towards the full validation of the proposed method as the basic tool for geoacoustic inversions. The results presented here, based on simulated signals in shallow water environments over a two-layered bottom, support our statement that the proposed technique can classify, with high probability, an underwater signal in the correct environment where it was recorded. It is also important to note that conclusions derived in previous studies with respect to the use of specific wavelets or the use of specific subbands are also derived here, which is an additional indication that the proposed classification scheme is robust in its behavior.

The next step in our study is the application of the signal classification scheme to actual inversion procedures involving multidimensional search spaces and, if possible, to acoustical signals from real experiments.

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