RECONSTRUCTION OF COMPRESSIVELY SAMPLED ULTRASOUND IMAGES USING DUAL PRIOR INFORMATION

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ABSTRACT

This paper introduces a new technique for compressive sampling reconstruction of biomedical ultrasound images that exploits two types of prior information. On the one hand, our proposed approach is based on the observation that ultrasound RF echoes are best characterised statistically using alpha-stable distributions. On the other hand, through knowledge of the acquisition process, the support of the RF echoes in the Fourier domain can be easily inferred. Together, these two facts inform an iteratively reweighted least squares (IRLS) algorithm, which is shown to outperform previously proposed reconstruction techniques, both visually and in terms of two objective evaluation measures.

Index Terms— Compressive sensing, ℓ_p norm, alpha-stable distributions, medical ultrasound

1. INTRODUCTION

Recent developments in medical ultrasound (US) imaging have led to commercial systems with the capability of acquiring Real-Time 3D (RT3D or 4D) image data sets. However, current scanners can only produce a few volume images per second, which is fast enough to see a fetus smile but not fast enough to see heart valves moving. Compressive sensing (CS) could prove to be a powerful solution to enhance US images frame rate by decreasing the amount of acquired data. In this context, in the last four years, a few research groups worked on the feasibility of compressive sampling in US imaging and several attempts of applying the CS theory may be found in the recent literature (for an overview see e.g. [1]).

In particular, in [2], we introduced a novel framework for CS of biomedical ultrasonic signals based on modeling data with symmetric alpha-stable ($S\alpha S$) distributions. Then, we proposed an ℓ_p norm-based minimization approach that employed the iteratively reweighted least squares (IRLS) algorithm, but in which the parameter p was judiciously chosen by relating it to the characteristic exponent of the underlying alpha-stable distributed data. The results showed a significant increase of the reconstruction quality when compared with previous ℓ_1 minimization algorithms. On the other hand, the effect of the random sampling pattern on the reconstruction quality, when working in the frequency domain (k-space) was studied in [3]. This was further exploited in [4] for the design of a US reconstruction technique similar to [2] but operating in the Fourier domain.

In this contribution, we further extend our techniques described in [2, 4] by supplementing the prior information available to an ℓ_p norm minimisation algorithm with the support of the RF echoes in the frequency domain. In ultrasound applications the latter can be easily inferred through knowledge of the acquisition frequency and transducer bandwidth.

In the following section, we provide a brief, necessary overview of the compressive sensing theory and of the heavy-tailed model that we employ for ultrasound data. Section 3 describes our proposed method that exploits dual prior information, while Section 4 illustrates its reconstruction performance. Finally, Section 5 summarises the main results.

2. BACKGROUND

2.1. Compressive sensing

It has been demonstrated [5, 6] that if a signal is K-sparse in one basis (meaning that it can be represented by K elements of that basis), then it can be recovered from $M = c.K << N$ fixed (non-adaptive) linear projections onto a second basis, called the measurement basis, which is incoherent with the sparsity basis, and where $c > 1$ is a small overmeasuring constant. The measurement model is

$$y = \Phi x \tag{1}$$

where x is the $N \times 1$ discrete-time signal, y is the $M \times 1$ vector containing the compressive measurements, and Φ is the $M \times N$ measurement matrix. Using the M measurements in the first basis and given the K-sparsity property in the other basis, the original signal can be recovered by taking a number of different approaches. In the context of this work, our

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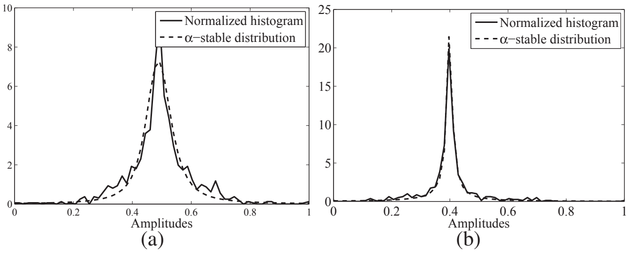


(a) (b)

Fig. 1. *Example RF signal modelling with SαS distributions. (a) An RF signal in time domain. (b) The real part of its* 1*D Fourier transform.*

interest lies in non-convex optimisation (re-weighted ℓ_p minimisation [7, 8]) strategies.

2.2. α-stable distributions as models for RF echoes

The ultrasound image formation theory has been long time dominated by the assumption of Gaussianity for the return RF echoes. However, the authors in [9] have shown that ultrasound RF echoes can be accurately modelled using a power-law shot noise model, which in [10] has been in turn shown to be related to alpha-stable distributions.

By definition, a random variable is called symmetric α-stable ($S\alpha S$) if its characteristic function is of the form:

$$\varphi(\omega) = \exp(j\delta\omega - \gamma|\omega|^{\alpha}), \tag{2}$$

where α is the *characteristic exponent*, taking values $0 < \alpha \leq 2$, δ $(-\infty < \delta < \infty)$ is the *location parameter*, and γ $(\gamma > 0)$ is the *dispersion* of the distribution. For values of α in the interval $(1,2]$, the location parameter δ corresponds to the mean of the $S\alpha S$ distribution, while for $0 < \alpha \leq 1$, δ corresponds to its median. The dispersion parameter γ determines the spread of the distribution around its location parameter δ, similar to the variance of the Gaussian distribution.

The alpha-stable tail power law provided one of the earliest approaches in estimating the stability index of real measurements [11]. The empirical distribution of the data, plotted on a log-log scale, should approach a straight line with slope α if the data is stable. Maximum likelihood methods developed by various authors are asymptotically efficient and have become amenable to fast implementations [12]. In Fig. 1 we show an example of an ultrasound RF echo modelled using $S\alpha S$ density functions both in time and in frequency domain.

3. IRLS WITH DUAL PRIOR INFORMATION

In reconstructing an ultrasound RF echo, we would ideally like to be able to reconstruct a vector x, with the smallest number of non-zero components, that is,with the smallest ℓ_0 norm. Since this is however an NP-hard problem, several sub-optimal strategies have been proposed with the majority solving a constrained optimization problem by employing ℓ_2 or ℓ_1 norms. CS reconstruction methods were also developed (e.g. [7, 8]) by employing ℓ_p norms with $p < 1$, with the goal of approximating the ideal ℓ_0 case. Specifically, the problem consists in finding the vector x with the minimum ℓ_p norm by minimising

$$\hat{x} = \min \|x\|_p \quad \text{subject to} \quad \Phi x = y \tag{3}$$

In [2] we have devised a principled strategy for choosing the optimal p by relating the ℓ_p norm minimisation to the actual $S\alpha S$ statistics of the RF signals. We achieved that by observing that for alpha-stable signals, which do not possess finite second- or higher-order moments, the minimum dispersion criterion [11] can be defined as an alternative to the classical minimum mean square error for Gaussian signals. This leads to a least ℓ_p norm estimation problem, an approach that we have shown to enhance the reconstruction of heavy-tailed RF signals from their measurement projections [2].

Here, our approach to RF signal reconstruction still relies on $S\alpha S$-IRLS [2] but is implemented in the frequency domain as in [4] and modified (following [8]) to incorporate information on the support of RF signals.

Specifically, denote by $X \in \mathbb{R}^{N \times J}$ an US RF image formed by J RF signals of length N, $x_1, x_2, \ldots x_J$. The 1D Fourier transforms of all individual RF echoes x_j can be written as

$$\xi_j = \mathscr{F} x_j,\ j \in \{1, 2, \ldots J\} \tag{4}$$

where $\mathscr{F} \in \mathbb{C}^{N \times N}$ is the 1D Fourier matrix. In the frequency domain, the measurement model becomes

$$m_j = \Phi_j \xi_j = \Phi_j \mathscr{F} x_j,\ j \in \{1, 2, \ldots, J\} \tag{5}$$

where Φ_j are Gaussian matrices of size $M \times N$ $(M \ll N)$ and $m_j \in \mathbb{C}^{M \times 1}$.

Now denote by Θ_j the subset of points in $\{1, 2, \ldots N\}$ that defines the support of ξ_j:

$$\hat{\xi}_{j,k} \neq 0,\ \forall k \in \Theta_j,\ j \in \{1, 2, \ldots, J\} \tag{6}$$

Following the arguments in [8], the information represented by (6) can be added to the IRLS algorithm for the minimisation of the l_p norm by solving the following problem instead of (3) (or its frequency domain equivalent)

$$\min_{\xi} \frac{1}{2} \sum_{\substack{k=1 \\ k \notin \Theta}}^{N} \left|\hat{\xi}_{j,k}\right|^p \text{ subject to } \Phi_j \xi_j = m_j,\ j \in \{1, 2, \ldots, J\} \tag{7}$$

Intuitively, (7) will offer a better solution than (3) because it will attempt to minimise the number of nonzero elements in ξ only outside the set Θ.

To solve (7) we use the modified IRLS algorithm proposed in [8]. Following the arguments in [2], the parameter p is set as $\alpha - 0.01$, where α is obtained by fitting an alpha-stable distribution to the data. Finally, the reconstructed RF lines are obtained by inverting the corresponding Fourier transforms:

$$\hat{x}_j = \mathscr{F}^{-1} \hat{\xi}_j,\ j \in \{1, 2, \ldots, J\} \tag{8}$$

4. RECONSTRUCTION RESULTS

In this section we present reconstruction experiments conducted using real data corresponding to an *in vivo* healthy thyroid. The image was acquired with a Siemens Sonoline Elegra scanner using a 7.5 MHz linear probe and a sampling frequency of 50 MHz. Various sections of the original image were cropped and patches of size 256×512 were obtained. These patches were then sampled line by line using linear projections of random Gaussian bases at two levels. The two levels correspond to the number of samples taken from the original signal (the echo lines); these are 33% and 50% (i.e. $M = 0.33N$ and $M = 0.5N$).

Reconstruction of the samples was achieved by using the proposed ℓ_p-norm minimization scheme (as described in Section 3) and for comparison, reconstructions using ℓ_p-norm minimization with $S\alpha S$ -IRLS [2] and $S\alpha S$-IRLS in the Fourier domain (FD-$S\alpha S$-IRLS) [4] are shown (Table I). The values of α for each line and so that of p (which is derived from α), were estimated directly from the ultrasound RF signal while for our new approach (IRLS-DP) the support was inferred through knowledge of the frequency of acquisition and transducer bandpass as detailed above.

An analysis of the results was undertaken in terms of reconstruction quality, which was measured by means of the *structural similarity index* (SSIM) [13] and *normalised root mean squared error* (NRMSE) of the reconstructed echoes ensemble compared with the original ensemble. SSIM resembles more closely the human visual perception, and as such, it is often preferred than the commonly used MSE-based metrics. For a given image I and its reconstruction Î the SSIM is defined by:

$$\mathrm{SSIM} = \frac{(2\mu_{\mathrm{I}}\mu_{\hat{\mathrm{I}}} + c_1)(2\sigma_{\mathrm{I}\hat{\mathrm{I}}} + c_2)}{(\mu_{\mathrm{I}}^2 + \mu_{\hat{\mathrm{I}}}^2 + c_1)(\sigma_{\mathrm{I}}^2 + \sigma_{\hat{\mathrm{I}}}^2 + c_2)}, \qquad (9)$$

where μ_{I}, σ_{I} are the mean and standard deviation of I (similarly for Î), $\sigma_{\mathrm{I}\hat{\mathrm{I}}}$ denotes the correlation coefficient of the two images, and c_1, c_2 stabilize the division with a weak denominator. In particular, when SSIM equals 0 the two images are completely distinct, while when the two images are matched perfectly SSIM is equal to 1.

It can be seen in Table I that according to both metrics employed, the best results are obtained using the proposed reconstruction algorithm, which exploits two types of prior information. The results support the fact that reconstructing ultrasound RF echoes in the Fourier domain produces better results than directly in time domain. We attribute this to the sparser representations that can be achieved for RF lines in the Fourier domain. Taking into account prior information of the signal in the form of its support is also confirmed to optimise reconstruction. We should note however that, unlike the observation made in [8], reducing further the order p leads actually to worse reconstruction results. The optimal value of p is thus confirmed to be related to the underlying alpha-stable distribution of the data.

$\frac{M}{N}$%	Metric	Method		
		$S\alpha S$-IRLS	FD-$S\alpha S$-IRLS	IRLS-DP
33	NMSRE	0.697	0.540	0.249
	SSIM	0.208	0.586	0.908
50	NMSRE	0.518	0.291	0.158
	SSIM	0.377	0.844	0.944

Table 1. Objective evaluation of three IRLS-based methods for reconstructing ultrasound images from RF frames with sampling rates of 33% and 50% relative to the original.

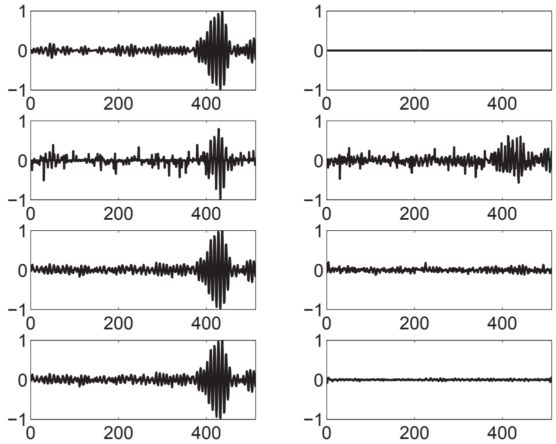
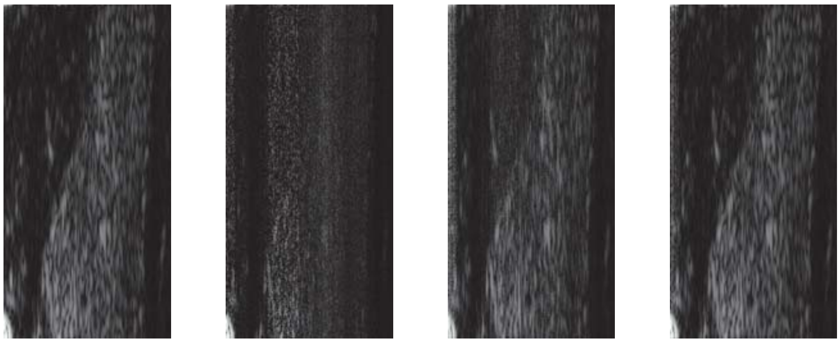


Fig. 3. Reconstruction errors for one RF line sampled at 33%. Top to bottom: original signal and reconstructed using $S\alpha S$-IRLS, $S\alpha S$-IRLS in the Fourier domain, and IRLS with dual prior respectively. Left column: RF lines; right column: the corresponding errors.

For a qualitative analysis, reconstruction results obtained with the three schemes discussed in this paper, with $M = 0.33N$ measurements are also presented in Fig. 2. Visually, it can be seen that the IRLS-DP reconstruction introduces the least distortion, clearly producing the best result compared to the original and confirming the results indicated by the NMRSE and SSIM values obtained.

5. CONCLUSIONS

In this paper, we extended our previously proposed framework for compressive ultrasound imaging. We have shown through simulations that RF echoes can be best reconstructed by driving an ℓ_p-norm minimisation problem with dual prior information: the value of the characteristic exponent of the RF line and its sparse support in the frequency domain. The latter plays certainly a central role in allowing a significant reduction in both the number of required measurements and



(a) (b) (c) (d)

Fig. 2. *Reconstruction results for a thyroid ultrasound image using 33% of the number of samples in the original. (a) B-mode ultrasound image. (b) SαS-IRLS reconstruction. (c) SαS-IRLS in the Fourier domain. (d) Fourier domain IRLS with dual prior.*

computational cost. Nevertheless, our experiments strongly suggest that the optimal value of p in a ℓ_p-norm minimisation procedure shouldn't be arbitrarily small but rather close to the characteristic exponent of the underlying alpha-stable distribution of the data. Our results are general but have been illustrated through only one example dataset due to space limitations.

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