Compressed Hyperspectral Sensing

Grigorios Tsagkatakis[a], Panagiotis Tsakalides[a,b]

[a] ICS - Foundation for Research & Technology - Hellas (FORTH), Crete, Greece
[b] Department of Computer Science, University of Crete, Greece

ABSTRACT

Acquisition of high dimensional Hyperspectral Imaging (HSI) data using limited dimensionality imaging sensors has led to designs with restricted capabilities thus hindering the proliferation of HSI. To overcome this limitation, novel HSI architectures strive to minimize the strict requirements of HSI by introducing computation into the acquisition process. The recently proposed framework of Compressed Sensing (CS) allows the integration of acquisition with computation. In this work, we propose a novel HSI architecture that exploits the sampling and recovery capabilities of CS to achieve a dramatic reduction in HSI acquisition requirements. In the proposed architecture, signals from multiple spectral bands are multiplexed before getting recorded by the imaging sensor. Reconstruction of the full hyperspectral cube is achieved by exploiting a dictionary of elementary spectral profiles in a unified minimization framework. Simulation results suggest that high quality recovery is possible from a small number of multiplexed frames, even from a single frame.

Keywords: Compressed Sensing, Hyperspectral Imaging, Snapshot Spectral Imaging.

1. INTRODUCTION

Hyperspectral Imaging is an essential technology for distinguishing different physical phenomena based on their spectral signatures, and it has been used in applications as diverse as the extraction of clinically meaningful information in medical imaging[1] , and anomaly detection in satellite imagery[2] . A fundamental problem that HSI architectures must address is the collection of the four-dimensional HSI data structure - two spatial, one spectral, and one temporal - using a single sensing element or a 1D/2D array of detectors. This discrepancy between the required and the available dimensionality of detectors has sparked different philosophies in HSI acquisition system design, each one with distinct capabilities in terms of spatial, temporal, and spectral resolution.

Typical HSI approaches utilize a push-broom type of sensors to obtain the complete spectral profile of a single spatial line. To collect the whole hyperspectral cube, the entire field-of-view has to be progressively scanned. In airborne HSI systems, the motion of the sensing platform (an aircraft or a satellite) is typically utilized to acquire consecutive scan lines[3] . However, when the imaging sensor is static during acquisition time, a mechanical device must be employed for line scanning. Spectral scanning methods, known as whisk-broom designs, focus on a specific pixel and collect the entire spectral profile at each frame. To obtain the complete hyperspectral cube, the process must be repeated along both spatial dimensions in a raster scan fashion. A significant limitation of this method is its susceptibility to motion blur and misalignments due to the scanning process, which can dramatically reduce the quality of the sensed hypercube data. Recently, frame designs which employ 2D arrays of monochromatic detectors and frequency selective optical filters have been proposed in order to capture data from multiple spatial locations at once, albeit from a single spectral band at each frame. Naturally, to generate the complete spectral profile of the scene, multiple frames, proportional to the required spectral resolution, have to be captured.

A key shortcoming shared by such designs lies in the large number of repetitive measurements that are required for generating the complete 3D hyperspectral datacube. In the case of spatial/spectral scanning, multiple lines/pixels have to be scanned, while for 2D frame scanning systems, multiple frames have to be acquired in order to obtain the complete spectral profile of the scene. This limitation is responsible for a number of issues that hinder HSI performance, including slow acquisition time and motion artifacts. To address these limitations,

Further author information: (Send correspondence to G.T.) E-mails: G.T.: greg@ics.forth.gr, P.T.: tsakalid@ics.forth.gr

Snapshot (or Simultaneous) Spectral Imaging (SSI) systems[4] acquire the complete spatio-spectral cube from a single or a small number of captured frames.

In this work, we propose a novel SSI architecture that can recover the complete hyperspectral profile of a scene from a small number of coded exposures. To achieve this goal, the proposed architecture utilizes a spectrally selective coding mask that multiplexes the incoming hyperspectral light signal before it is captured by a monochromatic sensor. While traditional imaging would require the acquisition of a number of frames equal to the number of the spectral bands, the proposed CHI system is able to extract the full spectral content of the scene by decoding a significantly smaller number of multiplexed measurements per pixel.

2. PREVIOUS WORK

The explicit manifestation of sensing constrains have sparked a large number of SSI approaches. The four-dimensional imaging spectrometer[5] employs a coherent fiber bundle in order to transform the 2D spatial information to 1D and it uses dispersive elements to capture the spectral information. Other approaches rely on replicating system components such as mirrors in the integral field spectroscopy with faceted mirrors[6] , arrays of properly placed microlenses in integral field spectrometry with lenslet arrays[7] , and sequences of multispectral filters[8] .

Recently, the problem of optimizing the amount of captured data by introducing carefully designed encoding strategies and state-of-the-art signal recovery algorithms has been addressed by the computational photography community[9] . The main premise of computational photography is that by combining optical elements such as lens arrays, dispersive elements, and controllable apertures, with sophisticated signal processing algorithms, one can efficiently address important image problems including limited depth of field and motion blurring. A key mathematical framework used in computation photography is the concept of Compressed Sensing (CS)[10–12] .

CS is a novel framework recently introduced for simultaneous sensing and compression and it presents a disruption to the famous and well-established Nyquist/Shannon sampling theorem. CS asserts that a sparse or compressible signal can be fully recovered from a limited number of incoherent measurements. An example of a SSI system built around the CS signal processing paradigm is the Coded Aperture Snapshot Spectral Imaging (CASSI)[13] architecture. CASSI employs a coded aperture, a single or a double dispersive element, and a 2D Focal Plane Array detector in order to acquire coded spatio-spectral mixtures of the 3D hyperspectral cube. The CASSI system was improved by introducing a coded mask and the acquisition of multiple snapshots[14] . More recently, a novel SSI architecture termed Spatial-Spectral Encoded Compressive HS Imager (SSCSI)[15] was proposed that combines a spectral dispenser and a random shearing mask, extending the single wavelength computational light field acquisition architecture proposed[10] . Despite the novelty of CASSI and SSCSI, we note that both systems employ random signal multiplexing in a single spatial axis through the dispenser, thus not fully exploiting the compressibility properties in both the spatial and the spectral dimensions.

3. COMPRESSED HYPERSPECTRAL IMAGING

The driving force behind this work lays in the impressive capabilities of CS in recovering randomly multiplexed sub-sampled signals. Focusing on the problem of HSI, we propose a novel HSI architecture that can achieve high quality reconstruction of a hypercube $\mathbf{I}(\mathbf{v}, \lambda)$ of spatial dimensions $\mathbf{v}$ and λ spectral bands from a limited number of frames, $\mathbf{y}$, without resorting to moving parts. The proposed architecture employs a multispectral per-pixel encoding of the incoming light, following a CS sampling paradigm. According to the CS framework, perfect reconstruction of a signal $\mathbf{s} \in \mathbb{R}^N$ is possible from a small number of random measurements $\mathbf{y} = \mathbf{\Phi s} \in \mathbb{R}^M$, far below the typical Shannon-Nyquist sampling limit, provided the signal $\mathbf{s}$ can be sparsely represented in a collection of elementary examples $\mathbf{D}$ and the sensing matrix Φ satisfies certain statistical properties. In one realization of our scheme, we assume that each pixel at spatial location $\mathbf{x}$ in a captured frame f_j represents a random linear combination of specific spectral responses such that:

$$\mathbf{y}(\mathbf{v}, f_j) = \sum_i \mathbf{\Phi_i I}(\mathbf{v}, \lambda_\mathbf{i}) \tag{1}$$

where the index i denotes the i-th spectral band. To increase the reconstruction SNR, one can acquire multiple such frames, in which case the measurements vector at spatial location $\mathbf{v}$ is given by $\mathbf{Y} = \mathbf{\Phi I}$, where the sensing matrix Φ contains the sensing matrices associated with different measurements. Figure 1 illustrates a schematic overview of the proposed architecture composed of the following elements: (i) a focusing lens, (ii) a tunable spectral filter, and (iii) a monochromatic imaging sensor.

Depending on the requirements, one straightforward approach of a practical implementation of the proposed scheme involves the temporal encoding of spectral responses via electronically tunable spectral filters such as Liquid Crystal Tunable Filters (LCTF) and Acousto-Optic Tunable Filters (AOTF). An alternative approach relies in exploiting the geometry of the system in order to optically multiplex multiple spectral responses. This can be achieved by placing the spectral filter mask in-front of the focal plane of the lens. As a consequence, multiple light rays will traverse the filter before reaching each detector element.

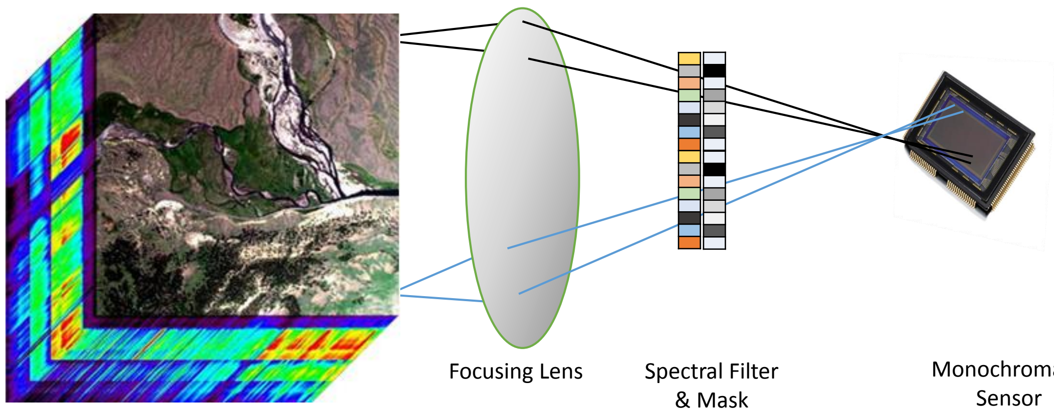


Figure 1: Image acquisition via the proposed scheme in two different sampling instances. During each sampling instance, the pattern of the coding mask allows the propagation of light from a specific spectral band which is optically multiplexed in the imaging sensor.

From a functional perspective, each acquired frame encodes a specific set of bands of light focused by the focusing lens on the imaging sensor. Multiple frames can be acquired in order to increase the quality of the reconstruction by changing the set of bands across frames. To reconstruct the spectral signal, we exploit prior knowledge that a hyperspectral signal can be sparsely, yet faithfully, represented in a dictionary of training exemplars.

4. RECOVERY FROM COMPRESSED HYPERSPECTRAL IMAGING

The CS[16,17] theory address the problem of recovering a k-sparse signal $\mathbf{x} \in \mathbb{R}^n$ from a small number of coded measurements $\mathbf{y} = \mathbf{\Psi x} \in \mathbb{R}^m$, where $m \ll n$. Recovering such a signal is possible by solving an l_0 constrained minimization problem given by:

$$\min ||\mathbf{x}||_0 \;\; \text{subject to} \;\; \mathbf{y} = \mathbf{\Psi x} \,. \tag{2}$$

where $\mathbf{\Psi}$ is the $m \times n$ sensing matrix. To ensure that the k-sparse solution in unique, the sensing matrix $\mathbf{\Psi}$ must satisfy the so-called Restricted Isometry Property (RIP), which is satisfied with high probability for sensing matrices whose elements are drawn randomly from appropriate distributions such as normalized mean bounded variance Gaussian[16] and Rademacher[18] distributions. In this work, we employ binary sparse matrices with a bounded number of non-zero elements per column[19] due to the physical realization capabilities of binary sensing in imaging systems. Although the l_0 minimization problem in (2) is an NP-hard combinatorial optimization problem, greedy approaches, including the Orthogonal Matching Pursuit (OMP)[20,21] have been very successful at recovering sparse signals in CS sampling. In addition to greedy approaches, the theory of CS suggests that, provided certain conditions are met, one can equivalently solve an l_1 constrained minimization problem called *Basis Pursuit* given by:

$$\min ||\mathbf{x}||_1 \;\; \text{subject to} \;\; \mathbf{y} = \mathbf{\Psi x} \,. \tag{3}$$

While the theory of CS has been primarily focused on sparse signals, a large number of signals are not naturally sparse, but can be sparsely represented in an $n \times l$ dictionary $\mathbf{D}$, *i.e.* $\mathbf{x} = \mathbf{D}\mathbf{s}$ where $\mathbf{s}$ is k-sparse. The l_1 minimization problem in (3) can thus be expressed as:

$$min||\mathbf{s}||_1 \text{ subject to } \mathbf{y} = \mathbf{\Psi}\mathbf{D}\mathbf{s} \ . \tag{4}$$

For compressible signals, the goal is not the exact reconstruction of the signal, but the reconstruction of a close approximation of the original signal. In this case, the problem is called *Basis Pursuit Denoising* and (4) becomes:

$$min||\mathbf{s}||_1 \text{ subject to } ||\mathbf{y} - \mathbf{\Psi}\mathbf{D}\mathbf{s}||_2 < \epsilon \ , \tag{5}$$

where ϵ is a bound on the residual error of the approximation which is related to the amount of noise in the data. The optimization in (5) can be efficiently solved by the lasso[22] algorithm for sparsity regularized least squares. The number of required measurements for the reconstruction is dictated by the *mutual coherence* between the sensing matrix $\mathbf{\Psi}$ and the dictionary $\mathbf{D}$, which is defined as the maximum of the inner product between columns of the dictionary and the sampling matrix. For a specific mutual coherence μ, recovery is possible from $m \geqslant C\mu^2(\mathbf{\Psi}, \mathbf{D})k\log(n)$ random measurements.

During each sampling instance, the imaging sensor captures light from the entire scene, however the recorded values correspond to a mixture of spectral bands, defined by the specific coding $\Psi(\mathbf{x}, t)$. To recover the spectral profile at location $\mathbf{x}$, one must solve the following minimization problem

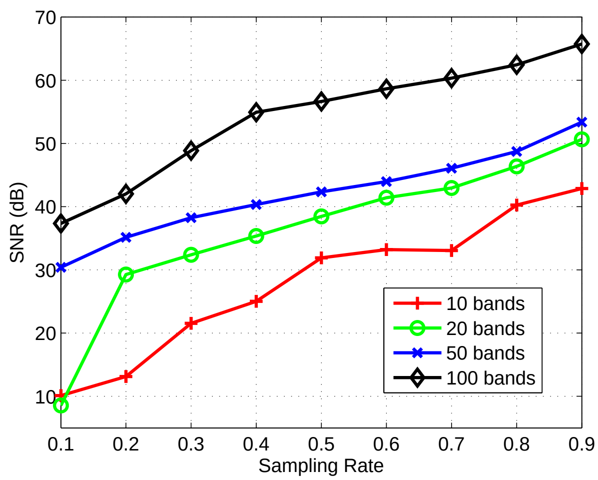
$$\mathbf{s}^*(\mathbf{x}) = \arg\min \|\mathbf{s}(\mathbf{x})\|_0 + \lambda\|\hat{\mathbf{y}}(\mathbf{x}, t) - \mathbf{\Psi}(\mathbf{x}, t)\mathbf{D}\mathbf{s}(\mathbf{x})\|_2 \ . \tag{6}$$

Each hyperpixel can then be retrieved by $\mathbf{I}(\mathbf{x}, \lambda) = \mathbf{D}\mathbf{s}^*(\mathbf{x})$. To achieve a high quality reconstruction, the dictionary must be able to support the sparse representation of the acquired signals. The problem of dictionary construction has been addressed under three main lines of research, namely parametric, non-parametric, and data-driven. In the first case, explicit transforms, including the 3D wavelet transform, have been used in hyperspectral image compression with success[23] . While such transformations are very generic in nature, existence of training examples can be employed during a sparsity-seeking dictionary approach such as the K-SVD algorithm[24] . Last, it has been shown that a random selection of training examples can be sufficient for achieving similar representation performance as dictionary learning, however at a dramatically reduced computational cost[25] . We adopt the last paradigm in our experiments using a smaller sample set for dictionary generation.

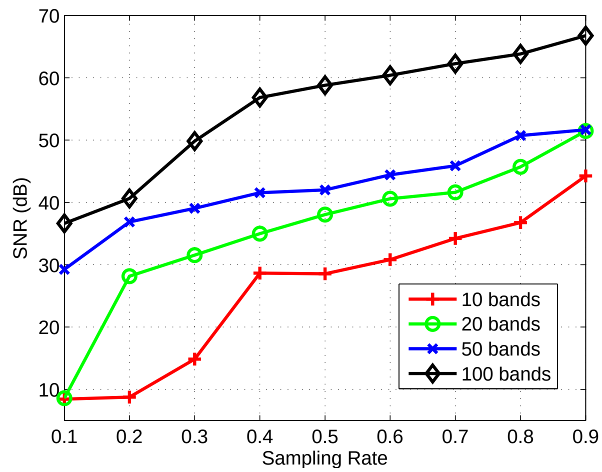
5. EXPERIMENTAL RESULTS

In order to investigate the imaging capabilities of the proposed HSI architecture, we consider hyperspectral data collected in 2001 with the AVIRIS sensor over the Salinas valley, California. We utilized the *salinasA* dataset for training the dictionary, while we tested the recovery performance on the *salinas* dataset. To evaluate the performance as a function of the number of spectral bands, we selected a subset from the 204 spectral bands of the datasets. To measure the reconstruction quality, we considered the spectral SNR, measured for each pixel and averaged over the spatial dimensions. To measure the encoding compression, we utilized the sampling rate, defined as the number of frames acquired over the number of spectral bands. Sampling rate equal to 1 corresponds to the case where the acquisition of frames equals in number to the spectral bands. In this case, one could forget CS altogether and perform traditional sampling where each frame encodes a specific spectral band. The other extreme corresponds to the SSI case where the system acquires a *single* frame, multiplexing some or even all spectral bands, and employs recovery mechanisms to estimate the full spectral profile of the scene. Naturally, one seeks to achieve the best possible performance from the lowest sampling rate possible.

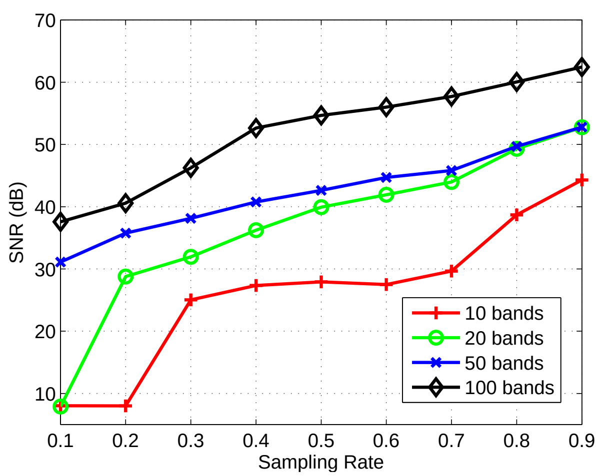
One of the key parameters that we investigate in our experiments is related to the amount of multiplexing introduced by the encoding strategy. More specifically, we consider different amounts of spectral multiplexing per acquired sample, which is defined by the ratio of active spectral bands per frame over the number of all spectral bands. We investigated multiplexing, where in the extreme case all spectral bands are multiplexed in each acquired frame. From an implementation perspective, we can identify two *optimal* cases, namely the lowest possible multiplexing and the largest possible multiplexing. The former case corresponds to the activation of a small number of spectral bands per frame. When the proposed HSI system is implemented via temporal encoding, reducing the number of active spectral bands per frame is translated to a smaller number of changes in the setting of the spectral filters, which implies shorter acquisition times and therefore better imaging of dynamic events. In the latter case, the system can be implemented without any temporal coding by allowing light from all spectral bands to be acquired simultaneously, thus leading to a more compact HSI design.



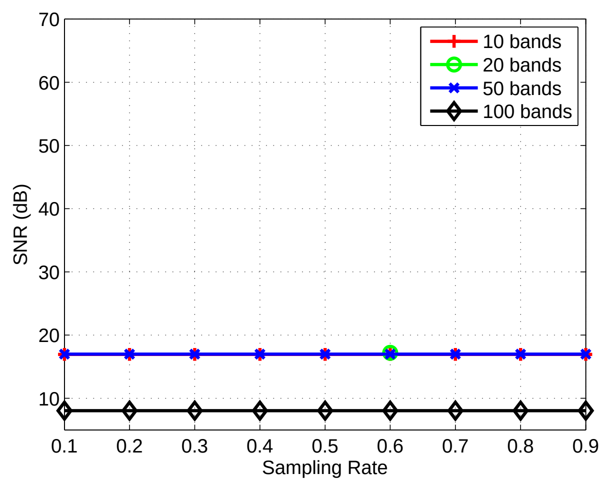
(a) 1 : 4 spectral multiplexing



(b) 2 : 4 spectral multiplexing



(c) 3 : 4 spectral multiplexing



(d) 4 : 4 spectral multiplexing

Figure 2: Reconstruction quality for spectral multiplexing equal to 1 : 4, 2 : 4, 3 : 4 and 4 : 4. Notice that the reconstruction quality typically deteriorates at higher mutliplexing scenarios.

Figure 2 showcases the performance of HSI data recovery using the proposed imaging architecture, where four settings for the multiplexing of the binary sampling patterns are employed, namely 1 : 4, 2 : 4, 3 : 4 and 4 : 4. Examining subfigures (a)-(c), we observe that the relative performance ordering of reconstruction with respect to the total number of spectral bands is the same for all multiplexing settings, and more specifically, we see that the larger the number of spectral bands, the higher the quality of the reconstruction. This is attributed to the fact that although performance is reported for each specific sampling rate, in practice this number depends on the amount of spectral bands, *e.g.* 0.1 sampling rate corresponds to 10 frames for the 100 spectral bands case, yet only to 1 frame for the 10 bands case. The relationship between the recovery performance at a given sampling rate and the total number of spectral bands, suggests that when the hyperspectral signals are characterized by a large number of spectral bands, the CS encoding/decoding strategy can exploit more efficiently the inherent redundancy of the data. The case of 4 : 4 multiplexing is an interesting one since we observe that no performance is gained by increasing the sampling rate. This is a natural characteristic of this specific setup where each pixel acquires all spectral responses during each frame, thus no additional information is offered by increasing the number of acquired frames. Note that this case corresponds to a non-CS based design and we can easily see that randomness of the encoding processes can have a dramatic impact on performance.

While Figure (2) demonstrates the positive effects of increased sampling rates, *i.e.* increased number of acquired frames, in SSI one must ask for the reconstruction of the HSI hypercube from a single coded image. Figures (3) and (4) present two examples of reconstructed spectral images, the first case corresponding to the reconstruction of a 5 spectral bands HSI and the second to a 50 spectral bands HSI, recovered from a *single* spectrally multiplexed image. For each case, we present the original and three reconstructed images with optical multiplexing equal to one-quarter, one-half, and full spectral multiplexing for each acquired frame.

The first case of 5 bands, shown in Figure (3), which corresponds to a small number of total spectral bands, shows that all three multiplexing schemes perform comparably, with a slight increase in performance at higher multiplexing ratios. These results are a good indicator that the proposed spectral multiplexing scheme can also support snapshot spectral imaging since we observe that most of the visual information contained in the original frame are kept in the reconstructed images. In addition to the case of a small number of spectral bands, we also examined the performance when a large number of spectral bands have to be recovered, in line with typical requirements in HSI.

For the case of 50 spectral bands, Figure (4) clearly demonstrates the impact of spectral multiplexing in the recovery capability. More specifically, we observe that increasing the amount of multiplexing has a negative effect on recovery in the case of a large number of bands. This effect is attributed to the observation that a single encoded frame corresponds to an extremely small sampling rate, unlike the case in Figure (3), thus limiting the recovery capabilities of the CS reconstruction algorithm. In such scenarios, maintaining a small multiplexing rate can be immensely beneficial.

As a general remark, the results presented in this section correspond to recovery with the greedy OMP approach. In addition to the greedy approach, we also considered recovery by l_1 minimization algorithms including the Least Angle Regression[26] and the Spectral Projected-Gradient for l_1[27] . However, the recovery performance was extremely erratic and no reliable conclusions could be drawn.

6. DISCUSSION

Addressing the disparity between the dimensionality of dynamic Hyperspectral Imaging and the dimensionality of the imaging sensors has sparked numerous approaches in HSI. In this work, we propose a novel HSI architecture, termed Compressed Hyperspectral Sensing (CHS), which employs state-of-the-art mathematical modeling for encoding and decoding higher dimensional images from a small number of encoded frames. The proposed scheme leverages the capabilities of CS in order to recover the complete spectral profile of a scene from a limited number of spectrally multiplexed images or even from a single image. Simulation results indicate that the proposed scheme is a viable HSI architecture that outperforms traditional frame-based approaches, offering the possibility of Hyperspectral video acquisition.

7. ACKS

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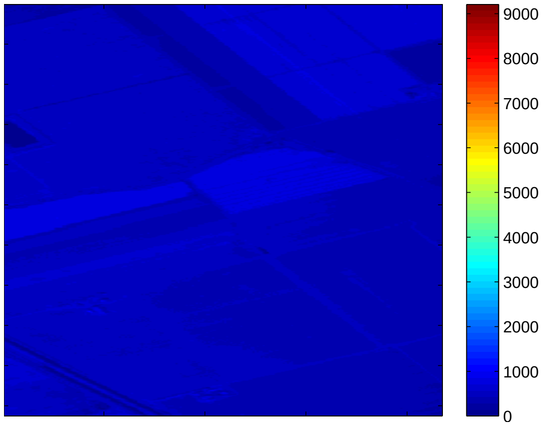
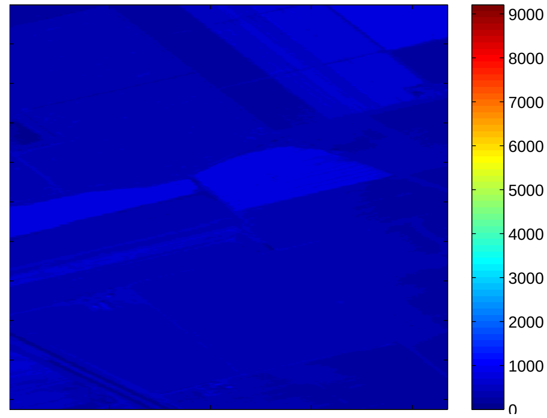
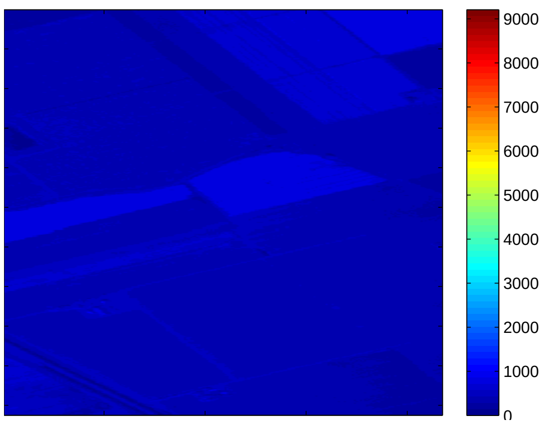


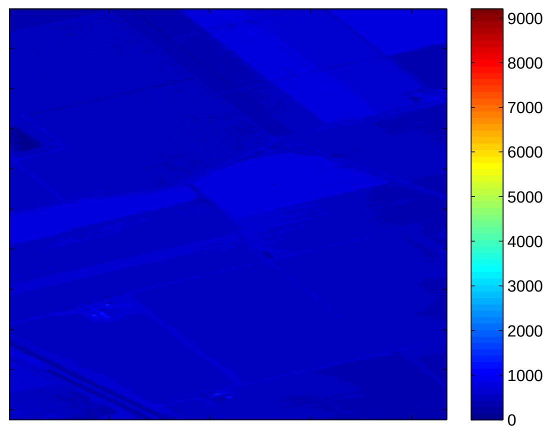
(a) Original 5^{th} band



(b) 1 : 4 spectral multiplexing



(c) 2 : 4 spectral multiplexing



(d) 4 : 4 spectral multiplexing

Figure 3: Snapshot HSI recovery of the (a) 5^{th} out of 5 bands with (b) 2 : 4, (c) 3 : 4 and (d) 4 : 4 spectral multiplexing. The SNR of the reconstructed images are (b) 18.2 dB, (c) 20.2 dB and (d) 20.5 dB.

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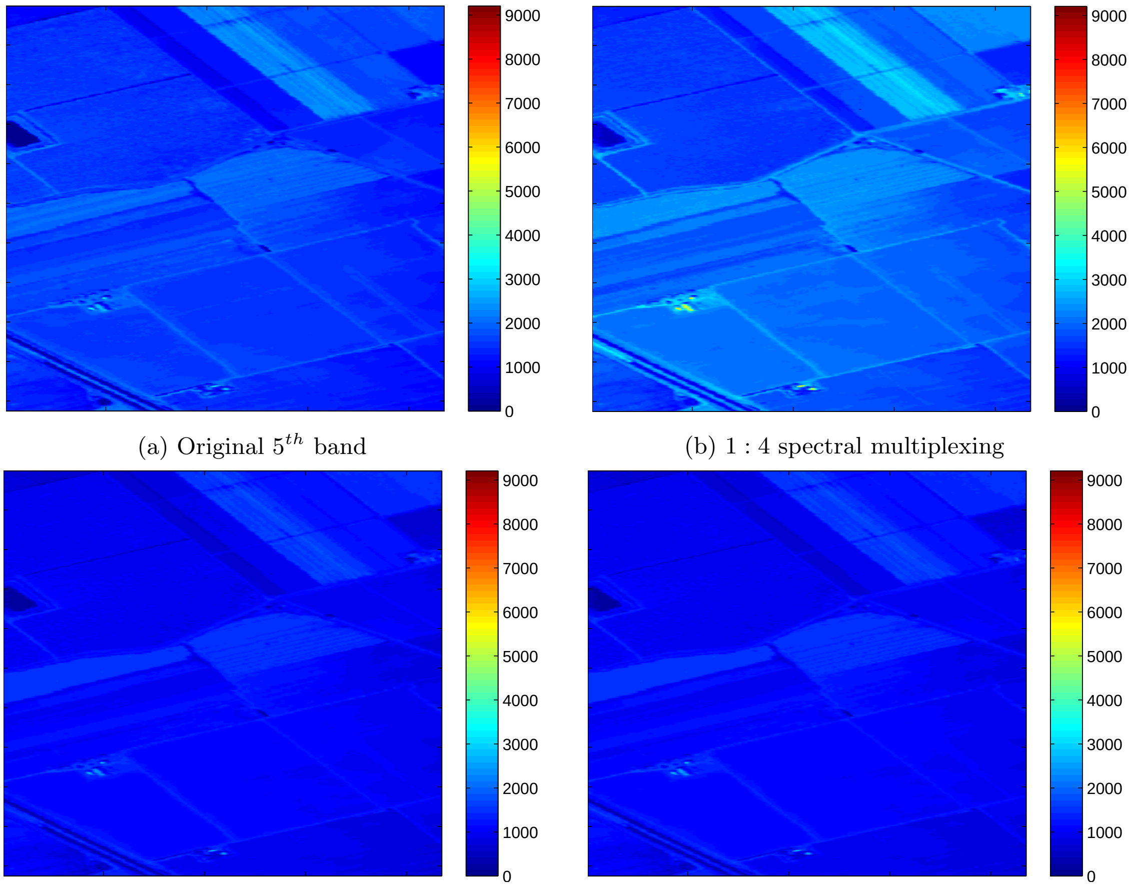

(a) Original 5^{th} band

(b) 1 : 4 spectral multiplexing

(c) 2 : 4 spectral multiplexing

(d) 4 : 4 spectral multiplexing

Figure 4: Snapshot HSI recovery of the (a) 5^{th} out of 50 bands with (b) 2 : 4, (c) 3 : 4 and (d) 4 : 4 spectral multiplexing. The SNR of the reconstructed images are (b) 15.4 dB, (c) 8.7 dB and (d) 8.8 dB.

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