Medical Image Fusion using the Convolution of Meridian Distributions

Mayank Agrawal, Panagiotis Tsakalides, and Alin Achim

Abstract—**The aim of this paper is to introduce a novel non-Gaussian statistical model-based approach for medical image fusion based on the Meridian distribution. The paper also includes a new approach to estimate the parameters of generalized Cauchy distribution. The input images are first decomposed using the Dual-Tree Complex Wavelet Transform (DT-CWT) with the subband coefficients modelled as Meridian random variables. Then, the convolution of Meridian distributions is applied as a probabilistic prior to model the fused coefficients, and the weights used to combine the source images are optimised via Maximum Likelihood (ML) estimation. The superior performance of the proposed method is demonstrated using medical images.**

I. Introduction

Image fusion is a technique to combine information from multiple images of the same scene in a single image that ideally contains all salient features from each of the individual input images. Medical imaging offers a powerful tool that helps physicians in the diagnosis process. As a result, there exist many distinct medical imaging modalities that provide important information of the human body (or parts and functions thereof) for clinical purposes. For example, magnetic resonance imaging (MRI) provides better information on soft tissue whereas computed tomography (CT) provides better information about denser tissue [1]. Fusing these two types of images creates a composite image which is more informative than any of the input signals provided by a single modality. For this reason, image fusion has become a common process used within medical diagnostics and treatment

Many image fusion schemes have been developed in the past. In general, these schemes can be roughly classified into pixel-based and region based. Lewis *et al.* [2] showed that comparable results can be achieved using both types of methods with added advantages for the region based approaches because they enable more intelligent fusion rules. On the other hand, pixel-based algorithms are simple and thus easier to implement.

In our paper, we propose a new image fusion technique based on Meridian convolution where the combination weights are optimised through maximum likelihood estimation. ML optimisation of weights [3] is much more reliable than a standard weighted averaging fusion rule [4, 5] as it does not require using ad-hoc threshold values for weight selection. In addition, we use the dual-tree complex wavelet transform, which provides near shift invariance and good directional selectivity. We provide a novel method for estimating the statistical parameters of the generalised Cauchy distribution (GCD) by using Mellin transform theory. Then, we employ the Meridian distribution, a special member of the GCD family, to model the wavelet coefficients of the input sub-bands and for weight optimisation during the image fusion process.

The paper is organized as follows: Section II gives a novel way for the GCD parameter estimation. Section III introduces a new framework for weighted average fusion, while in Section IV the convolution of Meridian distributions is derived. Section V presents the weights optimisation approach. Results are shown in section VI and conclusions are drawn in Section VII.

This work was partially funded by the Marie Curie TOK-DEV “ASPIRE” grant (MTKD-CT-2005-029791) within the 6th European Community Framework Programme.

M. Agrawal and A. Achim are with the Department of Electrical and Electronic Engineering, University of Bristol, UK
alin.achim@bristol.ac.uk
P Tsakalides is with the Institute of Computer Science, Foundation for Research and Technology-Hellas, Heraklion, Greece tsakalid@ics.forth.gr

II. Parameter estimation for GCD

A. The Generalized Cauchy Distribution

The generalized Cauchy density function proposed in [7] is given by:

$$f(x) = a\gamma(\gamma^p + |x|^p)^{(-2/p)} \qquad (1)$$

with $a = \dfrac{p\Gamma(2/p)}{2(\Gamma(1/p))^2}$. In this representation, γ is the scale parameter and p is the tail constant. The GCD family contains the Meridian [7] and Cauchy distributions as special cases with p = 1 and p = 2 respectively. For p < 2, the tail of the PDF decays slower than in the Cauchy distribution, resulting in a heavier-tailed probability density function (pdf). The advantage of GCD models consists in the availability of analytical expressions for their pdf, unlike the family of symmetric alpha stable distributions [2], as well as in the existence of simple and efficient parameter estimators.

B. Parameter estimation using Mellin Transform

Mellin transform as shown in [8] is a powerful tool for deriving the novel parameter estimation based on log-cumulants [9]. Let f be a function defined over $\mathbb{R}_+$. The Mellin transform of f is defined as:

$$\Phi(z) = M[f(u)](z) = \int_0^\infty u^{z-1} f(u)du \qquad (2)$$

where z is the complex variable of the transform.

Preprint submitted to 32nd Annual International IEEE EMBS Conference.
Received April 1, 2010.

By analogy with the way in which common statistics are deducted based on Fourier Transform, the following r[th] order second-kind cumulants can be defined, based on Mellin transform.

$$\bar{k}_r = \left.\frac{d^r \psi(z)}{dz^r}\right|_{z=1} \tag{3}$$

Where *Ψ (z) = log (Φ (z))*. Following the analogy further, the method of log-moments can be applied in order to estimate the two parameters of the PDF function ***f*** (GCD). To be able to do this, the first two second-kind cumulants are required. These can be estimated empirically from N samples x_i as follows:

$$\hat{\bar{k}}_1 = \frac{1}{N}\sum_1^N [\log(x_i)] \text{ and, } \hat{\bar{k}}_2 = \frac{1}{N}\sum_1^N [(\log(x_i) - \hat{\bar{k}}_1)^2] \tag{4}$$

C. Log-moments of GCD model

Plugging the expression of the GCD PDF given by (1) into (2) and after some straightforward manipulations, one gets:

$$\Phi(z) = \frac{\gamma\Gamma\left(\frac{s}{p}\right)\Gamma\left(\frac{2-s}{p}\right)}{4\,\Gamma\left(\frac{1}{p^2}\right)(\gamma^p)^{\left(\frac{2}{p}\right)}\left(\frac{1}{\gamma^p}\right)^{\frac{s}{p}}} \tag{5}$$

and ψ (z) = log *Φ (z)*, which is the second-kind second characteristic function of a GCD density. Calculating the first and second order second-kind cumulants (3):

$$\bar{k}_1 = \left.\frac{d\Phi(z)}{d(z)}\right|_{Z=1} = \log \gamma \tag{6}$$

and,

$$\bar{k}_2 = F(p) = \left.\frac{d^2\Phi(z)}{d(z)^2}\right|_{Z=1} = \frac{2\psi\left(1, \frac{1}{p}\right)}{p^2} \tag{7}$$

Where $\psi_n(t) = \frac{d^{n+1}}{dt^{n+1}} \log\Gamma(t)$ is the poly-gamma function.

The scale parameter, **γ**, can be found by solving (6) and we get:

$$\gamma = \exp\left(\bar{k}_1\right) = \exp\left(\frac{1}{N}\sum_1^N [\log(x_i)]\right) \tag{8}$$

The tail parameter, ***p*** is estimated by computing the inverse of the function ***F(p)*** in (7).

III. WEIGHTED AVERAGE FUSION

The proposed weighted average image fusion algorithm follows similar steps to the ones presented in [3]. First, the detail wavelet coefficients are fused using a weighted combination:

$$D_F = w_1 D_1 + w_2 D_2, \tag{9}$$

where D_1 and D_2 are the detail wavelet coefficients of two source images, D_F denotes the fused coefficients, and w_1 and w_2 are the weights for the two input images, respectively.

Secondly, the approximation wavelet coefficients of the fused image is found by taking the mean of the approximation wavelet coefficients of two source images, A_1 and A_2, respectively:

$$A_F = 0.5(A_1 + A_2). \tag{10}$$

IV. CONVOLUTION OF MERIDIAN DISTRIBUTIONS

The Meridian distribution is a member of the GCD family (when *p=1)* for modelling heavy-tailed non-Gaussian behaviour. The pdf of the Meridian model has the form [7]:

$$P(x;\mu,\gamma) = \frac{\gamma}{2((x-\mu)+\gamma)^2} \tag{11}$$

where μ ($-\infty < \mu < \infty$) is the location parameter, specifying the location of the peak of the distribution, and γ ($\gamma > 0$) is the scale parameter of the distribution that determines the spread of the distribution centred on μ. It can be shown that if two independent random variables X_1 and X_2 follow Meridian distributions with parameters (μ_1, γ_1) and (μ_2, γ_2), respectively, then the random variable $Y = X_1 + X_2$ follows the convolution of the distributions of X_1 and X_2, which is also a Meridian distribution:

$$P(y;\mu,\gamma) = P(x;\mu_1,\gamma_1) * P(x;\mu_2,\gamma_2)$$
$$= P(x;\mu_1+\mu_2,\gamma_1+\gamma_2) \tag{12}$$

More generally, the weighted sum of two independent Meridian random variables, $Z = w_1X_1 + w_2X_2$, also follows the Meridian distribution with parameters μ and γ defined as:

$$\mu = w_1\mu_1 + w_2\mu_2, \quad \gamma = w_1\gamma_1 + w_2\gamma_2 \tag{13}$$

In this paper, we assume that there are two input images, which are decomposed in the wavelet domain. Z represents the weighted combination of the two source images, whereas X_1 and X_2 are their corresponding wavelet coefficients.

V. OPTIMISING WEIGHTS VIA MERIDIAN CONVOLUTION

In this section, we introduce a new statistical optimisation approach using convolution of Meridian distributions to combine the wavelet coefficients, in which the distributions of input images are considered to be Meridian. It has been found that the wavelet transforms of real-world images tend to be sparse, resulting in a large number of small coefficients and a small number of large coefficients [10]. The problem of fusion can be posed as an optimisation problem of estimating appropriate combination weights as shown in [3], so that the composite image highlights salient information present in the input images. It is therefore sensible to assume that the fusion process maximizes the sparsity of the resulting image in the wavelet domain, which emphasizes the existence of strong coefficients in the transform, whilst suppressing small values. This should improve the visual quality of the fused images [11].

A. Optimization Scheme

The proposed approach intends to maximize the cost function derived from the convolution of Meridian distributions. The optimal weights determine how much each individual source image contributes into the fused image.

We assume that the distributions of wavelet coefficients corresponding to the input images are modelled by Meridian distributions. The pdf of the Meridian convolution is given below:

$$P(x;\mu,\gamma)=\frac{\gamma_1+\gamma_2}{2((x-(\mu_1+\mu_2))+(\mathrm{w}_1\gamma_1+\mathrm{w}_2\gamma_2))^2} \quad (14)$$

where (μ_1, γ_1) and (μ_2, γ_2) are the model parameters from the distributions of input sub bands, while w_1 and w_2 are the weights of the two source images, respectively.

The likelihood expression for Maximum Likelihood estimation is $L=-\log\,(P(x;\,\mu,\,\gamma))$. ML estimation can be performed by maximizing the cost function [4]:

$$C(w_1,w_2)=E[L] \quad (15)$$

Where the weights w_1 and w_2 remain always positive and they sum up to one, and $E\,[\cdot]$ is the expectation function. The optimal weights are those giving the maximum value to (15). After a few straightforward transforms, the partial derivatives of (15) with respect to w_1 and w_2 are:

$$\frac{\partial C(w_1,w_2)}{\partial w_1}=E\left\{-\frac{\gamma_1}{\gamma}+\frac{2Q(x_1-\mu_1)+2\gamma_1\gamma_2 w_2+2\gamma_1^2 w_1+Q_1}{(x-\mu)^2+\gamma^2}\right\}$$

$$\frac{\partial C(w_1,w_2)}{\partial w_2}=E\left\{-\frac{\gamma_2}{\gamma}+\frac{2Q(x_2-\mu_2)+2\gamma_1\gamma_2 w_1+2\gamma_2^2 w_2+Q_2}{(x-\mu)^2+\gamma^2}\right\} \quad (16)$$

with $Q=(x_1-\mu_1)\,w_1+(x_2-\mu_2)\,w_2$, (17)

$$Q_1=2(x_1-\mu_1)(2w_1\gamma_1+w_2\gamma_2)+2w_2\gamma_1(x_2-\mu_2)$$

and,

$$Q_2=2(x_2-\mu_2)(2w_2\gamma_2+w_1\gamma_1)+2w_1\gamma_2(x_1-\mu_1) \quad (18)$$

where x_1 and x_2 are the wavelet coefficients of the input images, while γ and μ are the model parameters defined in (13). The ML estimation can be implemented through the steepest ascent method. By using the instantaneous estimation of the expectation, the updating procedures are simplified as:

$$\begin{cases} w_{1,k+1}=w_{1,k}+\eta\dfrac{\partial C(w_1,w_2)}{\partial w_1} \\ w_{2,k+1}=w_{2,k}+\eta\dfrac{\partial C(w_1,w_2)}{\partial w_2} \end{cases} \quad (19)$$

where k refers to the iterating index and η is the learning rate. A value η of 0.05 is used. The above update rule is applied until the following stopping criterion is satisfied:

$$\left|w_{i,k+1}-w_{i,k}\right|\le\varepsilon \quad i=1,2 \quad (20)$$

ε above is the value of the error threshold. It must be very small and we have used 0.01 in our paper.

The scale parameter of GCD distribution was derived earlier in section II, where it can be seen that the scale parameter does not dependent on the tail pararmeter, p. Meridian distribution is a special case of GCD with $p=1$, so the scale parameter of Meridian distribution is same as shown in (8). In our work, similarly to [3] a square-shaped neighbourhood of size 7×7 around each reference coefficient is used to find:

$$X=\log\left|D_j\left(x,y\right)\right| \quad (21)$$

where W refers to the 7×7 window and $D_j\,(x, y)$ is the detail coefficient in the j_{th} subband at location (x, y). So the scale parameter for a square-shaped window is given by:

$$\gamma=\exp\,(mean\,[X]) \quad (22)$$

Since our developments are implemented in the wavelet domain, we assume that the location parameter $\mu=0$.

VI. Experimental Results and Discussions

Subjective tests and objective measures are used for qualitatively and quantitatively assessing the performance of the proposed methodology. In applications like medical image fusion, the main goal is to combine perceptually salient image elements such as edges and high contrast regions. Evaluation of fusion by visual assessment can be of paramount importance in many applications like this. Therefore, we have used MR and CT images to apply the algorithm, and visually evaluate the enhanced image in comparison with previously proposed fusion methods, including the weighted average (WA) method [4], fractional lower order moments (FLOM) [5], and the Cauchy convolution approach [3]. The proposed method was applied to different MR and CT image pairs, giving similar results. Results for one pair are shown in figure 1.

The proposed method provides improved fusion results when compared to the other fusion methods. It seems that our method works like a feature detector, retaining the salient features that are clearly distinguishable in each of the two input images. In addition edges seem to be well preserved in the fused image obtained using the proposed method.

An objective evaluation criterion should also be applied to compare results obtained using different algorithms. A quality measure which does not require a ground-truth was proposed by Piella and Heijmans [11] where the important edge information is taken into account to evaluate the relative amount of salient information conveyed into the fused image. We use their criterion to evaluate the fusion performance. The results in Table 1 show that the proposed method obtains higher metric values compared with the other methods, indicating that with our approach salient features are transferred better to the fused image from the input images. Thus, the quantitative results agree with the qualitative results observed when visually comparing the images.

VII. Conclusions and Future Work

The greatest challenges in statistical image fusion are to find suitable models, estimators and relationships between parameters and physical quantities. In this paper we proposed a novel technique to estimate the parameters of the GC distribution using Mellin transform theory and we developed a new image fusion method using convolutions of Meridian distributions in the wavelet domain.

The important features contained in the input images are captured and modelled by Meridian distributions, and the fused coefficients are found by maximizing the cost function derived from the Meridian convolution which takes into account this salient information. Maximum likelihood estimation is used to optimise the weights thus leading to an optimised fusion process. Future work will concentrate on using the generalised Cauchy distribution for designing a more general fusion algorithm.

References

[1] L. Chiorean and M-F. Vaida, **"**Medical Image Fusion Based on Discrete Wavelet Transform Using Java Technology", *Proc. of the ITI 31st Int. Conf.* 22-25 June 2009, pp. 55 – 60.

[2] J. J. Lewis, R. J. O'Callaghan, S. G. Nikolov, D. Bull, and N. Canagarajah, "Region-based image fusion using complex wavelets," *Proc. 7th Int. Conf. Inform. Fus.*, pp. 555-562, 2004.

[3] T. Wan, G. Tzagkarakis, P. Tsakalides, N. Canagarajah, and A. Achim, "Context enhancement through image fusion: a multiresolution approach based on convolution of Cauchy distributions," in *Proc. 2008 Int. Conf. on Acoustics, Speech, and Sig. Proc. (ICASSP '08),* Las Vegas, NV, March 30 - April 4, 2008.

[4] P.J. Burt and R.J. Kolczynski, "Enhanced image capture through fusion," *Proc. 4th Int. Conf Comp. Vis.*, 1993, pp. 173–182.

[5] A. M. Achim, C. N. Canagarajah, and D. R. Bull, "Complex wavelet domain image fusion based on fractional lower order moments," in *Proc. Int. Conf. Inform. Fus.*, vol. 1, p. 515.

[6] N. Mitianoudis and T. Stathaki, "Adaptive image fusion using ICA bases," in *IEEE Conf. Acoustics, Speech, and Signal Process.*, May. 2006, vol. 2, pp. 829–832.

[7] T. C. Aysal and K. E. Barner,' Meridian Filtering for Robust Signal Processing', *IEEE Trans. on* Sig. Proc. ,vol. 55, pp. 3949 – 3962, Aug 2007

[8] B Epstein, "Some applications of the Mellin transform in statistics," *The Annals of Mathematical Statistics*, vol. 19, pp. 370–379, Sep 1948.

[9] J M Nicolas, "Introduction aux statistiques de deuxieme espece: applications des log-moments et des log-cumulants `a l'analyse des lois d'images radar," *Traitement du Signal*, vol. 19, pp. 139–167, 2002.

[10] S. Mallat, "A theory for multiresolution signal decomposition: The wavelet representation," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 11, no. 7, pp. 674–692, Jul. 1989.

[11] Gemma Piella and Henk Heijmans, "A new quality metric for image fusion," in Proc. *IEEE Conf. Image Process.*, 2003, vol. 2, pp. 173–176.

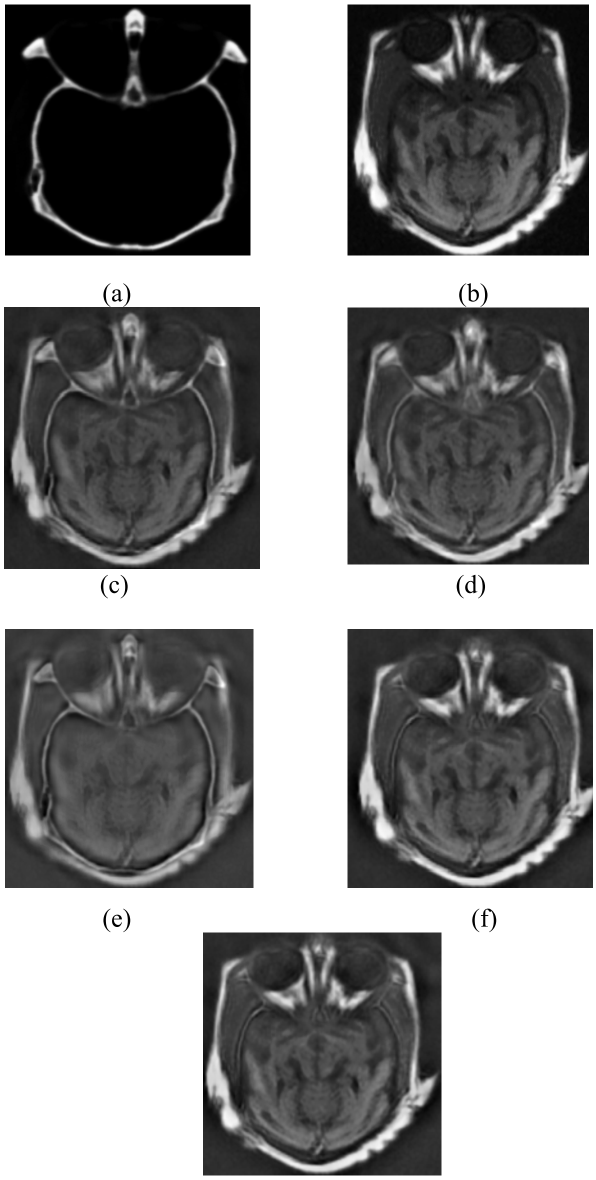

Fig. 1. Fusion of CT and MR images. (a) Original CT image. (b) Original MR image. (c) Image fused using Meridian convolution. (d) Image fused using Cauchy convolution. (e) Image fused using WA scheme rule. Image fused using FLOM. (g) Image fused using FLOM-Cauchy.

TABLE I Performance comparison using piella metric

Methods				
WA	FLOM	Flom-Cauchy	Cauchy-Convolution	Meridian Convolution
0.8053	0.6961	0.7199	0.7993	0.8219